ISSN : 0974 - 7435 Volume 8 Issue 6

BioTechnology
An Indian Journal
FULL PAPER
BTAIJ, 8(6), 2013 [872-879]

# Modeling and analysis of the dynamic behavior of piezoelectric bimaterials containing interacting interface crack and a circular cavity

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## ABSTRACT

This study is concerned with the treatment of the dynamic behavior of piezoelectric bimaterials containing interacting interface crack and circular cavity under time-harmonic anti-plane shearing. The exact electric boundary conditions at the edge of the circular cavity and the permeable conditions at the crack surface are used to enable the treatment. The theoretical solutions of the problem are formulated using Green's function method and conjunction technique. The resulting Fredholm integral equations are solved using the direct discrete method to provide the dynamic stress and electric fields. Numerical examples are provided to show the effect of the geometry parameters, the piezoelectric constants of the material and the frequency of the incident wave upon the dynamic stress concentration. The results show the significant effect of electromechanical coupling upon local stress distribution.


## KEYWORDS

Piezoelectric bimaterials;
Interface crack;
Circular cavity;
Dynamic stress concentration factor;
Dynamic stress intensity factor;
Green's function.

## INTRODUCTION

Due to their intrinsic electro-mechanical coupling behavior, piezoelectric materials have been widely used as sensors and actuators in smart components. However, failure often occurs in piezoelectric materials because of their brittleness and presence of faults. So the investigation of the failure behaviors caused by stress concentrations has become more important.

In recent years, Ou and Chen[1] investigated near-tip stress fields and intensity factors for an interface crack in metal/piezoelectric bimaterials. Zhong and Li[2] gave a closed-form solution for two collinear cracks in a piezoelectric strip. Li and Wang[3] studied the problem of an anti-plane shear crack normal to terminating at the interface of two boned piezoelectric ceramics. Zhou and Wang[4] obtained the basic solution of two parallel non-symmetric permeable cracks in piezoelectric materials. Wang[5] et al investigated the scattering of anti-plane shear wave by a piezoelectric circular cylinder with an imperfect interface. Wang and Gao[6] solved the stress intensity factor of the mode III cracks originating from the edge of a circular hole in a piezoelectric solid. It should be noted that most of the above documents are static, the dynamic cases have not been reported too many.

The objective of the present study is to provide a theoretical treatment of the dynamic interaction between the circular cavity and the interface crack in piezoelectric bimaterials under time-harmonic anti-plane shearing. The exact electric boundary conditions at the edge of the circular cavity and the permeable conditions at the crack surface are used in present paper. The dynamic electromechanical behavior is studied using Green's function method and conjunction technique. The present problem is reduced into the solutions for Fredholm integral equations of the first kind, which are solved by the direct discrete method. Numerical examples are provided to show the effect of the geometry parameters, the piezoelectric constants and the frequency of the incident wave upon the dynamic stress concentration.

## FORMULATION OF THE PROBLEM

Consider the problem of two bonded infinite piezoelectric materials containing a circular cavity near the interface and a crack subjected to a harmonic incident wave of frequency É with an incident angle $\pm_0$, as shown in Figure 1. Suppose the piezoelectric medium has been poled along the *z*-axis. *R* is the radius of the circular cavity in medium I. And *h* represents the distance of the cavity from the interface. The length of the interface crack is 2*A* along the *x* axis.

The steady-state mechanical and electrical fields corresponding to this incident wave will generally involve an exponential harmonic factor $e^{-iÉt}$. For the sake of convenience, this factor will be suppressed.

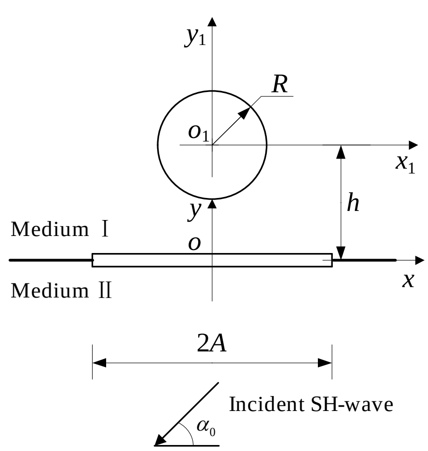


**Figure 1 : Interacting interface crack and a circular cavity in a piezoelectric bimaterials**

In the absence of body forces and free charges, the equilibrium equations of linear piezoelectric medium for a time-harmonic anti-plane shearing problem are given as [7]

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi + \rho\omega^2 w = 0,\quad e_{15}\nabla^2 w - \kappa_{11}\nabla^2\phi = 0 \qquad \textbf{(1)}$$

Where $\nabla^2$ stands for $\partial^2/\partial x^2 + \partial^2/\partial y^2$, and $w$, $\phi$, $\rho$ are anti-plane displacement, electric potential, mass density of the medium. While $c_{44}$, $e_{15}$, $\kappa_{11}$ are the elastic modulus, the piezoelectric constant and dielectric constant of the medium, respectively. Equation (1) can be simplified further

$$\nabla^2 w + k^2 w = 0,\quad \nabla^2\varphi = 0,\quad \phi = \frac{e_{15}}{\kappa_{11}}w + \varphi \qquad \textbf{(2)}$$

where the wave number $k$ is defined by $k^2 = \dfrac{\rho\omega^2}{c^*}$ with

$c^* = c_{44} + \dfrac{{e_{15}}^2}{\kappa_{11}} = c_{44}(1+\lambda)$ and $\lambda = \dfrac{{e_{15}}^2}{c_{44}\kappa_{11}}$.

