Modelling aerosol dispersion from Indian subcontinent into the oceanic region

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ABSTRACT

After observing the AOD (Aerosol optical depth) patterns measured by MODIS satellite sensors in the Arabian sea and Bay of Bengal which decay in radially outward direction from the Indian land mass, it was hypothesized that the AOD distribution pattern in the Arabian sea and Bay of Bengal at large time scales could be captured by using an atmospheric dispersion model considering the Indian subcontinent as a source of aerosol particles which are dispersed nearly equally in all directions into the oceanic region. In order to validate the hypothesis, a dispersion model with emission, advection and deposition was assumed and the broad goal of the current work was to solve it and estimate the AOD distribution in Arabian sea and Bay of Bengal. The parameters required for this analysis are: dispersion diffusivity, average wind direction and magnitude, mass extinction efficiencies and source and sink flux’s altitude variation for different aerosols (Black Carbon, Organic Carbon, Sea-salt, sulphate, dust). In this work, the average wind direction and magnitude was identified for the entire Indian subcontinent. The dispersion diffusivity values for eight different Indian locations were calculated at eight altitude levels. Also the average friction-velocities were estimated and an attempt was made to find the relation between dispersion diffusivity and friction-velocity. With the estimation of source and sink variation with altitude and mass extinction efficiencies of various aerosols, it will be possible to estimate the AOD patterns and validate whether atmospheric dispersion models can capture the aerosol distribution in Arabian sea and Bay of Bengal at large time-scales.

INTRODUCTION

Aerosols are extremely fine liquid-droplets or solid particles that remain suspended in air as fog or smoke and affect the climate by scattering or absorbing the solar radiation, generating a cooling or warming effect respectively. The aerosol dispersion is measured by finding the aerosol optical depth (AOD) which is a measure of transparency in the atmosphere. AOD can be measured using the satellite sensors and also using the General Circulation Models (GCM).

Atmospheric models of dispersion solve the math-
emathematical equations which predict the concentration of air pollutants or toxins emitted from sources such as industrial plants, vehicular traffic or accidental chemical releases. These models are generally used for studying dispersion at small length and time scales\textsuperscript{[2]}. However by observing the AOD patterns measured by MODIS satellite sensors in the Arabian sea and Bay of Bengal which decay in radially outward direction from the Indian land mass\textsuperscript{[3]}, it was hypothesized that the AOD distribution pattern in the oceanic region at large time-scales could be captured by using a simple atmospheric dispersion model considering the entire Indian subcontinent as a source of aerosol particles which are dispersed nearly equally in all directions into the oceanic region. Also by observing that the ECHAM5.5 GCM measurements of AOD over the Arabian sea and Bay of Bengal during the period from 18 March to 10 May 2006\textsuperscript{[3]} could not capture the AOD pattern as retrieved by the Moderate Resolution Imaging Spectro-Radiometer (MODIS) satellite sensors, it was hypothesized that if a simple dispersion model could predict the actual AOD behaviour then it would minimize the use of computationally intensive GCMs.

Thus, a dispersion model with emission, advection and deposition was assumed to capture the aerosol dispersion phenomenon at large time-scales. The broad goal of the current work was to solve this dispersion model and estimate the AOD distribution in Arabian sea and Bay of Bengal region and compare it with the predicted AOD distributions of HAM and MODIS\textsuperscript{[3]}.

**INPUT DATA**

The input data for the current project consisted of the daily values of wind velocity components: u-wind (wind velocity in x-direction), v-wind (wind velocity in y-direction) and w-wind (wind velocity in z-direction) of the past 10 yrs (2002-11) at varying altitudes. These values were obtained from NCEP Reanalysis (National Centres for Environmental Prediction, Earth System Research Laboratory). Using GrADS (Grid Analysis and Display System), the Indian subcontinent data (Latitudinal extent –6°N to 37°6’N and Longitudinal extent 60°E to 97°25’E) was extracted from the NCEP global data. The current project has used the NCEP velocity data of eight different altitude levels: 1000, 925, 850, 700, 600, 500, 400 and 300 mbar (from sea level to 9 km altitude).

