Moisture-dependent models for microwave complex permittivity and loss factor of spring oats kernels: A comparative study

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1. INTRODUCTION

The use of electrical properties of grains for moisture measurement has been one of the most prominent agricultural applications for dielectric properties data. The dielectric properties offer a potential means in making devices for sensing the moisture content of individual grain kernels, which help in preventing the spoilage of large blended lots stored in elevators, ships or mills [1-2].

Several efforts to model the dielectric properties of grains have been made [3-4]. The purpose of the present paper is to consider a more general approach towards modeling the dielectric properties of samples of spring oats (Avena sativa L.), using the data for them at a fixed frequency of 2.45 GHz at 24°C, to present empirical expressions which allow predictions of permittivity and loss factor. The electrical properties of grains are influenced by ionic conductivities and bound-water relaxations. The result of measured values is then a complicated function of the amount of water in the grain. However, all these effects disappear almost completely at higher microwave frequencies. Thus, microwaves offer a nondestructive, sensitive and feasible method for determining the water content in grains.

2. EXPERIMENTAL

Data for measured values of bulk density, decimal moisture content (m) and dielectric constant were taken from the TABLE 3 [5]. For deriving the values of bulk density \( \rho_b \), Kernel densities \( \rho_k \) and hence the volume fractions \( (=\rho_b/\rho_k) \) of the material in the mixture, the equa-
tions (7) and (8) of the same paper\cite{5} were used. The general quadratic and cubic models given by Nelson and Kraszewski\cite{2} connecting dielectric constant, moisture content and frequency of operation were used for their comparison with the corresponding new models proposed in the present study. The equations are:

\[ \varepsilon' = [1 + \{A_2 - B_2 \log f + (C_2 - D_2 \log f) M \} \rho]^2 \]  
\[ (1) \]

and

\[ \varepsilon'' = [1 + \{A_3 - B_3 \log f + (C_3 - D_3 \log f) M \} \rho]^3 \]  
\[ (2) \]

The solitary equation for the dielectric loss factor available for comparison is of the form:

\[ \varepsilon'' = 0.146 \rho^2 + 0.004615 \rho^2 \left[ 0.32 \log f + 1.743/\log f - 1 \right] \]  
\[ (3) \]

Where \( \rho = \rho_o = \text{bulk density of the material in gram} \times \text{cm}^{-3} \), \( M = \% \text{moisture content, wet basis} = 100 m \), \( f = \text{frequency of operation in MHz} \).

The values of constants viz. \( A_2, B_2, C_2, D_2 \) or \( A_3, B_3, C_3, D_3 \) of equations (1) and (2) for spring oats were taken from the TABLE 6 of Nelson’s paper\cite{2}.

3. Model development and evaluation of constants

Based on observations of almost linear plots obtained for the dependence of relative permittivity of grains and cereals with moisture content, especially in the microwave range, it was proposed to give quadratic as well as cubic models for such variations. On similar lines of the works of Noh and Nelson\cite{6} on rice samples, the second and a new term, called moisture density (product of decimal moisture content and bulk density), was also used. The third and the new term, called moisture specific volume (ratio of decimal moisture content to bulk density, \( m_v \)), in addition to \( m \) and \( m_d \) was also proposed to be incorporated in the composite model proposed in the present study. The composite models are:

Quadratic

\[ \varepsilon' = a_2 \left[ m_{d} \right]^2 + b_2 \left[ m_{d} \right] + c_2 \left[ m_{v} \right] + K_1 \]  
\[ (4a) \]

and

\[ \varepsilon'' = d_3 \left[ m_{d} \right]^2 + e_3 \left[ m_{d} \right] + f_3 \left[ m_{v} \right] + K_2 \]  
\[ (4b) \]

Cubic

\[ (5a) \]

\[ (5b) \]

The value of the constant \( K_1 \) was taken to be equal to the average of the relative permittivities as derived from equations (1) and (2) by putting \( M = 0 \) in them. The corresponding value of \( \rho \) was derived from equation (7) of the Nelson’s paper\cite{5} by putting \( m = 0 \) in it. Similarly, the value of \( K_2 \) was taken to be equal to the value of the loss factor from equation (3) of the present paper by putting the abovementioned value of \( \rho \) corresponding to \( M = 0 \). In this way

\[ K_2 = (\varepsilon'')_{m=0} = 0.146 (\rho_o)^2 \]  
\[ (6) \]

(\( \rho_o = \text{bulk density at } m = 0 \))

From the data for relative permittivity at different decimal moisture contents and bulk densities, the constants for the first part of each of the sets of models were evaluated using the method of least-squares-fit for nonlinear regression.

