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Misunderstandings of Galileos Law-From Galileo to Einstein

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Abstract

The approximative only nature of Galileo’s “law” has already been evidenced sporadically in the past but its status of “universal law” continues unchallenged in scientific textbooks. The traded misinterpretation of Galileo’s “law” of Free Fall is based on the overseeing that the translational acceleration measured in the Galilean setup (referred to the earth centre) is in fact the vector sum of two Newtonian accelerations, namely that of the falling test body mass m and that of the earth mass M towards the common centre of the participating masses. It can be shown that it is a composite acceleration incremented with respect to the Newtonian acceleration by the factor (M+m)/M correcting the distortion from the “simplest form” caused by the offset of the reference frame. Galileo’s law is therefore only approximately true for terrestrial fall situations with m/M typically in the order of 10-24. Several causes are discussed in the current paper, which has contributed to the missed review of Galileo’s “law”, persisting up to day.

Keywords: Magnetic field; London penetration depth; Josephson penetration depth; Sine-gordon equation; Parameters

Introduction

The approximative only nature of Galileo’s “law” has already been evidenced sporadically in the past but its status of “universal law” continues unchallenged in scientific text books. Several causes have contributed to the missed review of Galileo’s “law”, persisting up today [1].

The first of them is the fact that Galileo postulated his law of Free Fall many years before his greater scientific discovery, the Principle of Galilean invariance alias Galilean Relativity. Moreover, he was not

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aware of the mutuality of gravitational interaction. But when Newton published his gravitational laws almost a century later, nobody noticed that Galileo's law had to be revised in view of those findings.

Another cause contributing the missed review of Galileo's law is Euler's re-formulation of Newton's 2nd law of 1752 ("Force=Mass Times Translational Acceleration"), where he omitted the boundary condition "related to the common mass centre" inducing the fallacy, that the myriad of possible mass values and their complementary acceleration values to deliver one given force value, are directly comparable to each other. However, only that mass value obtained in (inertial frames related to) the common mass centre is directly comparable with another mass and it is comparable only with such one.

The purpose of the present paper is to clarify the approximative only nature of Galileo's "law" by considering Newton's laws and Newton's Gravitation Formula in a holistic interpretation at the first place. We shall therefore start with a review of Newton's laws and Newton's Gravitation Formula in a holistic interpretation. Based on this holistic interpretation we shall review Galileos "law" of Free Fall in order to make evident its approximative only nature in. This allows us to clarify the causes having contributed to the missed review so far in before we give our conclusions and final remarks.

Materials and Methods

Used symbols and definitions

- We shall use following conventions for denoting the central notions of the current paper
- Capital M refers to the earth mass. Test body masses are denoted with lowercase m
- Since Galileo and Newton have focused only translational movements, the term acceleration used without attribute refers only to translational acceleration
- C_{Mm} defines the common mass centre of M and m.
- C_M defines the mass centre of M.
- C_m defines the mass centre of m.
- amM defines the translational acceleration of m caused by the interaction with M.
- Apostrophized values of acceleration, e.g. $a^j mM$ refer to acceleration values measured in the frame of reference C_M of the Galilean fall experiment.
- F_{Mm} defines the force on M by interaction with m.

- Newton mass refers to the mass value valid in the inertial frame of reference C_{Mm} related to the common mass centre of M and m. It is symbolized without any index.

- Galilean mass refers to the mass value obtained by dividing the interaction force by the translational acceleration measured in the frame of reference C_M of the Galilean fall experiment. The Galilean mass of a body falling to Earth is denoted by the indexed symbol m_i.

- The attribute *natural* refers to the best possible mapping as viable only with the *preferred* reference system, which is that related to the common mass centre C_{Mm}. The attributes composite or apparent refer to a mapping related to C_M **FIG. 1.**

