Moment of inertia measurement based on displacement sensor

Chunfu Zhang[1], Jianguo Miao[2], Song Tang[2,] Guangwu He[2]
[1]School of Electrical Engineering, YanCheng Institute of Technology, Yancheng 224051, Jiangsu, (CHINA)
[2]Chenguang Calibration & Testing Center, Nanjing 210006, Jiangsu, (CHINA)

ABSTRACT

Moment of inertia (MI) is a significant characteristic parameter of spacecraft and MI measurement of the spacecraft during ground testing is of prime importance. There are various techniques to measure MI, on the basis of analyzing the MI measuring principle, the mathematical function of period and moment of inertia is deduced in torsional pendulum method, then a measuring method based on inductive displacement sensor is proposed. In a torsional pendulum procedure, the changes of displacement follow the physical laws, and the displacement acquired from the sensor are discrete electrical signals; direct comparison or median filtering algorithm are commonly used to process the signals, but these two methods have large errors and low precision. According to those characteristics, a local linear optimal fitting algorithm based on least square is presented to determine the period of torsional pendulum, all the theoretical analysis and experimental results indicates that this algorithm has higher resolving precision.

KEYWORDS

Moment of inertia; Torsional pendulum; Displacement; period; Local linear optimal fitting.

THE MEASUREMENT PRINCIPLE OF MOMENT OF INERTIA

Inertness of a rigid body to change its state of being in rotation is called moment of inertia. It is a significant characteristic parameter of aircraft's attitude control, the magnitude of it depends on the body's shape, mass of distribution and the position of spindle and can be described as (1) shows[1].

$$J = \int r^2 dm \tag{1}$$

Under normal conditions, a typical spacecraft consists of number of subsystems/payloads integrated to the main structure, the mass distribution, product appearance and other elements makes moment of inertia of aircraft be different from the theoretical values. Commonly, MI (Moment of Inertia) of a body can be measured using various techniques such as: inverted torsion pendulum, bifilar suspension, compound pendulum, spinning of specimen[2,3]. Among them, inverted torsion pendulum is widely used because of large-scale part measurement and high accuracy[4]. The test object rests on a precision rotary table attached to the top of the instrument; low friction bearings support the table and payload while constraining the motion of this torsion member to pure rotation. MI instruments have two kinds of structure of gas bearings and mechanical bearings, gas bearings provide the better performance because the effect of damping can generally be ignored. The basic principle of inverted torsion pendulum measurement based on gas bearings is shown in Figure 1[5].
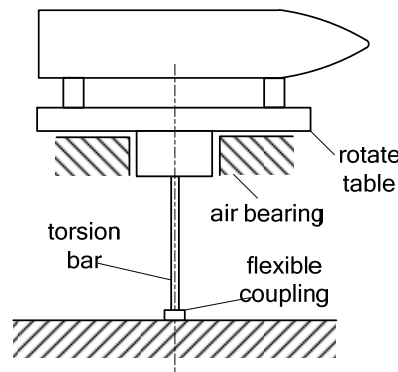
Figure1 : The schematic diagram of torsion pendulum measurement on a gas bearing rotary table

In inverted torsion pendulum measurement, if the oscillating angle of rotary table is θ, then the torsion pendulum of instrument can be described as the differential equation (2) shows.

$$\frac{d^2\theta}{dt^2} + 2\zeta\omega_n \frac{d\theta}{dt} + \omega_n^2\theta = 0 \tag{2}$$

Where ζ is the damping ratio, ω_n is the un-damped natural frequency, can be given by the torsional stiffness K of torsion rod and the moment of inertia J of the system in (3).

$$\omega_n = \sqrt{\frac{K}{J}} \tag{3}$$

The solution to this differential equation is

$$\theta = \theta_0 e^{-\zeta\omega_n t} \cos(\omega t + \varphi) \tag{4}$$

Clearly, this torsion pendulum is a damped oscillation process, and the oscillating angular frequency and period are given as (5) and (6) shows.