The same method was adopted using the data for dielectric loss factor derived from the works of S.O. Nelson\cite{7} i.e., from figure 1(b) for the operating frequency of 2450 MHz. The results have been presented in TABLE 1 and the evaluated constants are listed in TABLE 2.

In order to extend the applicability of the present models to grain kernels, the values of relative permittivity of the moist grain samples, (supposed to be an air-particle binary mixture), were proposed to be converted to those of solid materials (particles) with the help of eight dielectric mixture equations\cite{8-14}.

4. Brief Introduction of the dielectric mixture equations used

1. Rother-Lichtenecker formula or the logarithmic law of mixing for Chaotic mixture\cite{8}

\[ \ln \varepsilon_r = \sum_{i=1}^{n} f_i \ln \varepsilon_i \]  
\[ (7a) \]

Thus for an air-particle binary mixture

\[ \ln \varepsilon_r = f_1 \ln \varepsilon_1 + f_2 \ln \varepsilon_2 \]  
\[ (7b) \]
Thus, \( \varepsilon_r = \exp[1/f, \ln \varepsilon_r] \)  

2. Taylor’s formula for random angular distribution of needles\(^9\)

\[
3(\varepsilon_r - \varepsilon_n^0)/f = (\varepsilon_r - \varepsilon_n^0)(2\varepsilon_r + \varepsilon_n^0)
\]

\[
\varepsilon_r = 0.25 \left[ 2 + 3(f - 1) - \varepsilon_n^0 \right] + \left[ \left( 2 + 3f - 1 - \varepsilon_n^0 \right)^2 + 8\varepsilon_n^0 \right]^{1/2}
\]

(8a)

(8b)

(Taking only the positive root of the quadratic equation which the relation yielded).

3. Taylor’s formula for random angular distribution of disks\(^9\)

\[
[3(\varepsilon_r - \varepsilon_n^0)/(\varepsilon_r + \varepsilon_n^0)]/f = (\varepsilon_r - \varepsilon_n^0)(5\varepsilon_r + \varepsilon_n^0)
\]

(9a)

on similar pattern as 2

one gets,

\[
\varepsilon_r = 0.25 [ (1-3f) - (5-3f) \varepsilon_r ] + [(1-3f)
\]

\[
-(5-3f)\varepsilon_r^2 + 4(3f\varepsilon_r - (5-3f) \varepsilon_r)]^{1/2}
\]

(9b)

4. Lewin’s formula\(^10\)

Lewin proposed a formula for the computation of permittivity and permeability of mixture consisting of a homogenous material in which spherical particles are embedded. The formula is given as :

\[
(\varepsilon_r - \varepsilon_n^0)/\varepsilon_H = 3f(\varepsilon_r - \varepsilon_n^0)/(\varepsilon_H + (1+2f)\varepsilon_r)
\]

(10a)

which in the present case simplifies as

\[
\varepsilon_r = [(\varepsilon_r + 2\varepsilon_n^0) - (1-f)]/[(1+2f) - (1-f)]
\]

(10b)

5. Sillars formula\(^11\)

\[
\varepsilon_r = \varepsilon_n^0 + D(1-f) = \varepsilon_n^0 + D(1-f) \varepsilon_n^0]
\]

(11a)

where, \( D \) = depolarization factor depending on the shape of the particles.

\[
\Rightarrow \varepsilon_r = \left[ \left( \varepsilon_r - 1 \right)/f \right] - D(1-f) \left( \varepsilon_n^0 - 1 \right) + 1
\]

(11b)

(12b)

where \( D = 0.2 \)

Surprisingly enough, the data give the best fit for the value of \( D = 0.2 \), as derived for rutile particles, suggesting that the shape of the particles were the same in both the cases. Other wise, other values of \( D \) were to be tried for best fit.

6. Sadiku’s formula\(^12\)

\[
(f \varepsilon_r - 1)/(f \varepsilon_r + u) = (1-f)(\varepsilon_n^0 - 1)/(\varepsilon_r + u)
\]

(12a)

(12b)

In the above formula \( u \) = form no., supposed to be depending on the shape of the particles and the values of \( u = 5 \) for snow or ice taken from the literature, gave the best fit as \( D = 0.2 \) for rutile in the previous formula. It also led us to suppose that there must be a relation like \( D = 1/u \) between \( D \) and \( u \). Thus, by putting \( u = 5 \), the formula finally reduces to:

\[
\varepsilon_r + 2 = 3 [\varepsilon_r (1+f) + (5f-1)]/[(1+5f) - (1-f)]
\]

(12c)

Again, the limitation for the validity of the Weiner formula is that the particles should be small as compared to the wavelength used.