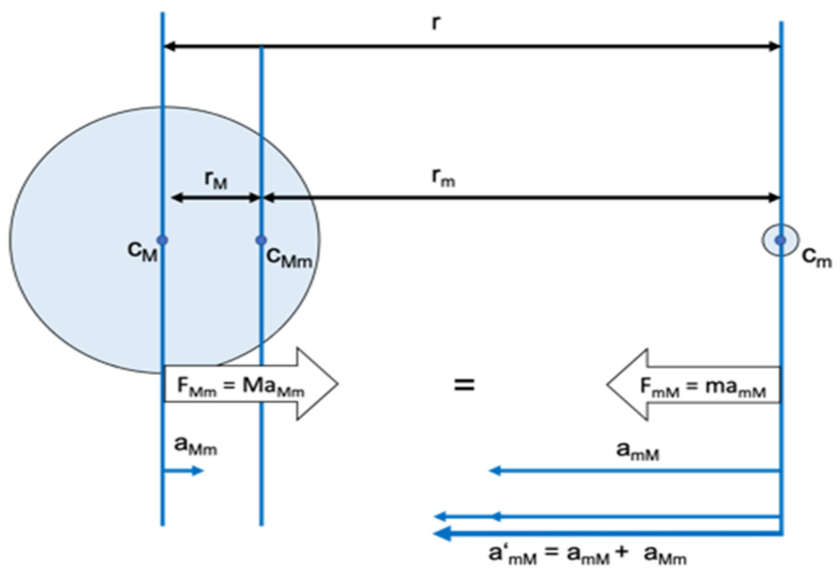

FIG.1. **Points of reference mentioned in this page.**

Revisiting Newton's laws and Gravitation Formula in a holistic interpretation

Newton's great merit has been to merge Galileo's postulations about terrestrial movements (the concept of "force" and its vectorial character; the exclusion of secondary effects like air resistance) with Kepler's theories about astronomic movements (the decrement of force with the square root of distance) with a minimalist mathematical apparatus. He intentionally only described features and effects, and consciously abstained from presumptions about causes. Newton has given his laws (published in 1687) lapidary formulations with a "polemic" intention. Each law focuses a paradigm (mostly of Aristotelian origin) predominant in his times and is frugal with detail specifications. Therefore, only a holistic interpretation of Newton's thought building is correct, when each law and the gravitation formula are read in the light of all the others. The core feature of the three laws and the gravitation force formula is that gravitation is a bilateral and permutation symmetric interaction. As mass is a key feature dealt with in this paper, it must be clarified that in Newton's formulations only one type of mass is mentioned and only in his gravitation interaction formula and none at all in his origi- nal Latin formulations of the three laws [2]. The mass of a body is in first instance just the ratio between the gravitation force and the translational acceleration of that body.

Newton's 1st law

Everybody persists in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.

- This statement of Newton is against the theory of Aristoteles, whereby the capability to change movement was exclusively a merit of the medium. He substitutes it by the entity "force" (first identified by Galilei) as the only agent for any change of the movement mode

- This law contains an allusion to the principle of least action by stating that the trajectory is "straight forward" (shortest possible). This law specifies the prerequisites of an inertial system but does not address the equivalence of laws in their simplest form in all other frames of reference obeying these prerequisites

- The intention of this law is not to define any inertial resistance of a body to change its motion. The persistence of the movement mode stated by this law is not a "laziness" but quite the contrary: it is the capability of any mass body to sustain movement without external impulsion, by storing and conserving kinetic energy, as long as it does not absorb more energy or cede some energy due to the interaction with another body

- The wording "it is compelled to change its state" induce the interpretation of a "lazy" body as compelled by of a totally external agent to abandon its quietness. But if we include the other laws and the Gravitation Interaction Formula in our consideration we discover, that the "victim" is an equally entitled co-determiner of any interaction force of the partner mass and on itself as well

- This law does not stipulate per se the prerequisite of an inertial reference systems for its validity

- This law does not yet include the mutuality character of gravitational force. In the 3rd law the term "force impressed" is substituted by the terms "action" and "reaction"[3]

Newton's 2nd law

The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed. In 1750 (63 years later) Euler reformulated it to Force=Mass times Acceleration" [4].

- This statement of Newton is against the theory of Aristoteles accord- ing to which uniform velocity was the sole mode of movement and establishes the novel concept of translational acceleration ("alteration of motion") as the relevant response to gravitational interaction force. This law also emphasizes its vectorial character ("in the direction") and states a proportionality between the translational acceleration and the "impressed force" (alias the mutual attraction force as per the gravitation interaction force formula). Euler has later specified the pro- portionality factor as "mass" and reformulated the law as mentioned above.

If we include the core concept "mutual attraction" of the gravitation in- teraction formula in the interpretation of the 2nd law, it does not state that matter is "lazy" but that it performs the dosage of the movement caused by any gravitational interaction, in order to achieve the con- junction of the interacting masses at a preferred point of space (the common mass centre), at a preferred point of time (simultaneity) and along the preferred trajectory (the shortest path). The interaction part- ner with the larger mass speeds up less, not because it has more "lazy mass", but because it is geometrically nearer to the preferred point and is not "allowed" to reach it earlier than the more distend and lighter interaction partner.