$$\omega = \omega_n \sqrt{1 - \zeta^2} \tag{5}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\omega_n \sqrt{1 - \zeta^2}} \tag{6}$$

Under usual conditions, the damping ratio ζ is much smaller than 1, and can be ignored. Hence the oscillation of system turns into constant amplitude and period one, the approximation relation of oscillation period T and J can be given as (7) shows.

$$J = \frac{K}{4\pi^2} T^2 \qquad\qquad (7)$$

Hence, if the oscillation period of can be measured, the moment of inertia can be obtained from (7). At present we have two approaches: using photoelectric timing sensor or measuring the displacement. To be specific, in the former process, the oscillation is converted to digital pulses, then the width of pulse, the period we want, can be measured by gating counter; it is simple and real-time, but the width of photoelectrical barrier (or slit) affects the measuring precision. Measuring the displacement actually measure the amplitude of oscillation, then indirectly calculate the time period. This method is more complicated but has higher accuracy, so in this paper we will discuss it thoroughly.

MEASUREMENT OF DISPLACEMENT SIGNAL

To measure the amplitude of torsional pendulum, a baffle should be mounted on the rotary table along radial direction, meanwhile, displacement sensor are fixed tangentially. As Figure.2 shows, if the torsional pendulum angle is small enough, the circumferential oscillation can be converted to tangential displacement approximately. Apparently, the error is largest in the position of maxim amplitude, and approximate 0 in the zero position, therefore the displacement period should take the median point as reference to analyze.
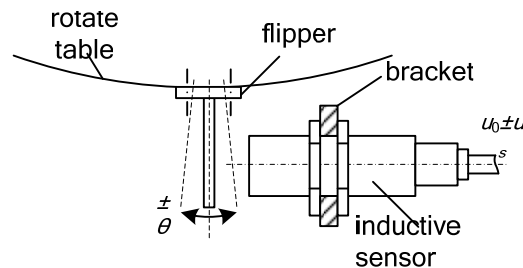

Figure 2 : The measurement diagram of displacement

During the installation of sensor, the static distance between it and baffle need to be adjusted to match the sensor's features as much as possible, and then we can acquire the optimal linear range and best repeatability, minimize the output distortion to avoid the measuring accuracy loss. When the rotary table is at equilibrium position, the electrical output of sensor is defined as the static reference line; besides, to link the displacement and period, the system must adopt the hardware trigger sample mode of equitime intervals.

TIME PERIOD DETERMINATION AND DATA PROCESSING

Theoretically, when the test voltage (displacement signal) goes through static zero line twice, the time spent is half of oscillating period, as Figure 3 shows. For this reason, we can determine the period of torisional pendulum rapidly by directly comparison; however, because of the existence of noise and measuring error, the actual voltage cannot satisfy the approximately monotone linearity around the static zero position, at this moment, the direct comparison will lead to great error. Although the well-known FFT algorithm can do frequency analysis fast to periodical discrete signal[6], the conflict between the quantity of measuring points and arithmetic error cause it doesn't appropriate to determine the period. Taking the factors above, and considering the random errors such as noise and etc, we can acquire precise periodic quantity by further optimal fitting based on traditional median filtering algorithm.

The median filtering algorithm can be described as (8) shows:

$$\overline{u}_k = \sum_{j=k-L}^{k+L} u_j \qquad\qquad (8)$$

The main idea of the median filtering algorithm is to run through the signal entry by entry, replacing each entry with the median of neighboring entries[7]. The pattern of neighbors is called the "window", which slides, entry by entry, over the entire signal. For the signal in Figure.3, the most obvious window is just the first few preceding and following entries,