**Atmospheric fluid motion analysis (large time-scale)**

The atmospheric fluid motion analysis was done for eight locations, situated at varying geographic and climatic regions of the Indian subcontinent: Mumbai (western-coastal plain), Delhi (semi-arid), Gangtok (mountain), Jaisalmer (arid), Chennai (eastern-coastal plain), Kolkata (gangetic-plain), Bangalore (western-ghat) and Thiruvananthapuram (southern-coastal plain). It was assumed that the average atmospheric behaviour for the entire Indian subcontinent could be captured by averaging the behaviour at these eight locations.

In order to study the atmospheric fluid motion at large time-scales, the wind velocity components were plotted for consecutive years. Figure 1 shows the u-wind values from January 1, 2002 to December 31, 2007 in Mumbai at 1000 mbar altitude which clearly shows the periodicity of atmospheric fluid motion at large time-scales with fluctuations about a mean value. After observing such periodicity for all eight locations, it was concluded that the atmospheric fluid motion has an average behaviour that is periodic over yearly time scale. This is like expressing the velocity component as:

\[
\bar{u}_j = \bar{u}_j + u'_j
\]

where \(\bar{u}_j\) is the mean velocity and \(u'_j\) is the fluctuation about that mean velocity.

**Atmospheric dispersion model to estimate AOD**

Aerosol optical depth is estimated by\textsuperscript{[2]}:

\[
\text{AOD} = \sum_{i} \alpha_i c_i h
\]

where; \(\alpha_i\) : mass extinction efficiency of \(i^{th}\) aerosol \(c_i\) : concentration of \(i^{th}\) aerosol; \(h\) : path length through the aerosol layer and summation is about all aerosols (Black Carbon, Organic Carbon, dust, sea salt and sulphate).

In order to estimate the concentration of aerosols at large time-scales (for calculating AOD), the following dispersion model with advection, emission and deposition was considered\textsuperscript{[2]}:
\[
\frac{\partial c}{\partial t} + \frac{\partial (uc)}{\partial x} + \frac{\partial (vc)}{\partial y} = \frac{(F_E - F_D)dx dy}{dx dy dz} \tag{3}
\]

where: \(c\): concentration of aerosol at location \((x, y, z)\) and time \(t\); \(u\) and \(v\): \(x\) and \(y\) components of wind velocity; \(F_E\) and \(F_D\): emission and deposition fluxes of aerosol.

In equation 3, \(z\)-direction velocity was neglected as its values are negligible compared to \(x\) and \(y\) direction velocities (as shown in TABLE 1). At large time-scales the wind velocity \(u_j\) can be written in terms of fluctuations \(u_j'\) about the mean value \(\bar{u}_j\) (as proved in equation 1):

\[
\bar{u}_j = \bar{u}_j + u_j' \tag{4}
\]

Similarly,

\[
c = \bar{c} + c' \tag{5}
\]

where \(c' = u_j' = 0\).

Substituting the above values (equation 4, 5) in equation 3 and using K theory equation (small time-scales)\(^{[2]}\):

\[
-u_j'c' = -K_{jj} \frac{\partial c}{\partial x_j} \tag{6}
\]

where; \(K_{jj}\): eddy diffusivity (small time-scales) in \(j^{th}\) direction.