From the preferred point of space of a gravitational interaction, i.e. the common mass centre $C_{Mm} = (Mx_M + mx_m) / \left(M + m\right) M$ of the two masses at position x_M and x_m respectively, one can calculate in a weighted manner the following relations once we consider the distance relation $r = r_M + r_m$ and C_{Mm} as origin of the reference frame, i.e. by setting C_{Mm} = (0, 0, 0) and choosing x_M = (r_M , 0, 0) and x_M = (r_M , 0, 0) as illustrated in Fig.1:

$$Mr_M = mr_m \quad (1)$$

$$\frac{rM}{r} = \frac{m}{M+m} \quad (2)$$

$$\frac{r_m}{r} = \frac{M}{M + m}$$

Formulas (2) and (3) represent mass proportionality factors. They imply:

- That every unit of matter involved in the interaction contributes in an equitable manner to the weighting of distances
- That each offset of a mass from the common mass centre C_{Mm} is defined by the relation of the mass of the interacting partner, divided by the sum of both masses
- For the typical fall cases on Earth the deviation of the mass proportionality factor from 1 is extremely small: in the case of a falling apple ($\sim 10^{-1}$ kg) it is in the order of $\sim 10^{-24}$

Newton’s 3rd law

To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

This law of Newton is against the everyday experience, where the mechanical interaction of two masses is restricted to falling of masses to the Earth surface and to collisions, with one “active” body hitting a “passive” body with generally asymmetric consequences. This law is to state, that any gravitational interaction (also the remote gravitational interaction unknown before his theory) is perfectly symmetric.

Newton introduced in this law the terms “action” and “reaction” in+stead of the term “force impressed” used in his 1st law.

Another core concept of this law is that any gravitational interaction force is always a joint result of both interacting masses; there is no “active only mass” nor a “passive only mass”.

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$$M_{TM} = mr_m \qquad (1)$$

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- That each offset of a mass from the common mass centre C_{Mm} is defined by the relation of the mass of the interacting partner, divided by the sum of both masses.
- For the typical fall cases on Earth the deviation of the mass proportionality factor from 1 is extremely small: in the case of a falling apple ($\sim 10^{-1}$ kg) it is in the order of $\sim 10^{-24}$.

Newton's Gravitation Interaction Formula

Every mass attracts every other mass by a force acting along the line intersecting the two mass centres. The force is proportional to the product of the two masses, and inversely proportional to the square of the distance between them

$$F_{Mm} = F_{mM} = G\frac{Mm}{r^2} \tag{4}$$

The Gravitation Interaction Formula repeats several concepts of the three laws detailing and/or quantifying them, and adds some new concepts:

- The formula details the mutuality aspect of gravitation of the 3rd law by stating that both masses contribute in an equitable manner (pro- portional to each mass value) to a "communitarian" attraction force, which acts on both masses with the same value.
- The formula incorporates the discovery made by Kepler, that the in- teraction force weakens with the square of the distance between the interacting masses.
- The formula implies a novel potentiating feature of mass:
    - The contribution a mass is giving to the interaction force not only depends on its own quantity of mass but also on its quantitative relation to the mass of the interaction partner.
    - The interaction force is not proportional to the sum of the involved masses but to their product.
    - Consequently, the interaction force is all the larger, the more evenly the involved total mass is distributed on both sides of the inter- action. This can be seen as follows. By considering the mass relation $m = \alpha M$ for any proportional factor α = m/M we may set the sum of the masses to unity up to a constant mass value $M + m = M + \alpha M = 1$With $M = 1/(1 + \alpha)$ it is possible to rewrite the mass product as a function of α. This function has a global maximum at $\alpha = 1$.This hints towards a model of the gravitational force as maximum pairing of overlapping clouds of gravitational particles. As far as we know, such a model has not been considered so far.

$$Mm = \alpha M^2 = \alpha / (1 + \alpha)^2$$

Newton's Gravitation Interaction Formula (4) can be grouped as follows, to get a more precise quantification of the 2nd law

$$F_{Mm} = F_{mM} = G\frac{M}{r^2}\mathrm{m} = a_{mM}\mathrm{m} = a_{Mm}M \quad (5)$$

and to quantify the translational accelerations

$$a_{Mm} = \frac{F_{Mm}}{M} = G\frac{Mm}{Mr^2} = G\frac{m}{r^2} \quad (6)$$

$$a_{mM} = \frac{F_{mM}}{m} = G\frac{Mm}{r^2 m} = G\frac{\mathrm{M}}{r^2} \quad (7)$$

The formal content of formulas (6) and (7) is the same as the claim of Galileo for his geocentric frame of reference: "the translational acceleration does not depend from the own mass m". In addition to that both formulas specify that "the acceleration of any mass m only depends from the mass M of the interaction partner". The equality of translational accelerations of any value of m within a frame of reference related to the common mass centre C_{Mm} of M and m seems to confirm the existence of two types of mass: an "inertial mass" versus. a "gravitational mass". But the "simplest form" is to consider two mutually tuned performance features of the unique entity "mass": one generating movement and one dosing the acceleration in such a way, that the privileged point of meeting (C_{Mm}) is reached at the same time as the interaction partner does.

