Journal of Physics and Astronomy

Hypothesis|Vol9 Iss7

Modification of the Interior Solution of Einstein's G22 Field Equation for A Homogeneous Spherical Massive Bodies Whose Fields Differ in Radial Size, Polar Angle, and Time

Rilwan Usman[1*], Abba Umar Maisalatee[2], and M. Alpha[1]

[1]Department of Physics, Nigerian Army University, Nigeria

[2]Department of Physics, Nasarawa State University, Nigeria

[*]**Corresponding author**: Rilwan Usman, Department of Physics, Nigerian Army University, PMB 1500 Biu, Borno State, Nigeria, Tel: 08037772627/07055324002; E-mail: rilwanusmanloko4@gmail.com

Received date: July 22, 2021; **Accepted date:** July 04, 2021; **Published date:** July 11, 2021

Abstract

In the general theory of relativity, Einstein's field equations relate the geometry of space-time with the distribution of matter within it. Research has shown that the tensors for spherical massive bodies are not functions of radial distance only as shown by Schwarzchild; they depend on other factors such as polar angle, azimuthal angle, and time. In this article, we formulate the analytical solution of Einstein's field equation interior to a homogeneous spherical body whose tensor field varies with time, radial distance, and polar angle using weak field and slow-motion approximation. The obtained result converges to Newton's dynamical scalar potential with additional time factors not found in the well-known Newton's dynamical theory of gravitation which is a profound discovery with the dependency on three arbitrary functions. The result obtained can be used in the study of rotating astrophysical bodies such as stars. Our result obeyed the equivalence principle of Physics.

Keywords: Schwarzschild metric; Einstein Equation; radial size; polar angle; Einstein tensor

Introduction

According to Einstein's theory of gravitation published in 1915/1916, which unifies Special Relativity and Sir Isaac Newton's law of universal gravitation, that gravitation is not due to a force but rather a manifestation of curved space and time, with the curvature being produced by the mass-energy and momentum content of the space-time. This is termed General Relativity and is the most widely accepted theory of gravitation [1,2]. The equations are in the form of a tensor equation that related the local space-time curvature (expressed by the Einstein tensor) with the local energy and momentum within that space-time (expressed by the stress-energy tensor) [3]. Einstein's General Relativity is the leading theory of space-time and gravity. Exact solutions of Einstein's equations thus model gravitating systems and enable exploration of the Mathematics and Physics of the theory. After the publication of Einstein's geometrical gravitational field equations in 1915, the search for their exact and analytical solutions for all the gravitational fields in nature began [4,5]. Schwarzchild first constructed the exact solution to this field equation in static and pure radial spherical polar coordinates in 1916 by considering astrophysical bodies such as the sun and the stars. In Schwarzchild's metric, the tensor field varies with radial distance only [6].

Citation: Rilwan Usman, Abba Umar Maisalatee and M. Alpha. Modification of the Interior Solution of Einstein's G22 Field Equation for A Homogeneous Spherical Massive Bodies Whose Fields Differ in Radial Size, Polar Angle, and Time, J Phys Astron.2021;9(7):228.

A new method and approach were introduced to formulate exact analytical solutions as an extension of Schwarzschild's method. This new approach took into consideration the fact that the tensor field of astrophysical bodies does not depend on radial distance only as indicated in Schwarzschild's equation [7]. This new approach was used in several studies of Einstein's geometrical field equations such as. This method would help in the study of Ceres, Pluto, Makemake, Haumea, the Ouort Cloud, and other astrophysical bodies. In this research work, we show how the exact analytical solution of the interior field equation can be constructed in the limit of c^{-2} in a gravitational field for time-varying spherical massive bodies using the new method and approach [8].

Einstein's Equation

Einstein's field equation interior to a homogeneous spherical distribution of mass is given generally as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G T_{\mu\nu}}{c^4} \quad (1)$$

Where $\Gamma_{\mu\nu}$ is the energy-momentum tensor due to any distribution of mass or pressure [9].

G is the universal gravitational constant.