By introducing a complex variable $\eta = x + iy = re^{i\theta}$ and its conjugate $\bar{\eta} = x - iy = re^{-i\theta}$, the equation (2) can be rewritten as

$$\frac{\partial^2 w}{\partial\eta\partial\bar{\eta}} + \frac{1}{4}k^2 w = 0,\quad \frac{\partial^2\varphi}{\partial\eta\partial\bar{\eta}} = 0,\quad \phi = \frac{e_{15}}{\kappa_{11}}w + \varphi \qquad \textbf{(3)}$$

While the constitutive relations can be rewritten as

$$\begin{aligned}
\tau_{rz} &= c_{44}\left(\frac{\partial w}{\partial\eta}e^{i\theta} + \frac{\partial w}{\partial\bar{\eta}}e^{-i\theta}\right) + e_{15}\left(\frac{\partial\phi}{\partial\eta}e^{i\theta} + \frac{\partial\phi}{\partial\bar{\eta}}e^{-i\theta}\right),\\
\tau_{\theta z} &= ic_{44}\left(\frac{\partial w}{\partial\eta}e^{i\theta} - \frac{\partial w}{\partial\bar{\eta}}e^{-i\theta}\right) + ie_{15}\left(\frac{\partial\phi}{\partial\eta}e^{i\theta} - \frac{\partial\phi}{\partial\bar{\eta}}e^{-i\theta}\right),\\
D_r &= e_{15}\left(\frac{\partial w}{\partial\eta}e^{i\theta} + \frac{\partial w}{\partial\bar{\eta}}e^{-i\theta}\right) - \kappa_{11}\left(\frac{\partial\phi}{\partial\eta}e^{i\theta} + \frac{\partial\phi}{\partial\bar{\eta}}e^{-i\theta}\right), \qquad \textbf{(4)}\\
D_\theta &= ie_{15}\left(\frac{\partial w}{\partial\eta}e^{i\theta} - \frac{\partial w}{\partial\bar{\eta}}e^{-i\theta}\right) - i\kappa_{11}\left(\frac{\partial\phi}{\partial\eta}e^{i\theta} - \frac{\partial\phi}{\partial\bar{\eta}}e^{-i\theta}\right).
\end{aligned}$$

In which $\tau_{rz}$ and $\tau_{\theta z}$ are anti-plane shear stress components, $D_r$ and $D_\theta$ are in-plane electric displacement components, respectively.

The boundary conditions of the present problem can be written as

$$\begin{cases} D_\theta^{\mathrm{I}}(\eta,\bar\eta)=D_\theta^{\mathrm{II}}(\eta,\bar\eta), \ \tau_{\theta z}^{\mathrm{I}}(\eta,\bar\eta)=0 \\ \tau_{\theta z}^{\mathrm{II}}(\eta,\bar\eta)=0, \ \phi^{\mathrm{I}}(\eta,\bar\eta)=\phi^{\mathrm{II}}(\eta,\bar\eta) \end{cases} \tag{5}$$

at $\eta = x, -A \le x \le A$

$$\begin{cases} \tau_{rz}^{\mathrm{I}}(\eta_1,\bar\eta_1)=0 \\ \phi^{\mathrm{I}}(\eta_1,\bar\eta_1)=\phi^{c}(\eta_1,\bar\eta_1) \ , \qquad |\eta_1|=R \\ D_r^{\mathrm{I}}(\eta_1,\bar\eta_1)=D_r^{c}(\eta_1,\bar\eta_1) \end{cases} \tag{6}$$

The superscript I, II and $c$ refer to the variable in the medium I, medium II and the cavity respectively.

## SOLUTIONS OF GREEN'S FUNCTION

The displacement Green's function and the electric potential Green's function in medium I are the solutions for an isotropic piezoelectric half space with a circular cavity impacted by an out-plane harmonic line source loading at the horizontal surface.

The fundamental solutions, which satisfy the governing equation (3) and the boundary condition equations (6), can be decomposed into two parts: one is the disturbance impacted by a line source loading and the other is the scattered wave excited by the circular cavity. The first part can be expressed as [8]

$$G_w^{(i)} = \frac{i}{2c_{44}^{\mathrm{I}}(1+\lambda_{\mathrm{I}})} H_0^{(1)}(k_{\mathrm{I}}|\eta-\eta_0|), \quad G_\phi^{(i)} = \frac{e_{15}^{\mathrm{I}}}{\kappa_{11}^{\mathrm{I}}} G_w^{(i)} \tag{7}$$

Where $H_0^{(1)}(*)$ expresses the zero-order Hankel function of the first kind. And the second part can be written as

$$G_w^{(s)} = \sum_{n=-\infty}^{\infty} A_n[\chi_n^{(1)} + \chi_n^{(2)}]$$
$$G_\phi^{(s)} = \frac{e_{15}^{\mathrm{I}}}{\kappa_{11}^{\mathrm{I}}} G_w^{(s)} + \sum_{n=1}^{\infty}[B_n\chi_n^{(3)} + C_n\chi_n^{(4)}] \tag{8}$$

Where,

$$\begin{cases} \chi_n^{(1)} = H_n^{(1)}(k_{\mathrm{I}}|\eta|)[\eta/|\eta|]^n \\ \chi_n^{(2)} = H_n^{(1)}(k_{\mathrm{I}}|\eta+2ih|)[(\eta+2ih)/|\eta+2ih|]^{-n} \\ \chi_n^{(3)} = \eta^{-n} + (\bar\eta-2ih)^{-n} \\ \chi_n^{(4)} = \bar\eta^{-n} + (\eta+2ih)^{-n} \end{cases}$$

So the expressions of the Green's functions in medium I can be written as

$$G_w^{\mathrm{I}} = G_w^{(i)} + G_w^{(s)}, \quad G_\phi^{\mathrm{I}} = G_\phi^{(i)} + G_\phi^{(s)} \tag{9}$$

The cavity is assumed to be vacuum or filled with homogeneous gas of dielectric constant $\kappa_0$, and free of forces and surface charges [9]. Only electric field exists in the cavity. The infinite expression of the electric potential which is satisfied the Laplace equation inside the cavity should be

$$G_\phi^{c} = D_0 + \sum_{n=1}^{\infty}[D_n\eta^n + E_n\bar\eta^n] \tag{10}$$

The Unknown Coefficients $D_n$ and $E_n$ can be obtained by boundary condition equations (6).