With these assumptions equation 3 was simplified:

\[
\frac{\partial \bar{c}}{\partial t} + \frac{\partial (uc)}{\partial x} + \frac{\partial (vc)}{\partial y} = \frac{\partial (K_{xx} \frac{\partial \bar{c}}{\partial x})}{\partial y} + \frac{\partial (K_{yy} \frac{\partial \bar{c}}{\partial y})}{\partial x} + \frac{\partial (F_E - F_D)}{\partial z} \tag{7}
\]

where, \(K_{xx}\) and \(K_{yy}\) are the dispersion diffusivities (not eddy diffusivities) in \(x\) and \(y\) direction as the current work was done for large time-scales whereas eddy diffusivities can only be used for small time-scales. So, \(K_{xx}\) and \(K_{yy}\) have been called dispersion diffusivities (for large time-scales).

The main goal of the current work is to solve equation 7 (atmospheric dispersion model) for Arabian sea and Bay of Bengal at large time-scales in order to validate the hypothesis.

Estimation of the average wind pattern about the Indian subcontinent

In order to solve equation 7, \(\bar{u}\) and \(\bar{v}\) were estimated by finding the average wind direction and magnitude (past 10 years) for the Indian subcontinent region (Latitudinal extent: 6\(^{\circ}\)N to 37\(^{\circ}\)6\('\)N and Longitudinal extent 60\(^{\circ}\)E to 97\(^{\circ}\)25\('\)E). TABLE 1 summarizes the average wind magnitude and direction at eight selected altitudes. From the table it can be seen that \(w\) is negligible compared to \(\bar{u}\) and \(\bar{v}\), so it was neglected while finding the average wind direction.

Estimation of dispersion diffusivity
In order to solve equation 7, $K_{xx}$ and $K_{yy}$ were estimated using the velocity autocorrelation functions (explained below).

(a) Autocorrelation function

If $N$ observations of a variable (say $x$) are given, autocorrelation function is the correlation between one time series and the same series lagged by one or more time units. The first-order autocorrelation coefficient is the simple correlation coefficient of the first $N-1$ observations, $x_t, t=1, 2, \ldots, N-1$ and the next $N-1$ observations, $x_{t+k}, t=1, 2, \ldots, N$. The correlation function between $x_t$ and $x_{t+1}$ is given by:

$$r(1) = \frac{\sum_{t=1}^{N-1} (x_t - \overline{x})(x_{t+1} - \overline{x})}{N-1}$$

Above equation can be generalized to give correlation between observations separated by k time steps:

$$r(k) = \frac{\sum_{t=1}^{N-k} (x_t - \overline{x})(x_{t+k} - \overline{x})}{N-k}$$

(b) Relation between autocorrelation function and dispersion diffusivity

Dispersion diffusivity represents the area under autocorrelation function vs. time plot:

$$K_{xx} = \lim_{t \to \infty} \int_0^t r_u(t-t')dt'$$

where, $K_{xx}$ and $K_{yy}$ are the dispersion diffusivities in $x$ and $y$ directions and $r_u$ and $r_v$ are the $u$-wind and $v$-wind velocity autocorrelation functions respectively. Using equation 10, the autocorrelation functions were plotted for both $u$-wind and $v$-wind at the eight selected locations (at eight altitudes). Figure 2 shows the autocorrelation function for $u$-wind in Chennai at 1000mbar altitude. All the autocorrelation curves so obtained showed a continuous portion for a small interval of time followed by noise (as it can be seen in Figure 2). Using equation 11 and 12, the dispersion diffusivities were estimated by calculating the area under these autocorrelation curves. TABLE 2 and 3 represent the $K_{xx}$ and $K_{yy}$ values obtained for the Indian locations at varying altitudes. The last column in
TABLE 2 and 3 represents the average $K_{xx}$ and $K_{yy}$ values (of eight locations) which were assumed as the average dispersion diffusivity values for the entire Indian-subcontinent.

**Estimation of relation between dispersion diffusivity and friction-velocity (at large time-scales)**

An attempt was made to find relationship between dispersion diffusivity and friction-velocity at large time-scales as there are well-known correlations of friction-velocity with eddy diffusivity at small time-scales\(^2\).