Now, let us assume that the homogeneous mass distribution is in a weak field limit. We can neglect the contribution from the source, thus define the energy-momentum tensor given as

$$T_{\mu\nu} = \frac{1}{2} p_o c^4 \quad (2)$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{4\pi G p_o}{c^2} \quad (3)$$

Where p_o is the density

c is the speed of light in a vacuum.

It was observed by (Misner) that the exterior field equations along the G_{00}, G_{22} and G_{33} converge within the exterior field, similarly along the interior field.

For mathematical convenience, we choose G_{22}

Hence the non-trivial field equation is

$$R_{22} - \frac{1}{2} R g_{22} = \frac{4\pi G p_o}{c^2} \quad (4)$$

The affine connections of this field constructed by Lumbi were used to construct the Ricci tensor R_{22} and the curvature scalar R given respectively as [10,11]:

$$R_{22} = R^{o}_{220} + R^{1}_{221} + R^{2}_{222} + R^{3}_{223} \quad (5)$$

$$R = g^{00} R_{00} + g^{11} R_{11} + g^{22} R_{22} + g^{33} R_{33} \quad (6)$$

The expanded form of equations (5) and (6) are given as

$$R_{22} = \Gamma^{0}_{20.2} + \Gamma^{0}_{20}\Gamma^{0}_{02} - \Gamma^{1}_{22}\Gamma^{0}_{10} + \Gamma^{1}_{21.2} - \Gamma^{1}_{22.1} + \Gamma^{1}_{21}\Gamma^{1}_{12} - \Gamma^{1}_{22}\Gamma^{1}_{11} - \Gamma^{2}_{12}\Gamma^{1}_{22} + \Gamma^{2}_{22.2} - \Gamma^{2}_{22.2} + \Gamma^{0}_{22}\Gamma^{2}_{02} - \Gamma^{0}_{22}\Gamma^{2}_{02}$$
$$+ \Gamma^{1}_{22}\Gamma^{2}_{12} - \Gamma^{1}_{22}\Gamma^{2}_{12} + \Gamma^{2}_{22}\Gamma^{2}_{22} - \Gamma^{2}_{22}\Gamma^{2}_{22} + \Gamma^{3}_{22}\Gamma^{2}_{32} - \Gamma^{3}_{22}\Gamma^{2}_{32} + \Gamma^{3}_{23.2} - \Gamma^{1}_{22}\Gamma^{3}_{13} + \Gamma^{3}_{23}\Gamma^{3}_{32} \quad (7)$$

$$R_{22}=g^{00}\left\{\Gamma^0_{00.0}-\Gamma^0_{00.0}+\Gamma^0_{00}\Gamma^0_{00}-\Gamma^0_{00}\Gamma^0_{00}+\Gamma^1_{00}\Gamma^0_{10}\Gamma^1_{01.1}-\Gamma^1_{00}\Gamma^0_{10}-\Gamma^2_{00}\Gamma^0_{20}-\Gamma^2_{00}\Gamma^0_{20}+\Gamma^3_{00}\Gamma^0_{30}-\Gamma^3_{00}\Gamma^0_{30}-\Gamma^1_{00.1}+\Gamma^0_{01}\Gamma^1_{00}\right.$$

$$\left.-\Gamma^0_{00}\Gamma^1_{01}+\Gamma^0_{01}\Gamma^1_{10}-\Gamma^1_{00}\Gamma^1_{11}+\Gamma^2_{00}\Gamma^1_{21}-\Gamma^2_{00.2}+\Gamma^0_{02}\Gamma^2_{00}-\Gamma^1_{00}\Gamma^2_{12}-\Gamma^1_{00}\Gamma^3_{13}-\Gamma^2_{00}\Gamma^3_{23}\right\}$$

$$+g^{11}\left\{\Gamma^0_{10.1}-\Gamma^0_{11.0}+\Gamma^0_{10}\Gamma^0_{01}-\Gamma^0_{11}\Gamma^0_{00}+\Gamma^1_{10}\Gamma^0_{11}-\Gamma^1_{11}\Gamma^0_{10}-\Gamma^2_{11}\Gamma^0_{20}+\Gamma^1_{11.1}-\Gamma^1_{11.1}+\Gamma^0_{11}\Gamma^1_{01}\right.$$