7. Formula obtained from effective medium theory\(^13\)

\[
\varepsilon_r = \varepsilon_n^0 [(1+2f) \varepsilon_r + 2 \varepsilon_n^0 (1-f)]/[(1+f) + (2f+1-\varepsilon_r) (1-f)]
\]

(13a)

\[
\varepsilon_r = [(2+f) \varepsilon_r - 2(1-f)]/[(1+2f) - (1-f)\varepsilon_r]
\]

(13b)

In the above formula, particulate material has been taken as the first component and air as the second one under the limiting case of small concentration of the component A in the binary system AB-opposed to those taken in other formulae.

8. Skipetrov formula\(^14\)

\[
\varepsilon_{ef} - \varepsilon_r [1{\{3f(\varepsilon_r - \varepsilon_n^0)\}}/[(2+f)+\varepsilon_r(1-f)]]
\]

(14a)

The equation finally gives :

\[
\varepsilon_r = 1 + [3f(\varepsilon_r - 1)]/[(2+f)+\varepsilon_r(1-f)]
\]

(14b)

In all the above equations, \( \varepsilon_n^0 = 1 \) and \( \varepsilon_r = \varepsilon_2^0 \)

The above expression has been claimed by the investigator to be an original expression for the effective dielectric function of dilute suspension of spherical beads of diameter \( d \ll \lambda \). Further, it has been claimed that the above formula is expected to be more appropriate for the interpretation of the experiments and behavior at higher volume fractions.

Using any measured value of \( \varepsilon_r \), the corresponding value of volume fraction of the particle, \( f = (\varepsilon_r) \), the value of the permittivity of the particles, \( \varepsilon_2 \ (= \varepsilon_2^0 \), say) was calculated choosing any of the eight equations, say the first one. The constants of the first set of equations concerning relative permittivity versus \( m \) (say) for the quadratic or the cubic model, as the case may be, were used to compute the value of \( m \), (say). Using these values of \( m \) and the constants evaluated for the second set of equations (concerning loss factor versus moisture content, say), the value of loss factor of the particles (kernels), \( \varepsilon_2'' \) were calculated. Thus one gets the values of \( \varepsilon_2' \) and \( \varepsilon_2'' \) for a given computed value of \( m \) (say). The same process was repeated for different values of volume fractions of a given sample. A similar process was adopted by taking another dielectric mixture equation one by one, to get the data points. The same process was repeated for computation of \( \varepsilon_2' \) and
\( \varepsilon'' \) as functions of \( m_d \) and \( m_v \) for both types of the proposed models. It was expected to achieve the estimates of \( \varepsilon' \) and \( \varepsilon'' \) of spring oats kernels as functions of \( m, m_d \) and \( m_v \).

### 3. RESULTS AND DISCUSSION

The data of measured values of relative permittivity and dielectric loss factor of spring oats (Avena sativa) L., bulk samples at nine different moisture contents ranging from 8.2 % to 25.4%, wet basis and bulk and kernel densities corresponding to 24°C and 2.45 GHz as taken from the literature are presented in TABLE 1. TABLE 2 presents the constants and model parameters for the present quadratic and cubic models relating relative permittivity and dielectric loss factor to \( m, m_d \) and \( m_v \) evaluated with the help of least-squares-fit method for non-linear regression analysis. The same TABLE also presents the quantitative comparative performances of the Nelson’s moisture-dependent quadratic and cubic models for relative permittivity and similar models proposed in the present study. The same TABLE also presents the same comparative performance analysis of the present models for dielectric loss factor with that of Kraszeweski-Nelson model for the same. It also presents the average percentage errors of prediction for the different models with respect to the measured values of two dielectric properties (\( \varepsilon' \) and \( \varepsilon'' \)) along with the coefficients of determination (\( r^2 \)). Graphical presen-
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Figure 2(a): Variation of relative permittivity and loss factor of spring oats kernels at 2.45GHz and 24°C as functions of moisture-density in the light of quadratic model

Figure 2(b): Variation of relative permittivity and loss factor of spring oats kernels at 2.45GHz and 24°C as functions of moisture-density in the light of cubic model

Figure 3(a): Variation of relative permittivity and loss factor of spring oats kernels at 2.45GHz and 24°C as functions of moisture-specific volume in the light of quadratic model

(Figure 3b)??

Graphical presentation of data points for moisture-density (m) dependent quadratic and cubic models for \( \varepsilon'_2 \) and \( \varepsilon''_2 \) are given in Figures 2a and 2b, respectively. Similar graphical presentations of respective data points for moisture-specific volume (m) dependent quadratic and cubic models are given in figures 3a and 3b.

