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Modeling of different shaped microcantilevers for sensing applications

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ABSTRACT

We present the comparison of frequency response of different shaped microcantilever beams (namely rectangular, T-shaped and V-shaped) which are commonly used for sensing applications. Performance was measured on the basis of both resonant frequency and geometrical parameters. It was found that the overall cantilever design that produced optimum sensitivity for sensing applications is the rectangular shaped microcantilever. After optimizing the shape of microcantilever we have performed the analysis based on material properties. It is observed that out of all possible candidates silicon carbide possesses highest value of resonant frequency.

KEYWORDS

Microcantilever;
Sensor.

INTRODUCTION

Micromechanical cantilever based sensors have been proved to be quite useful and highly sensitive devices .They have been reported to be used in varied applications ranging from physical, chemical to even biological fields[1-5]. The magnitude of surface-stresses involved in micro cantilever-based sensing in these applications is very small. Consequently, it is important to find out ways of enhancing the sensitivity of the micro cantilever. This can be achieved by adjusting various parameters like choice of the materials, surface properties and geometric parameters. It is desirable to have sensitive micro cantilevers made of commonly and commercially available material. Hence we need to consider factors like shape of the cantilever which greatly affects the resonant frequency and deflection produced.

In recent years, there has been a considerable increase in the study of different shapes of micro cantilevers and their applications[6-8]. Out of the various shapes reported in literature, the most popularly used shapes are: Rectangular shaped, T-shaped and V-shaped[9-11]. However each of these shapes has its own advantages and disadvantages. The growing use of V-shaped cantilever is because they are less susceptible to lateral twisting and rolling as the micro cantilever jumps into and out of contact with the surface[11]. It was shown that a T-shaped beam having a larger area at the free end could increase the oscillation amplitude and it makes the measurement of resonant frequency easier[12]. But, use of rectangular beams is more popular because of their linear behavior and simple geometry. Further studies have shown that displacement sensitivity, surface stress sensitivity, deflection under stress and resonant frequency depend greatly on the geometry of microcantilever beams[10]. Hence it is important to select the optimum geometry suiting our application.

In our study, various parameters related to geom-

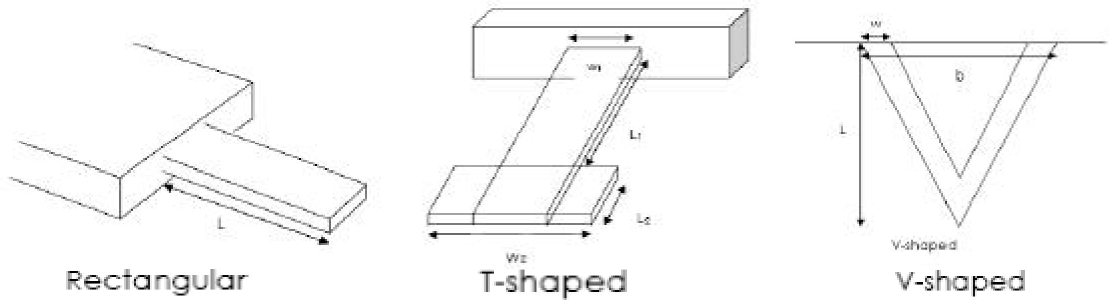

Figure1: Schematic view of each of the microcantilever shapes and the geometrical parameters used in this study

etry of the beam and beam material was varied to attain a generalized understanding of frequency response for various shapes. The dependence of frequency was studied analytically so as to compare the response of above mentioned shaped beams.

THEORY

Figure (1) shows the schematic view of each of the microcantilevers shapes and the geometrical parameters used in this study .The important design parameter is resonant frequency because this usually determines the maximum measurement bandwidth for a cantilever. The resonant frequency, *f*, for a simple cantilever can be expressed as,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m^*}} \tag{1}$$

where k denotes spring constant and m* denotes the effective mass of the cantilever. This equation shows that resonant frequency increases as a function of increasing spring constant and decreasing mass of the cantilever. The spring constant of a material is defined by *Hooke's law* as the force required for producing unit deflection. i.e.

F = -kx

where F is the force applied and x is the deflection produced due to the force.

Thus deflection produced is inversely related to spring constant and in order to produce higher deflection a lower value of spring constant is desired.