$$-\Gamma^0_{11}\Gamma^1_{01}+\Gamma^1_{11}\Gamma^1_{11}-\Gamma^1_{11}\Gamma^1_{11}+\Gamma^2_{11}\Gamma^1_{21}-\Gamma^2_{11}\Gamma^1_{21}+\Gamma^3_{11}\Gamma^1_{31}-\Gamma^3_{11}\Gamma^1_{31}+\Gamma^2_{12.1}-\Gamma^2_{11.2}+\Gamma^1_{12}\Gamma^2_{11}-\Gamma^1_{11}\Gamma^2_{12}$$

$$\left.+\Gamma^2_{12}\Gamma^2_{21}+\Gamma^3_{13.1}-\Gamma^1_{11}\Gamma^3_{13}-\Gamma^2_{11}\Gamma^3_{23}+\Gamma^3_{13}\Gamma^3_{31}\right\}$$

$$+g^{22}\left\{\Gamma^0{}_{20,2}+\Gamma^0{}_{20}\Gamma^0{}_{02}-\Gamma^1{}_{22}\Gamma^0{}_{10}+\Gamma^1{}_{21,2}-\Gamma^1{}_{22,1}+\Gamma^1{}_{21}\Gamma^1{}_{12}-\Gamma^1{}_{22}\Gamma^1{}_{11}-\Gamma^2{}_{12}\Gamma^1{}_{22}\right.$$

$$+\Gamma^2{}_{22,2}-\Gamma^2{}_{22,2}+\Gamma^0{}_{22}\Gamma^2{}_{02}-\Gamma^0{}_{22}\Gamma^0{}_{02}+\Gamma^1{}_{22}\Gamma^2{}_{12}-\Gamma^1{}_{22}\Gamma^2{}_{12}+\Gamma^2{}_{22}\Gamma^2{}_{22}-\Gamma^2{}_{22}\Gamma^2{}_{22}+\Gamma^3{}_{22}\Gamma^2{}_{32}-\Gamma^3{}_{22}\Gamma^2{}_{32}$$

$$\left.+\Gamma^3{}_{23,2}-\Gamma^1{}_{22}\Gamma^3{}_{13}+\Gamma^3{}_{23}\Gamma^3{}_{32}\right\}+g^{33}\left\{-\Gamma^1{}_{\infty}+\Gamma^3{}_{13}-\Gamma^2{}_{\infty}\Gamma^3{}_{23}-\Gamma^1{}_{33,1}-\Gamma^1{}_{33}\Gamma^1{}_{11}-\Gamma^2{}_{33}\Gamma^1{}_{21}+\right.$$

$$\Gamma^3{}_{31}\Gamma^1{}_{33}-\Gamma^2{}_{33,2}-\Gamma^1{}_{33}\Gamma^2{}_{12}+\Gamma^3{}_{32}\Gamma^2{}_{33}+\Gamma^3{}_{33,3}-\Gamma^3{}_{33,3}+\Gamma^0{}_{33}\Gamma^3{}_{03}-\Gamma^0{}_{33}\Gamma^3{}_{03}$$

$$\left.+\Gamma^1{}_{33}\Gamma^3{}_{13}-\Gamma^1{}_{33}\Gamma^3{}_{13}+\Gamma^2{}_{33}\Gamma^3{}_{23}-\Gamma^2{}_{33}\Gamma^3{}_{23}+\Gamma^3{}_{33}\Gamma^3{}_{33}-\Gamma^3{}_{33}\Gamma^3{}_{33}\right\} \tag{8}$$