The Green's functions of medium II are the fundamental solutions for a half space impacted by an out-plane harmonic line source loading at the horizontal surface. They can be expressed as

$$G_w^{\mathrm{II}} = \frac{i}{2c_{44}^{\mathrm{II}}(1+\lambda_{\mathrm{II}})} H_0^{(1)}(k_{\mathrm{II}}|\eta-\eta_0|), \quad G_\phi^{\mathrm{II}} = \frac{e_{15}^{\mathrm{II}}}{\kappa_{11}^{\mathrm{II}}} G_w^{\mathrm{II}} \tag{11}$$

## SOLUTIONS OF INTERACTING BETWEEN THE CAVITY AND THE INTERFACE CRACK

Consider the incident wave is a harmonic wave which is directed at an angle $\pm_0$ with the interface. The incident, reflected and transmitted waves can be generally expressed as [10]

$$w^{(i)} = w_0 \exp\left\{-i\frac{k_{\mathrm{I}}}{2}\left[\eta e^{-i\alpha_0} + \bar\eta e^{i\alpha_0}\right]\right\}$$
$$\phi^{(i)} = \phi_0 \exp\left\{-i\frac{k_{\mathrm{I}}}{2}\left[\eta e^{-i\alpha_0} + \bar\eta e^{i\alpha_0}\right]\right\} \tag{12}$$

$$w^{(r)} = w_1 \exp\left\{-i\frac{k_{\mathrm{I}}}{2}\left[\eta e^{i\alpha_0} + \bar\eta e^{-i\alpha_0}\right]\right\}$$
$$\phi^{(r)} = \phi_1 \exp\left\{-i\frac{k_{\mathrm{I}}}{2}\left[\eta e^{i\alpha_0} + \bar\eta e^{-i\alpha_0}\right]\right\} \tag{13}$$

$$w^{(t)} = w_2 \exp\left\{-i\frac{k_{\mathrm{II}}}{2}\left[\eta e^{-i\alpha_2} + \bar\eta e^{i\alpha_2}\right]\right\}$$
$$\phi^{(t)} = \phi_2 \exp\left\{-i\frac{k_{\mathrm{II}}}{2}\left[\eta e^{-i\alpha_2} + \bar\eta e^{i\alpha_2}\right]\right\} \tag{14}$$

The detailed expressions of $w_j$ and $\phi_j$ (j=0,1,2), which are the amplitudes of these waves, are fully described in Wang's work[10].

Now, the incident wave and the reflected wave are applied to the half space of medium I which contains the circular cavity. To satisfy the traction free and the impermeable conditions at the surface, the scattered anti-plane displacement and electric potential in the medium I can be expressed as

$$w^{(\mathrm{Is})}=\sum_{n=-\infty}^{\infty}F_n[\chi_n^{(1)}+\chi_n^{(2)}]$$
$$\phi^{(\mathrm{Is})}=\frac{e_{15}^{\mathrm{I}}}{\kappa_{11}^{\mathrm{I}}}w^{(s)}+\sum_{n=1}^{\infty}[P_n\chi_n^{(3)}+Q_n\chi_n^{(4)}] \quad \textbf{(15)}$$

The infinite expression of electric potential inside the cavity should be expressed as

$$\phi^{(\mathrm{I}c)}=S_0+\sum_{n=1}^{\infty}[S_n\eta^n+T_n\bar{\eta}^n] \quad \textbf{(16)}$$

The value of the Unknown Coefficients $S_0$, $S_n$ and $T_n$ can be obtained by the boundary conditions (6).

Similarly, the transmitted waves are applied to the half space of medium II. The scattered waves do not exist because there is no defect in medium II.

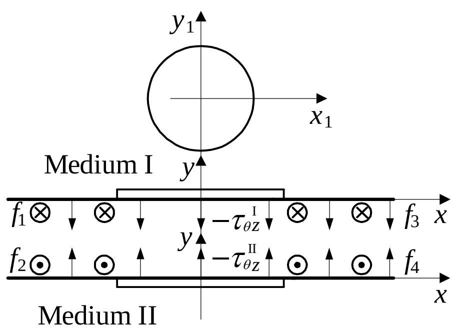


**Figure 2 : Conjunction of two semi-infinite piezoelectric media**

Based on the obtained Green's functions, the anti-plane displacements and the electric potentials in the half spaces of medium I and II, the interacting solutions of the circular cavity and the interface crack in piezo-electric bimaterials can be constructed by using the conjunction technique, as shown in Figure 3.

The construction process as follows. Firstly, the piezoelectric bimaterials are divided into two parts: the upper half space of medium I and the lower half space of medium II along the interface at y=0. The total anti-plane displacement $w^{\mathrm{I}}$, the total electric potential field $\phi^{\mathrm{I}}$, the total shear stress $\tau_{\theta z}^{\mathrm{I}}$ and the total electric displacement $D_{\theta}^{\mathrm{I}}$ on the surface of medium I can be respectively expressed as

$$w^{\mathrm{I}}=w^{(i)}+w^{(r)}+w^{(\mathrm{Is})},\phi^{\mathrm{I}}=\phi^{(i)}+\phi^{(r)}+\phi^{(\mathrm{Is})} \quad \textbf{(17)}$$

$$\tau_{\theta z}^{\mathrm{I}}=\tau_{\theta z}^{(i)}+\tau_{\theta z}^{(r)}+\tau_{\theta z}^{(\mathrm{Is})},D_{\theta}^{\mathrm{I}}=D_{\theta}^{(i)}+D_{\theta}^{(r)}+D_{\theta}^{(\mathrm{Is})} \quad \textbf{(18)}$$