(a) Friction-velocity $u_*$

Friction-velocity is a measure of mechanical turbulence. It has typical values ranging from 0.05 m/s in light winds to about 1 m/s in strong winds\(^7\). Friction-velocity, $u_*$ is\(^2\):

$$u_* = \left( \frac{u'^2 + v'^2 + w'^2}{3} \right)^{1/4}$$  \hspace{1cm} (13)

where; $u'$, $v'$ and $w'$ : u-wind, v-wind and w-wind fluctuation from their mean value.

The $u_*$ values were obtained by using the past 10 years wind velocity data in equation 13 where the mean value of a wind-velocity component was assumed as its average value in past 10 years. Figure 3 shows the friction velocity values for the chosen locations vs. altitude. From the figure, it was observed that $u_*$ increases with the increase in altitude.

**Relation between friction-velocity and dispersion diffusivity**

At small time-scales, eddy diffusivities are related to friction-velocity by [Appendix A]:

$$\frac{K_{xx}'}{u_*^2} \Omega \sin(lat) = \text{constant}$$  \hspace{1cm} (14)

where; $K_{xx}'$ : eddy diffusivity ($K_{xx}$ and $K_{yy}$); $\Omega$ : earth’s rotation rate; $u_*$ : friction-velocity; $\Omega$ : earth’s rotation rate;
In order to verify if the same holds for longertime scales, was plotted for different Indian loca
tions at varying altitudes as shown in Figure 4 and 5. Through Figure 4 and 5, it could be seen that no such relation holds for $K$ and $\sigma$ at large time-scales but the order of magnitude could be estimated as:

$$K_{yy} \approx 0.03$$  \hspace{1cm} (16)

$$K_{xx} \approx 0.08$$  \hspace{1cm} (15)

**RESULTS**

Results of the current work are shown in the following TABLES 1 to 3 and Figures 3 to 5:

**Appendix A**

(a) Relation between friction-velocity and eddy diffusivity

From\(^2\),

$$K_{yy} = \frac{\left(\sigma v \cdot tF_{yy}\right)^2}{2t}$$  \hspace{1cm} (17)

where; $K_{yy}$: Eddy diffusivity in $y$-direction; $z_i$: Mixing-length; $L$: Monin-Obukhov length; $u_*$: Friction-velocity; $lat$: latitude-coordinate; $\Omega$: earth’s rotation rate; $t$: time

$$\sigma_v = 1.78u_* \left[1 + 0.059 \left(-\frac{z_i}{L}\right)\right]^{1/3}$$  \hspace{1cm} (18)