suppose the length of entry is L, take average value of the points in the region, if the difference between the value and static zero value was minimum, as (9) shows:
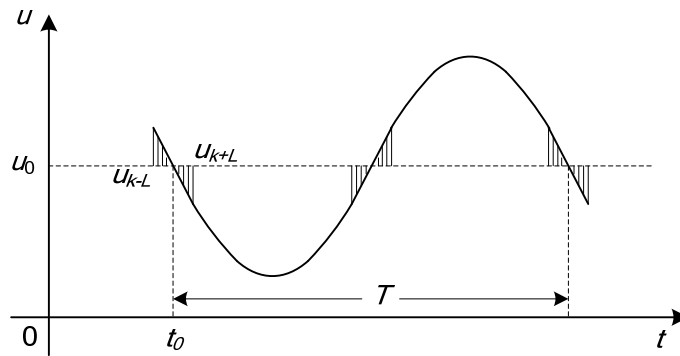

Figure 3 : The period determination of discrete displacement signal

$$\left|\overline{u}_k - u_0\right| \rightarrow \min \tag{9}$$

Then the kth point will be a static zero point, via the arithmetic circulate, we can acquire the subsequent static zero points, and on this basis, we can obtain the period of every torsional pendulum motion (uniformly spaced sampling). The median filtering can remove noise to some extent, moreover, due to the constraint of this algorithm, all the periods obtained are abound to be integral multiple of sampling period, hence the sampling frequency must be improved as much as possible, which increase the hardware cost; although this frequency constraint still leads to a considerable measuring error, this algorithm can accurately determine the interval in which the displacement pass the static zero line.

Based on the median filtering algorithm mentioned above, a local optimal fitting algorithm is given to bring down the hardware requirement, in which the sampling frequency can be lower without precision loss. As we know, the sine displacement signal is approximately linear and symmetrical around the static zero line, according to this characteristic, the principle of this algorithm is as follows: first in accordance with the zero section calculated in median filtering algorithm, a multipoint redundant least square fitting algorithm is carried on to acquire the linear regression function of displacement-time, then the accurate zero cross point can be work out, that is to say, the crossing point of static zero line and displacement curve. The function of displacement-time can be given as (10) shows.

$$u = Kt + u_{t0} \tag{10}$$

Where k is the slope of voltage-time line, u_{t0} is the initial offset.
Then the function of static zero line can be described as (11)shows.

$$u = u_0 \tag{11}$$

Based on (10) and (11), the cross point of the displacement curve and static zero line t_c can be obtained in this zero section as Figure.4 shows; then the subsequent cross points come through the algorithm circulate and precise period of pendulum can be acquired finally.
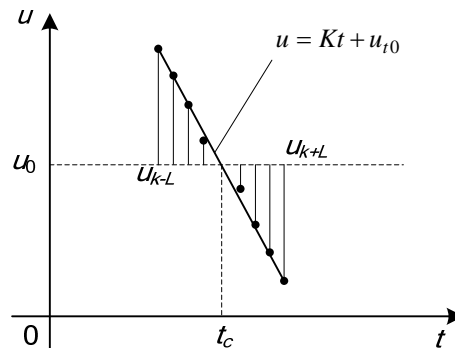

Figure 4 : The schematic diagram of the local linear optimal fitting

THE EXPERIMENT AND RESULT ANALYSIS

The linear working range of the displacement sensor used in this experiment is 1.5±0.5mm, reference distance has been decided in the assembling process, and the working range is decided by air cylinder of the rotary table. The sensitivity of sensor is 8V/mm, the output voltage is 5±4V, the characteristic curve is given in Figure 5.
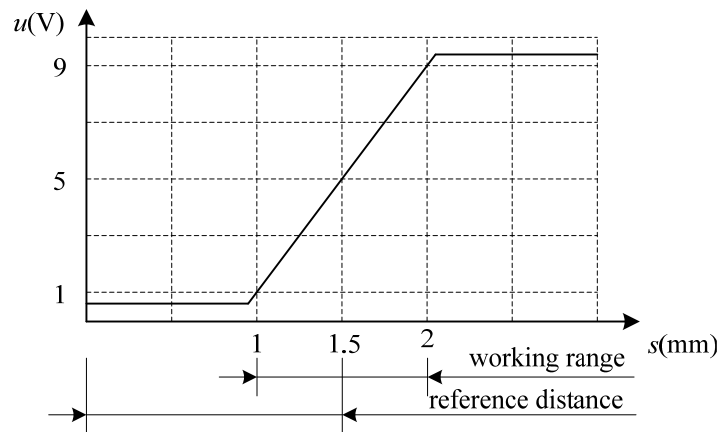
Figure 5 : The characteristic curve of inductive displacement sensor