And $w^{\mathrm{II}}$, $\phi^{\mathrm{II}}$, $\tau_{\theta z}^{\mathrm{II}}$, $D_{\theta}^{\mathrm{II}}$ on the surface of medium a! can be written respectively as

$$w^{\mathrm{II}}=w^{(t)},\phi^{\mathrm{II}}=\phi^{(t)},\tau_{\theta z}^{\mathrm{II}}=\tau_{\theta z}^{(t)},D_{\theta}^{\mathrm{II}}=D_{\theta}^{(t)} \quad \textbf{(19)}$$

Secondly, a pair of opposite sheares $-\tau_{\theta z}^{\mathrm{I}}$ and $-\tau_{\theta z}^{\mathrm{II}}$ are upper and lower surfaces at n, respectively. And additional shear stresses $f_1(r_0,\theta_0)$, $f_2(r_0,\theta_0)$ and additional electric displacement $f_3(r_0,\theta_0)$, $f_4(r_0,\theta_0)$ are also applied on the surfaces respectively, so as to meet the continuity conditions of the surface *y*=0 while conjunction of two parts, as shown in Figure 3. This can create the traction free and electrically permeable crack.

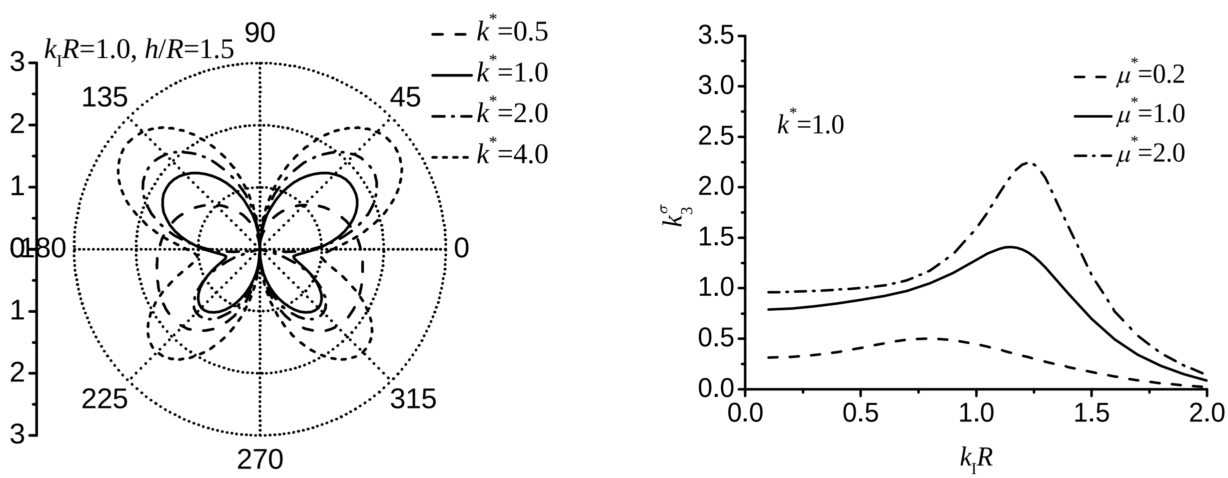


**Figure 3 : The results in elastic bimaterials**

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By employing the following continuity conditions

$$\tau_{\theta z}^{\mathrm{I}}\cos\theta_0 + f_1(r_0,\theta_0) = \tau_{\theta z}^{\mathrm{II}}\cos\theta_0 + f_2(r_0,\theta_0) \quad \text{at}(r_0 > A, \theta_0 = 0,\pi) \tag{20}$$

$$w^{\mathrm{I}}(r,\theta) + w^{(f_1)}(r,\theta) + w^{(c_1)}(r,\theta) = w^{\mathrm{II}}(r,\theta) + w^{(f_2)}(r,\theta) + w^{(c_2)}(r,\theta) \quad \text{at}(r > A, \theta = 0,\pi) \tag{21}$$

$$D_\theta^{\mathrm{I}}\cos\theta_0 + f_3(r_0,\theta_0) = D_\theta^{\mathrm{II}}\cos\theta_0 + f_4(r_0,\theta_0) \tag{22}$$

$$\phi^{\mathrm{I}}(r,\theta) + \phi^{(f_3)}(r,\theta) = \phi^{\mathrm{II}}(r,\theta) + \phi^{(f_4)}(r,\theta) \tag{23}$$

Where,

$$w^{(f_1)} = \int_A^\infty f_1(r_0,\pi)G_w^{\mathrm{I}}(r,\theta;r_0,\pi)dr_0 + \int_A^\infty f_1(r_0,0)G_w^{\mathrm{I}}(r,\theta;r_0,0)dr_0$$

$$w^{(f_2)} = -\int_A^\infty f_2(r_0,\pi)G_w^{\mathrm{II}}(r,\theta;r_0,\pi)dr_0 - \int_A^\infty f_2(r_0,0)G_w^{\mathrm{II}}(r,\theta;r_0,0)dr_0$$

$$w^{(c_1)} = \int_0^A \tau_{\theta z}^{\mathrm{I}}(r_0,\pi)G_w^{\mathrm{I}}(r,\theta;r_0,\pi)dr_0 - \int_0^A \tau_{\theta z}^{\mathrm{I}}(r_0,0)G_w^{\mathrm{I}}(r,\theta;r_0,0)dr_0$$

$$w^{(c_2)} = -\int_0^A \tau_{\theta z}^{\mathrm{II}}(r_0,\pi)G_w^{\mathrm{II}}(r,\theta;r_0,\pi)dr_0 + \int_0^A \tau_{\theta z}^{\mathrm{II}}(r_0,0)G_w^{\mathrm{II}}(r,\theta;r_0,0)dr_0$$

$$\phi^{(f_3)} = \int_A^\infty f_3(r_0,\pi)G_\phi^{\mathrm{I}}(r,\theta;r_0,\pi)dr_0 + \int_A^\infty f_3(r_0,0)G_\phi^{\mathrm{I}}(r,\theta;r_0,0)dr_0$$

$$\phi^{(f_4)} = -\int_A^\infty f_4(r_0,\pi)G_\phi^{\mathrm{II}}(r,\theta;r_0,\pi)dr_0 - \int_A^\infty f_4(r_0,0)G_\phi^{\mathrm{II}}(r,\theta;r_0,0)dr_0$$