Examination of data in Table 2 reveals that Nelson’s moisture-dependent quadratic and cubic models for relative permittivity provided values of average percentage errors \( \approx 3-4 \). Corresponding values for the new cubic and quadratic models are \( \approx 3-6 \). The \( r^2 \)-values for the two old models are almost similar (\( \approx 0.99 \)) whereas they are \( \approx 1 \) for both the new models (quadratic and cubic). Solitary Kraszewski–Nelson Model (KNM) as well as the new cubic model (PSM) for loss factor provides higher average percentage errors \( \approx 15 \) and \( 12 \) and almost the same order of values for \( r^2 \) (\( \approx 0.993 – 0.994 \)). On the contrary, the present quadratic model (PSM) provides best fit with experimental data in having \( r^2 = 1 \) and average error \( \approx 6 \% \).

The cubic model for the variation of \( \varepsilon'_2 \) and \( \varepsilon''_2 \) as function of decimal moisture content (m) provided a poorer fitting as compared with the similar quadratic model, but the trend of variation is the same as in the density-dependent models. As regards the moisture-density dependent quadratic model, none other than the third formula provided acceptable data points for computed values of \( m_d \) and hence for \( \varepsilon''_2 \), for different computed values of \( \varepsilon'_2 \), corresponding to different values for bulk material samples, though seven different dielectric mixture equations were tried in the present study (Figure 2a). However, the trend of variation is similar as above. As regards the plot corresponding to the cubic model for \( m_v \) (Figure 3b), it is seen that although the trend of variation is similar as in other plots, a critical computed value of \( m_v \) (\( \approx 0.35 \)) is reached beyond which the dielectric loss factor became too large so that it overtook the relative permittivity and then continued to be ahead of relative permittivity for subsequent values of \( m_v \).
In case of \(m_v\)-dependent quadratic model for \(\varepsilon'\) and \(\varepsilon''\) similar to the \(m_d\)-dependent quadratic model, it was found that none other than the third formula provided acceptable data points for computed values of \(m_v\) and hence for \(\varepsilon''\) corresponding to different computed values of \(\varepsilon'\). Further, the trend of variation was similar to that for \(m_d\)-dependent quadratic model. As regards the last model i.e., the cubic model for the variation of computed values of \(\varepsilon'\) and \(\varepsilon''\) corresponding to different values of \(m_v\) (computed), the trend of variation is similar to that offered in the other models and the fitting was excellent over the entire range of variation of \(m_v\).

### TABLE 3(A)

<table>
<thead>
<tr>
<th>Nelson’s model (NM) for relative permittivity</th>
<th>Prasad - Singh Model (PSM) for relative permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>QM*</td>
<td>QM*</td>
</tr>
<tr>
<td>CM*</td>
<td>CM*</td>
</tr>
<tr>
<td>Predicted values</td>
<td>Predicted values</td>
</tr>
<tr>
<td>(r^2) Average % error</td>
<td>(r^2) Average % error</td>
</tr>
<tr>
<td>1.81 - 1.82</td>
<td>1.52 - 1.62</td>
</tr>
<tr>
<td>1.96 - 1.95</td>
<td>1.68 - 1.78</td>
</tr>
<tr>
<td>2.07 0.9956</td>
<td>1.80 1.91</td>
</tr>
<tr>
<td>2.21 - 2.20</td>
<td>2.02 - 2.09</td>
</tr>
<tr>
<td>2.29 4.01</td>
<td>3.85 2.18 6.08</td>
</tr>
<tr>
<td>2.40 - 2.40</td>
<td>2.31 - 2.31</td>
</tr>
<tr>
<td>2.58 - 2.58</td>
<td>2.49 - 2.42</td>
</tr>
<tr>
<td>2.61 - 2.61</td>
<td>2.62 - 2.61</td>
</tr>
<tr>
<td>2.70 - 2.70</td>
<td>2.82 - 2.61</td>
</tr>
</tbody>
</table>

### TABLE 3(B)

<table>
<thead>
<tr>
<th>Kraszewski-Nelson model (KNM) for loss factor</th>
<th>Prasad - Singh Model (PSM) for loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>QM</td>
<td>QM</td>
</tr>
<tr>
<td>CM</td>
<td>CM</td>
</tr>
<tr>
<td>Predicted values</td>
<td>Predicted values</td>
</tr>
<tr>
<td>(r^2) Average % error</td>
<td>(r^2) Average % error</td>
</tr>
<tr>
<td>0.071 - 0.068</td>
<td>0.115 - 0.139</td>
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<td>0.091 0.9929</td>
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<td>0.204 14.96</td>
<td>0.205 5.94 0.198 12.09</td>
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<td>0.219 - 0.223</td>
</tr>
<tr>
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<td>0.238 - 0.264</td>
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<td>0.261 - 0.255</td>
<td>0.255 - 0.300</td>
</tr>
<tr>
<td>0.283 - 0.274</td>
<td>0.274 - 0.353</td>
</tr>
</tbody>
</table>

*Quadratic model, **Cubic model

### 4. REFERENCES