The subsequent sampling unit is implemented by PCI-1712L, the input range is set as 0~10v, the sampling precision is 12bits, and the sample rate is 10ksps. TABLE I shows the processing results of three kinds of measurement under the condition of 50 periods.

TABLE 1 : The measuring result of 50 periods (ms)

	Average value	RMS	Limiting deviation
Direct comparison	1267.974	0.236	0.596
Median filtering	1268.004	0.054	0.103
Local linear optimal fitting	1268.002	0.039	0.077

In TABLE 1, the period's average values, root-mean-squares, limiting deviations of the three algorithms are all listed to compare. We can see that the average value of direct comparison is clearly less than the other two ways, which is because of the interference of noise, the displacement will pass the zero line earlier than the actual time, and meanwhile, repeating fluctuation's minimum period will be filtered out, the period acquired is small. Besides, the randomness of noise will cause the RMS and limiting deviation greater than the later two.

Median filtering algorithm and the local linear optimal fitting algorithm have great consistency, which states that both of the two methods eliminate the effect of random noise, gross errors don't turn up. Furthermore, the influence of sampling period is reduced in the latter, so the local linear optimal fitting has best precision, the limiting deviation is less than 0.06‰ even under 50 periods.

CONCLUSION

The local linear optimal fitting algorithm has been applied to shenck M7 moment of inertia measuring system, and has good measurement performance; Although the relative measurement precision of algorithm (In TABLE 1) is better than 0.06 per thousand, the overall measuring accuracy of the system also depends on the stability of the mechanical structure, temperature compensation and other factors; In experiments we found that the temperature drift and the consistency of the torsional pendulum's initial motivation had significant influence on the accuracy, and the follow work will be focus on it.

REFERENCES

[1]   Rudra Gowda, Pravesh Mathur, Pathak Swapnil, etc; Techniques for Measurement of Centre of Gravity & Moment of Inertia and their Importance in Spacecraft Attitude Control, 2nd national Conference on Advance in Metrology, February **(2012)**.
[2]   Hung Heng-Chuan; A practical method to improve Moment of Inertia Measurement accuracy for FORMOSAT-3 satellites., 69th International Conference on Mass Properties, 87-96 **(2010).**
[3]   Swank, J.Aaron, Hardham, etc; Moment of inertia measurement using a five-wire torsion pendulum and optical sensing,

Proceedings of the 21st Annual ASPE Meeting, Monterey, CA, United states.

[4] Tang Wen-yan, Li Hui-peng, Zhang Chun-fu; Measurement of Flight Object's Moment of Inertia Using Torsion Pendulum, Journal of Nanjing University of Science and Technology, **32(1)**, 69-72.

[5] Li Hui-peng, Tang Wen-yan, Zhang Chun-fu; A Measuring System of Inertia Moment of Missile and Its Error Analysis, ACTA ARMAMENTARII, **28(2)**, 206-208.

[6] Rairan, Danilo, Garzon, H.Max; Period estimation for adaptive reconstruction of (quasi-) periodic signals, International Journal of Adaptive Control and Signal Processing Published online in Wiley Online Library (wileyonlinelibrary.com) **(2013)**.

[7] Ramos Pedro M. da Silva, Manuel Fonseca, Martins Raúl Carneiro, etc; Simulation and experimental results of multiharmonic least-squares fitting algorithms applied to periodic signals. IEEE Transactions on Instrumentation and Measurement, **55(2)**, 646-651 **(2006)**.