Unknown functions $f_1(r_0,\theta_0)$, $f_2(r_0,\theta_0)$, $f_3(r_0,\theta_0)$, $f_4(r_0,\theta_0)$ can be obtained by solving the following integral equations with kernels of Green's functions

$$\begin{aligned}&\int_0^\infty f_1(r_0,\pi)[G_w^{\mathrm{I}}(r,\theta;r_0,\pi) + G_w^{\mathrm{II}}(r,\theta;r_0,\pi)]dr_0 \\ &+\int_0^\infty f_1(r_0,0)[G_w^{\mathrm{I}}(r,\theta;r_0,0) + G_w^{\mathrm{II}}(r,\theta;r_0,0)]dr_0 \\ &= -w^{(s)}(r,\theta) - \int_0^A \tau_{\theta z}^{\mathrm{I}}(r_0,\pi)G_w^{\mathrm{I}}(r,\theta;r_0,\pi)dr_0 \quad (\theta = 0,\pi) \\ &+\int_0^A \tau_{\theta z}^{\mathrm{I}}(r_0,0)G_w^{\mathrm{I}}(r,\theta;r_0,0)dr_0 \\ &-\int_0^A \tau_{\theta z}^{\mathrm{II}}(r_0,\pi)G_w^{\mathrm{II}}(r,\theta;r_0,\pi)dr_0 \\ &+\int_0^A \tau_{\theta z}^{\mathrm{II}}(r_0,0)G_w^{\mathrm{II}}(r,\theta;r_0,0)dr_0\end{aligned} \tag{24}$$

$$\begin{aligned}&\int_0^\infty f_3(r_0,\pi)[G_\phi^{\mathrm{I}}(r,\theta;r_0,\pi) + G_\phi^{\mathrm{II}}(r,\theta;r_0,\pi)]dr_0 \\ &+\int_0^\infty f_3(r_0,0)[G_\phi^{\mathrm{I}}(r,\theta;r_0,0) + G_\phi^{\mathrm{II}}(r,\theta;r_0,0)]dr_0 \\ &= -\phi^{(s)}(r,\theta), \quad \theta = 0,\pi\end{aligned} \tag{25}$$

The shear stress at the circular cavity's edge can be expressed as

$$\begin{aligned}\tau_{\theta z} = \tau_{\theta z}^{\mathrm{I}} &+ c_{44}^{\mathrm{I}} i\int_0^\infty f_1(\eta_0)\left(\frac{\partial G_w^{\mathrm{I}}}{\partial\eta}e^{i\theta} - \frac{\partial G_w^{\mathrm{I}}}{\partial\bar{\eta}}e^{-i\theta}\right)d|\eta_0| \\ &+ e_{15}^{\mathrm{I}} i\int_0^\infty f_3(\eta_0)\left(\frac{\partial G_\phi^{\mathrm{I}}}{\partial\eta}e^{i\theta} - \frac{\partial G_\phi^{\mathrm{I}}}{\partial\bar{\eta}}e^{-i\theta}\right)d|\eta_0|\end{aligned} \tag{26}$$

The dynamic stress concentration factor (DSCF) $\tau^*$ is defined by

$$\tau^* = \left|\frac{\tau_{\theta z}|_{\eta = Re^{i\theta}}}{\tau_0}\right| \tag{27}$$

Where $\tau_0 = -ik_{\mathrm{I}}(c_{44}^{\mathrm{I}}w_0 + e_{15}^{\mathrm{I}}\phi_0)$ stands for the amplitude of the shear stress corresponding to the incident wave.

The dynamic stress intensity factor at the crack's tip can be introduced as follows

$$k_{\mathrm{III}} = \lim_{r_0 \to A} f_1(r_0,\pi)\sqrt{2(r_0 - A)} \tag{28}$$

A dimensionless dynamic stress intensity factor (DSIF) $k_3^\sigma$ in the application is defined as

$$k_3^\sigma = \left|\frac{k_{\mathrm{III}}}{\tau_0 Q}\right| \tag{29}$$

Where $Q = \sqrt{A}$ refer to the characteristic dimension of the crack.

## NUMERICAL EXAMPLES

As examples, some of the calculating results for DSCFs and DSIFs are plotted from Figure 4 up to Figure 10 based on formula (27) and formula (29). It should be recognized that the effect of some parameters will be expressed by using the following dimensionless components

$k^* = k_{\mathrm{II}}/k_{\mathrm{I}}$ , $\mu^* = c_{44}^{\mathrm{II}}/c_{44}^{\mathrm{I}}$ , $\lambda^* = \lambda_{\mathrm{I}}/\lambda_{\mathrm{II}}$