The spring constant for a rectangular shaped microcantilever, derived from Euler-Bernoulli beam theory[13] is given by

$$k = \frac{Ewh^3}{4L^3} \tag{2}$$

where E is the modulus of elasticity of the material and w is the width, h is the thickness and L is the length.

The fundamental resonant frequency for a rectangular beam is given by the following relation[13]:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{E}{\rho}\frac{h}{L^2}} \tag{3}$$

The spring constant for a T shaped microcantilever is given by[9] the following equation:

$$k = \frac{Eh^3}{4}\frac{w_1 w_2}{{l_2}^3 w_1 + {l_1}^3 w_2 + 3l_1^2 l_2 w_2 + 3l_1 l_2^2 w_2} \tag{4}$$

where l_1, w_1 and h refer to the length, width and thickness of the resonator beam respectively and l_2 and w_2 refer to the length and width of the extra mass.

The resonant frequency of the T shaped beam is given by the following relation[9]:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{Eh^3 w_1 w_2}{4(l_2^3 w_1 + l_1^3 w_2 + 3l_1^2 l_2 w_2 + 3l_1 l_2^2 w_2)(.24m_1 + m_2)}} \tag{5}$$

The above expression shows the nonlinear relation between the resonant frequency and the geometrical parameters in this case unlike the rectangular shaped resonator making the analysis considerably complex.

For the V-shaped cantilever, the simplifying assumption made is that the cantilever is equivalent to two parallel beams with the same dimensions. This "parallel beam approximation" was initially put forth by Albrecht et. al.[14] and later considered by Butt[15]. Later Sader recognized the ambiguity in defining w and L for V-shaped cantilevers and offered his own modifications to the earlier stated results, giving the final form:[16,17]:

$$k = \frac{Ewh^3}{2L^3}\cos\theta\left[1 + \frac{4w^3}{b^3}\left(3\cos\theta - 2\right)\right]^{-1} \tag{6}$$

where b is the width at the base of the "V", θ is half the angle between the two legs, w is the width of the legs measured parallel to the front edge of the substrate (not perpendicular to the leg edge) and L is the length of the cantilever measured straight out to the apex from the substrate and not parallel to the leg.

Combining the above expression with equation (1) we can get the resonant frequency for a V shaped micro cantilever. m* as mentioned in equation (1) can be expressed as the product of the volume, V and density, ρ where volume depends on the geometrical parameters and density is a material property.

Now V= surface area*thickness =area*h. (7)

The area of the V-shaped cantilever was calculated by subtracting the area of enclosed by the inner triangle from the area occupied by the outer triangle. The following expression for surface area was obtained:

$$\text{Area}=\frac{w(b-w)}{\tan\theta} \tag{8.1}$$

$$\text{Volume}=\frac{w(b-w)h}{\tan\theta} \tag{8.2}$$

$$\text{Thus; } m=\frac{w(b-w)h\rho}{\tan\theta} \tag{8.3}$$

Using the above expression, the resonant frequency for the V shaped micro cantilever.

$$f=\frac{1}{2\pi}\sqrt{\frac{Eh^2\sin\theta}{2L^3\left[1+4\frac{w^3}{b^3}(3\cos\theta-2)\right](b-w)\rho}} \tag{9}$$

RESULTS AND DISCUSSION

Figure (2) shows the variation of resonating frequency response as a function of *h/L* ratio for different shapes (rectangular shaped,V shaped and T shaped) of microcantilever. This ratio was chosen here because it completely specifies all the dimensions of the resonator on which the resonant frequency and deflection produced depend. Here we have taken silicon carbide as a material for micro cantilever beam (Young's modulus: 450GPa and density: 2800 kg/m^3) as it is widely used because of high durability and easy availability[18]. In our analysis we are keeping material parameters (and also environmental conditions like ambient pressure and temperature) constant and varying the geometrical parameters to get different h/L ratios for different shapes of microcantilevers. For each shape we have obtained different resonating frequencies for different values of h/L. Hence, for a particular value of h/L, frequency response of all the shapes could be directly compared. As suggested by the formulae, frequency response of

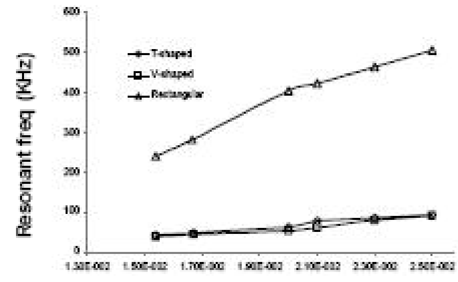

Figure 2 : Computed variation of resonant frequency as a function of h/L for different shapes of microcantilever

rectangular shaped beam varies linearly with h/L whereas V-shaped and T-shaped micro cantilevers showed non-linear relationship with h/L.