Explicitly equations (7) and (8) are given as

$$R_{22}=\frac{2}{c^4}\left(1+\frac{2f\left(t_\circ r_\circ\theta\right)}{c^2}\right)^{-2}\left(\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial\theta}\right)^2+\frac{2r}{c^2}\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial r}+\frac{2f\left(t_\circ r_\circ\theta\right)}{c^2} \tag{9}$$

$$R=\frac{8}{c^4}\left(1+\frac{2f\left(t_\circ r_\circ\theta\right)}{c^2}\right)^{-3}\left(\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial t}\right)^2-\frac{8}{c^2r}\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial r}-\frac{2}{c^2}\left(1+\frac{2f\left(t_\circ r_\circ\theta\right)}{c^2}\right)^{-2}\left(\frac{\partial^2 f\left(t_\circ r_\circ\theta\right)}{\partial t^2}\right)$$

$$-\frac{2}{c^2}\left(\frac{\partial^2 f\left(t_\circ r_\circ\theta\right)}{\partial r^2}\right)-\frac{2}{c^4r^2}\left(1+\frac{2f\left(t_\circ r_\circ\theta\right)}{c^2}\right)^{-2}\left(\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial\theta}\right)^2-\frac{4f\left(t_\circ r_\circ\theta\right)}{c^2r^2} \tag{10}$$

Substituting (9) and (10) into (4) gives

$$\nabla^2 f\left(t_\circ r_\circ\theta\right)+\left(\frac{\partial^2 f\left(t_\circ r_\circ\theta\right)}{\partial t^2}\right)-\frac{1}{c^2r^2}\left(\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial\theta}\right)^2+\frac{4f\left(t_\circ r_\circ\theta\right)}{c^4r^2}\left(\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial\theta}\right)^2-\frac{4}{c^2}\left(\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial t}\right)^2$$

$$+\frac{24f\left(t_\circ r_\circ\theta\right)}{c^4}\left(\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial t}\right)^2-\frac{4f\left(t_\circ r_\circ\theta\right)}{c^2}\left(\frac{\partial^2 f\left(t_\circ r_\circ\theta\right)}{\partial t^2}\right)=\frac{4\pi Gp_o}{c^2} \tag{11}$$

$$\nabla^2 f\left(t_\circ r_\circ\theta\right)+\left(\frac{\partial^2 f\left(t_\circ r_\circ\theta\right)}{\partial t^2}\right)-\frac{1}{c^2r^2}\left(\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial\theta}\right)^2-\frac{4}{c^2}\left(\frac{\partial f\left(t_\circ r_\circ\theta\right)}{\partial t}\right)^2\frac{4f\left(t_\circ r_\circ\theta\right)}{c^2}\left(\frac{\partial^2 f\left(t_\circ r_\circ\theta\right)}{\partial t^2}\right)^2=\frac{4\pi Gp_o}{c^2} \tag{12}$$

To the weak field limit of c^o, the equation reduces to

$$\nabla^2 f\left(t_\circ r_\circ \theta\right) + \frac{\partial^2 f\left(t_\circ r_\circ \theta\right)}{\partial t^2} = \frac{4\pi G p_o}{c^2} \quad (13)$$

Conclusion

The result obtained in is the Newton dynamical scalar field equation with an additional time factor which signifies the dynamical nature of the system, it is indeed a profound discovery, it confirms our assumption in that Newton Dynamical Theory of Gravitation NDTG is a limiting case of Einstein's geometrical gravitational field equations EGGFE, and this should clear the objection in 7, 38,41. The experimentally established equivalence principle of physics is shown with the dependency of the scalar function on time, radial distance, and polar angle.

If the pressure is negligible compared to mass density, hence

$p_o \equiv 0$

Our scalar potential will be

$$\nabla^2 f\left(t_\circ r_\circ \theta\right) + \frac{\partial^2 f\left(t_\circ r_\circ \theta\right)}{\partial t^2} = -\frac{GM_0}{r} \quad (14)$$

Where $GM_0 = k$

The EFE reduces to Newton's law of gravitation by using both weak-field approximation and slow-motion approximation, equation (3) will thus further splits into four non-linear equations analogous to Maxwell's equation and could thus be applied in the study of the Gravitoelectric and Gravitomagnetic coupling phenomena.

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