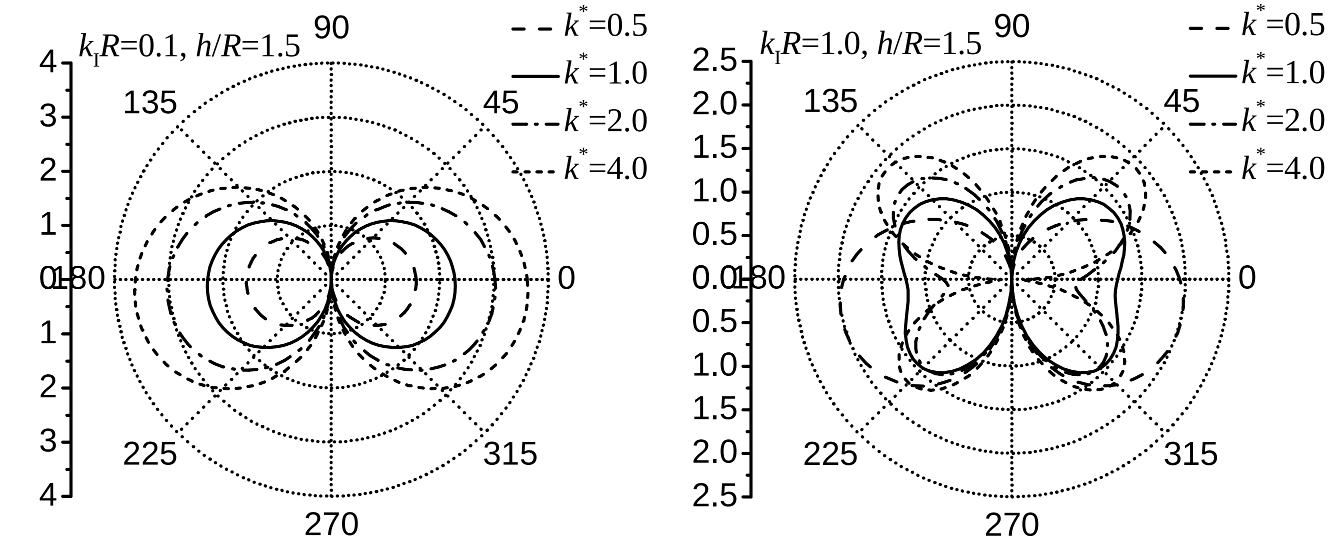


**Figure 4 : Variation of DSCF vs. density**

It can be seen from Figure 3 that the results of the present paper coincide well to document [11] while the two dissimilar piezoelectric media is reduced to the elastic bimaterials.

Figure 4 and Figure 5 show variations of DSCFs at the edge of the circular cavity with material constants under vertical incidence, respectively. Figure 5 shows that DSCFs increase with the increment of $k^*$, and the locations for maximum value are obviously different according to $k^*$. And Figure 6 implies that the influence of the material mismatch on DSCFs is more significant at higher incident frequencies.

Figure 6 and Figure 7 show variations of DSCFs with the geometry parameters under vertical incidence, respectively. Figure 6 shows that DSCFs increase with the increment of $h/R$ at lower frequencies, but the phenomenon is not exist at higher frequencies. Figure 7 implies that DSCFs decrease approximately with the increment of $A/R$. The maximum value of DSCF is obtained at $A/R$=2.0.

Figure 8-10 exhibits the variation of DSIFs against the materials' constants, geometric parameters and the frequencies of incident wave under vertical incidence, respectively. Figure 8 displays that DSIF attains its maximum in the region of $k_IR$=1.15-1.25 for different $\lambda^*$. And the DSIFs increase with the increment of $\lambda^*$ when $k_IR$<1.2. The mismatch of the two materials will de-