It is observed from figure (2) that the frequency response of rectangular microcantilever shows the better response as compared to that of V-shaped and T-shaped ones for the same values of h/L. Although the frequency response of rectangular cantilever is found to be better than the other shapes such as V-shape and T-shape, the other two shapes are also widely used in many applications. V-shaped cantilevers show excellent lateral stability[11] whereas T-shaped micro cantilever beams have reduced deflection offset and are useful for chemical and biomedical detection[19].

Also, from the graph it can be concluded that in the h/L ratio's range of 1.54×10^{-2} to 2×10^{-2} and 2.3×10^{-2} to 2.5×10^{-2},T-shaped and V-shaped cantilever beams gave almost equal frequency response. But in the range of 2×10^{-2} to 2.3×10^{-2}, response of T-shaped beam was better than that of V-shaped one. So this outcome can be explored to use the appropriate shape in the different h/L ranges.

Having observed that rectangular shape gives better response than V-shaped or T-shaped micro cantilever beam, we now try to find the suitable material that can be employed to optimize the result. In order to select the suitable material, using the expression for the fundamental resonating frequency for rectangular micro cantilever, the resonant frequencies for different materials namely Si, SiC, SU-8, Diamond, GaAs and Si_3N_4 were plotted against various surface area values. Each of these materials has been reported in literature to be suitable as microcantilever. Here we assume the environmental conditions like ambient temperature and pressure to be constant and air damping to be negli-

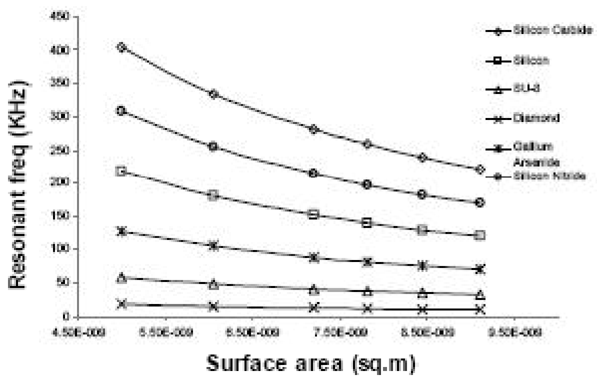

Figure 3: Computed variation of resonant frequency versus surface area of a rectangular microcantilever for different materials

TABLE 1: Material parameters used in the study

Material	Young's Modulus of Elasticity(GPa)	Poisson's Ratio	Density (kg/m^3)
SiC	450	0.15	2800
Si	110	0.24	2330
SU-8	4.02	0.22	1190
Diamond	3520	0.2	122
GaAs	85.5	0.31	5320
Si_3N_4	290	0.24	3100

gible.

Figure (3) shows the computed variation of surface area of a rectangular microcantilever for different materials. Plot depicts that for the value of surface area lying in the range 5×10^{-9} m^2 to 9.5×10^{-9} m^2 resonant frequencies for different materials noted above were found to be present in the range of 11Khz to 420 KHz. The best response was observed in the case of silicon carbide (SiC) whereas diamond displayed poor response. Similar results were obtained by plotting frequency response against surface area.

As resonant frequency is found to be in direct proportion with $\sqrt{(E/\rho)}$ and silicon carbide having the highest E/ρ ratio gives the best response. It also has the advantage of high thermal conductivity and durability because of which it can be operated under harsh operating conditions. It can also be used in both bulk and thin film forms. Although silicon nitride was observed to be the second best material, because of the cost factor it is not as widely used as SiC.

CONCLUSIONS

The frequency response of V-shaped, T-shaped and rectangular shaped microcantilever beams were studied as a function of h/ L where h represents the thickness of the beam and L represents the length of the beam. It was observed that rectangular beam gives the best frequency response for all h/L values. To further optimize the results, the frequency response of rectangular micro cantilever beam for different materials namely Si, SiC, SU-8, Diamond, Si_3N_4, GaAs was studied as a function of surface area. It was observed that out of all possible candidates silicon carbide gives the best response.

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