**TABLE 3: $K_{yy}$ values (m$^2$/s) for eight Indian locations at varying altitudes and their average**

<table>
<thead>
<tr>
<th>Altitude (mbar)</th>
<th>Mumbai</th>
<th>Jaisalmer</th>
<th>Gangtok</th>
<th>Delhi</th>
<th>Kolkata</th>
<th>Bangalore</th>
<th>Chennai</th>
<th>Thiruvananthapuram</th>
<th>Average $K_{yy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>5.39</td>
<td>5.95</td>
<td>13.55</td>
<td>3.51</td>
<td>5.70</td>
<td>2.16</td>
<td>5.62</td>
<td>4.23</td>
<td>5.76</td>
</tr>
<tr>
<td>925</td>
<td>10.97</td>
<td>8.35</td>
<td>12.95</td>
<td>4.11</td>
<td>10.81</td>
<td>4.85</td>
<td>8.72</td>
<td>8.59</td>
<td>8.67</td>
</tr>
<tr>
<td>850</td>
<td>10.62</td>
<td>6.92</td>
<td>17.96</td>
<td>6.48</td>
<td>11.76</td>
<td>7.82</td>
<td>9.18</td>
<td>35.89</td>
<td>13.33</td>
</tr>
<tr>
<td>700</td>
<td>16.87</td>
<td>10.37</td>
<td>13.73</td>
<td>13.50</td>
<td>13.99</td>
<td>14.95</td>
<td>17.01</td>
<td>15.69</td>
<td>14.51</td>
</tr>
<tr>
<td>600</td>
<td>26.03</td>
<td>18.25</td>
<td>13.98</td>
<td>28.21</td>
<td>21.89</td>
<td>14.87</td>
<td>16.09</td>
<td>10.91</td>
<td>18.78</td>
</tr>
<tr>
<td>500</td>
<td>39.89</td>
<td>34.07</td>
<td>23.39</td>
<td>42.67</td>
<td>35.66</td>
<td>19.54</td>
<td>18.96</td>
<td>12.53</td>
<td>28.34</td>
</tr>
<tr>
<td>400</td>
<td>67.02</td>
<td>76.61</td>
<td>53.51</td>
<td>64.54</td>
<td>63.83</td>
<td>25.75</td>
<td>24.85</td>
<td>12.13</td>
<td>48.53</td>
</tr>
<tr>
<td>300</td>
<td>122.76</td>
<td>164.94</td>
<td>164.94</td>
<td>133.8</td>
<td>114.91</td>
<td>31.29</td>
<td>30.58</td>
<td>18.39</td>
<td>97.70</td>
</tr>
</tbody>
</table>
Figure 4: $K_{xx} \Omega \sin(lat)/u_0^2$ (sec) for eight Indian locations at varying altitudes (m). Observation: unlike small time-scales, $K_{xx} \Omega \sin(lat)/u_0^2$ is not constant at large time-scales but the order of magnitude was estimated as: $K_{xx} \Omega \sin(lat)/u_0^2 \sim 0.08$

Figure 5: $K_{yy} \Omega \sin(lat)/u_0^2$ (sec) for eight Indian locations at varying altitudes (m). Observation: unlike small time-scales, $K_{yy} \Omega \sin(lat)/u_0^2$ is not constant at large time-scales but the order of magnitude was estimated as: $K_{yy} \Omega \sin(lat)/u_0^2 \sim 0.03$
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\[
F_y = \left[ 1 + \left( \frac{t}{T_i} \right) \right]^{-1} \tag{19}
\]

\[
T_i^{-1} = \frac{2.5 u_*}{z_i} \left[ 1 + 0.0013 \left( \frac{-z_i}{L} \right)^{1.5} \right] \tag{20}
\]

From equation 18, 19 and 20:

\[
\sigma_y \sim u_* \tag{21}
\]

\[
F_y \sim T_i^{-1/2} \left( \frac{z_i}{u_*} \right)^{1/2} \tag{22}
\]

From\[2^3\],

\[
z_i \sim \frac{u_*}{\Omega \sin \text{lat}} \tag{23}
\]

Therefore, from equation 22 and 23:

\[
F_y \sim \left( \frac{1}{\Omega \sin \text{lat}} \right)^{1/2} \tag{24}
\]

Substituting above values in equation 17:

\[
K_{yy} \left( \frac{\Omega \sin \text{lat}}{u_*^2} \right) = \text{constant} \tag{25}
\]

Similarly it can be proved that:

\[
K_{xx} \left( \frac{\Omega \sin \text{lat}}{u_*^2} \right) = \text{constant} \tag{26}
\]

CONCLUSION

In equation 7 (aerosol dispersion model), the following quantities were estimated for the Indian subcontinent:

\( \lambda, \ \gamma \): average wind direction
\( \bar{u}, \ \bar{v} \): average wind magnitude
\( K_{xx}, K_{yy} \): dispersion diffusivities

FUTURE WORK

The deposition and emission fluxes \( F_D \) and \( F_E \) at different altitudes have to be found in order to compute. Also the mass extinction efficiency (which depends on effective radius of aerosol particles and wavelength) of aerosols needs to be found in order to find the AOD patterns in Arabian sea and Bay of Bengal.

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REFERENCES