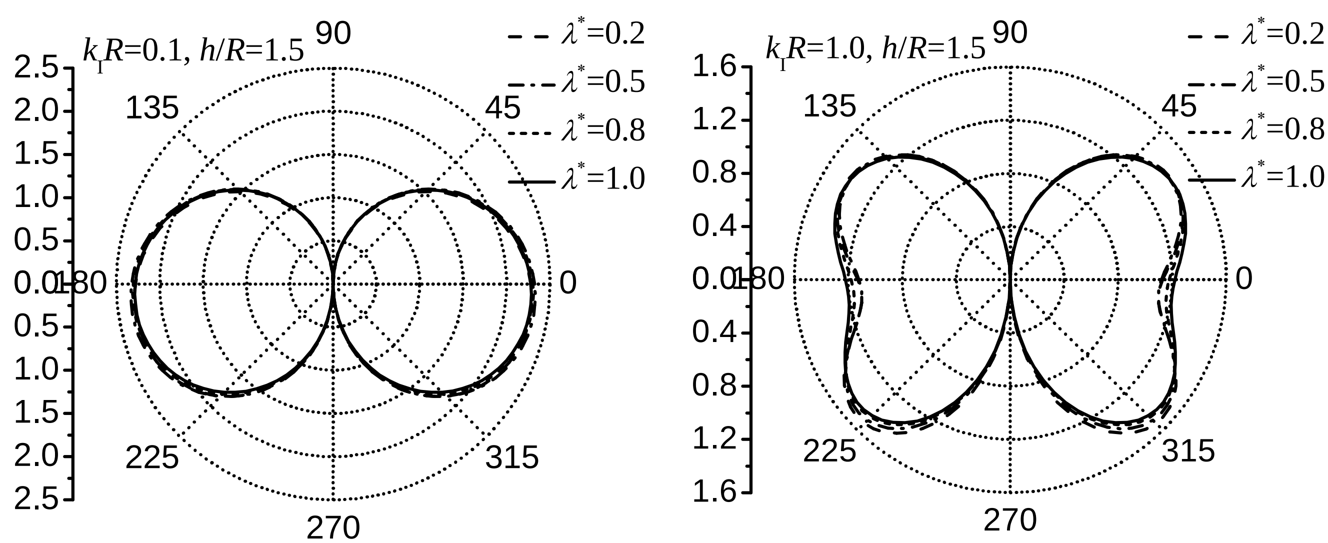


**Figure 5 : Variation of DSCF vs. $\lambda^*$**

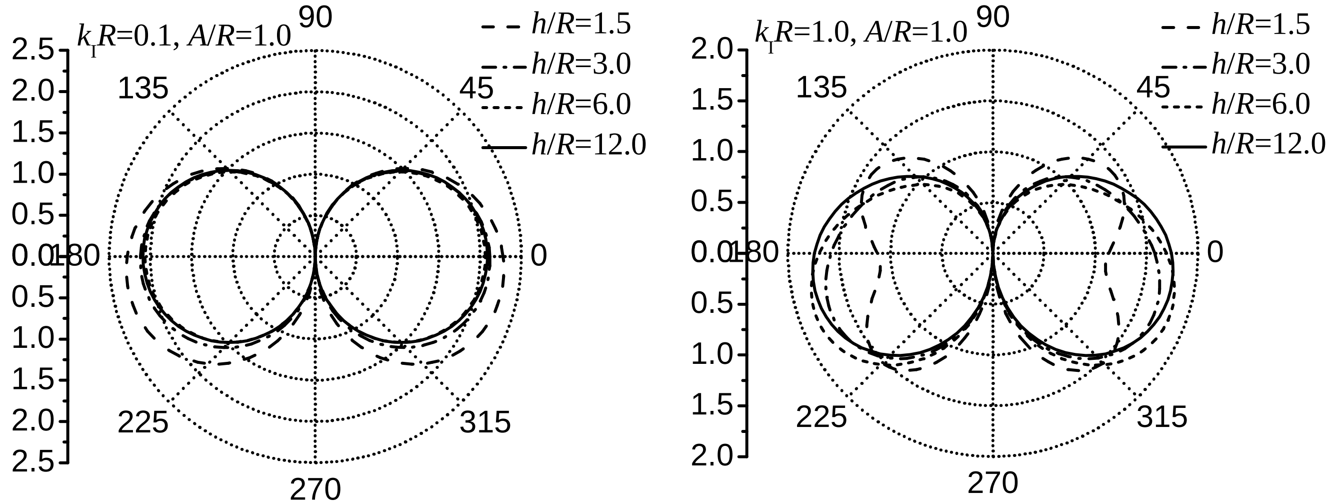


**Figure 6 : Variation of DSCF vs. *h*/*R***

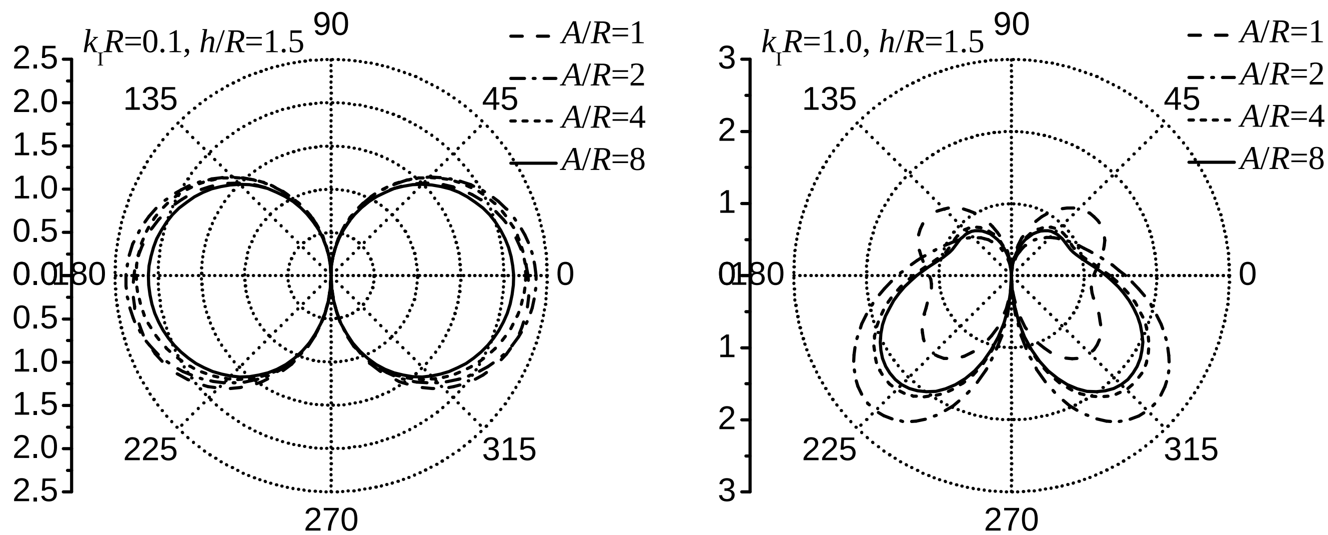


**Figure 7 : Variation of DSCF vs. *A*/*R***

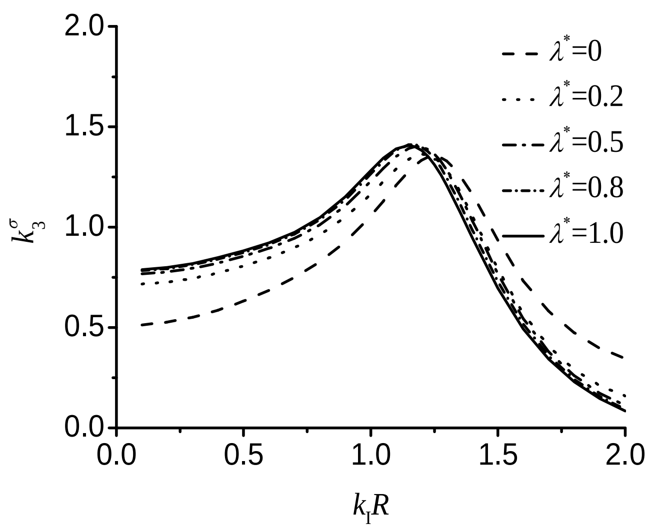


**Figure 8 : Variation of DSIF vs. wave number and $\lambda^*$**

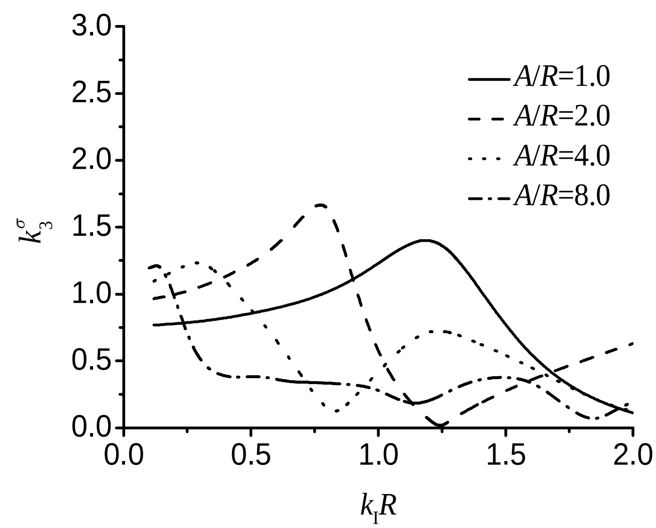


**Figure 9 : Variation of DSIF vs. wave number and *A*/*R***

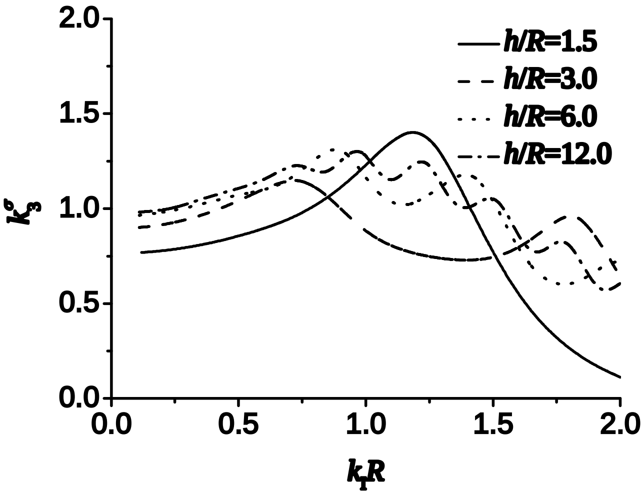


**Figure 10 : Variation of DSIF vs. wave number and *h/R***

crease the sensitivity of the stress intensity factor at lower frequencies. Figure 9 shows that the peak values of DSIFs always appear at lower frequencies which decrease with the increment of *A/R*. Figure 10 shows that oscillation phenomenon of DSIFs is more significant at higher incident frequencies. The biggest peak value is obtained when *h/R*=1.5, indicating the complicated scattering of elastic waves due to the interaction between the circular cavity and the interface crack.

## CONCLUSIONS

A general solution is provided to the dynamic interaction between a circular cavity and the interface crack in piezoelectric bimaterials under time-harmonic antiplane shearing. The analysis is based on Green's function and conjunction technique. The effect of the material constants, the geometry parameters and the frequency of the incident wave upon the dynamic stress concentration factor and the dynamic stress intensity factor is examined and discussed in the present paper. Dynamic analyses in piezoelectric bimaterials are more important than those on homogenous piezoelectric medium, because the former may have larger dynamic stress intensity factor. While the oscillating phenomena of dynamic stress intensity factors should also be paid attention, especially in high-frequency situation. The results reveal that the material mismatch isn't invariably increasing the failure possibility. The stress concentration at the edge of the cavity will decrease if the appropriate parameters are chosen.

## ACKNOWLEDGEMENT

This work was supported by the National Science Fund for Young Scholars of Hebei Education Deparment (Q2012031).

## REFERENCES

[1] Z.C.Ou, Y.H.Chen; Near-tip stress fields and intensity factors for an interface crack in metal/piezoelectric bimaterials; Int. J. Engineering Science, **42**, 1407-1438 **(2004)**.

[2] X.C.Zhong, X.F.Li; Closed-form solution for two collinear cracks in a piezoelectric strip; Mechanics Research Communications, **32**, 401-410 **(2005)**.

[3] X.F.Li, B.L.Wang; Anti-plane shear crack normal to terminating at the interface of two boned piezoelectric ceramics; Int. J. Solids and Structures, **44**, 3796-3810 **(2007)**.

[4] Z.G.Zhou, B.Wang; Basic solution of two parallel non-symmetric permeable cracks in piezoelectric materials; Applied Mathematics and Mechanics, **28(4)**, 417-428 **(2007)**.

[5] W.Y.Tian, K.T.Chau; Arbitrarily oriented crack near interface in piezoelectric bimaterials; Int.J.Solids and Structures, **40**, 1943-1958 **(2003)**.

[6] Y.J.Wang., C.F.Gao; The mode III cracks originating from the edge of a circular hole in a piezoelectric solid; Int.J.Solids and Structures, **45**, 4590-4599 **(2008)**.

[7] S.A.Meguid, X.D.Wang; Dynamic antiplane behaviour of interacting cracks in a piezoelectric medium; International Journal of Fracture, **91**, 391-403 **(1998)**.

[8] D.K.Liu, H.Lin; Scattering of SH-waves by an interacting interface linear crack and a circular cavity near bimaterial interface; Acta Mechanica Sinica, **20(3)**, 317-326 **(2004)**.

[9] H.Sosa, N.Khutoryansky; New developments concerning piezoelectric materials with defects; International Journal of Solids and Structures, **33(23)**, 3399-3414 **(1996)**.

[10] X.D.Wang; On the dynamic behaviour of interacting interfacial cracks in piezoelectric media; International Journal of Solids and Structures, **38**, 815-831 **(2001)**.

[11] D.K.Liu, H.Lin; Scattering of SH-waves by an interacting interface linear crack and a circular cavity near bimaterial interface; Acta Mechanica Sinica, **20(3)**, 317-326 **(2004)**.