Summary of Newton's laws review

We can resume the core statements of the three laws and the Gravitation Interaction Force Formula in a most abstract way as follows:

- Gravity is exclusively interaction. There is no isolated gravitational action without gravitational re-action.
- Mass has always a twofold active and passive capability to interact, which can never be dissociated. There is no "acting-only mass" and no "subjected-only mass".
- The gravitational interaction of masses targets to nullify the distance between the interaction partners (attraction).
- Gravitational interaction breaks the homogeneity of space and time by a trifold privileging

- of the common centre of mass as conjunction point,
- of the common point of time for the conjunction,
- of the shortest path as trajectory of the masses.

- The effects of both the active and passive performance features of a mass m not only depended on the quantity of m, but also and essentially from its quantitative relation to the interacting mass M .

Reviewing Galileo's "law" of Free Fall with Newton's laws and Newton's Gravitation Interaction Formula

The present paper in no way questions the usefulness of Galileo's "law" of Free Fall ("all masses fall on Earth from the same height the same time") for all practical terrestrial applications but confutes (within the realm of Classical Mechanics, i.e., for masses much larger than atoms, moving at speeds well below velocity of light) its status of a universal physical law. When Galileo postulated this "law" in 1589 he had not yet made his greatest discovery, the Principle of Reference alias Principle of Invariance alias of Relativity ("mechanical laws are invariant in their simplest form only in stationary frames or in those moving with constant translational speed"). Galileo Galilei,who died in the year Newton was born, did not know the mutuality of gravita- tional force formulated by Newton. Therefore, he could not know that the translational acceleration of a falling mass he measured with respect to the Earth mass centre (C_M) was incremented by the translational acceleration of the Earth mass (M) towards the common centre of masses (C_{Mm}). Galilei led "his people to the Promised Land but not enter in himself." [5]. Beside some rare alerts, e.g. in, the status of Galileo's "law" of Free Fall as "Universal law" is uncontested till today. But the following review of his setup and results with Newton's laws and gravitational formula reveals that it is not true -for any relation of the interacting masses.

Interaction of two Masses

Let's consider two masses M and m. They mutually attract by the same but opposite forces $F_{Mm} = F_{mM}$ and move toward their common mass centre C_{Mm} with a translational acceleration which can be derived from the 2nd law as:

Hence,

$$a_{Mm}M \ = \ a_{mM}m \qquad (8)$$

$$a_{mM} \ = \ a_{mM} \ \left(M \ / \ m\right) \qquad (9)$$

Let’s call it Newtonian translational acceleration figure.

In the Galilean setup (referencing to C_M) however, the measured transla- tional acceleration of m is the sum of a_{mM} and a_{Mm} because it includes also the translational acceleration of M toward C_{Mm} . What Galileo measured was this aggregate translational acceleration “. Let’s apostrophize it and call it Galilean translational acceleration figure:

$$\overset{\circ}{a}_{mM} \ = \ a_{mM} \ + \ a_{Mm} \ = \ a_{Mm}\left(M \,/\, m\right) \ + \ a_{Mm} \ = \ a_{Mm}\left(M \ + \ m\right)/\,m \qquad (10)$$

or

$$\overset{\circ}{a}_{mM} \ = \ a_{mM} \ + \ a_{Mm} \ = \ a_{mM} \ + \ a_{mM} \ \left(m\,/\,M\right) \ = \ a_{mM} \ \left(M \ + \ m\right)/\,M \qquad (11)$$

$$\overset{\circ}{a}_{mM} \ a_{mM} \ = \ \frac{\mathrm{M} + \ m}{M} \qquad (12)$$

Formula (12) shows that the Galilean translational acceleration figure is incremented with respect to the Newtonian translational acceleration figure by a mass-proportion factor

$$(M \ + \ m)\,/\,M$$

with a similar structure to the mass proportionality factor (3) for the offset of C_{Mm} from C_M. Also the factors to define the preferred location of C_{Mm} or the “reduced mass” to solve two-body problems [6] have a similar structure. As the lighter interaction partner is more distant from C_{Mm} it must speed up faster than the heavier interaction partner in order to arrive there simultaneously and therefore the denominator is the heavier one of the two masses. The forces $F_{Mm} = F_{mM}$ are invariant to any inertial frame, but their decomposition into a mass and a translational acceleration figure is not. We can only derive the genuine Newtonian mass figure in its “simplest form” (as the Galilean Principle of Invariance requires), if we divide the force figure by a translational acceleration measured with reference to C_{Mm} , which is the only truly inertial (not accelerated) point during the interaction.

Implications on the distinguishment of gravitational and inertial mass

The approximative character of Galileo's "law" calls for a review on the tradi- tional distinguishment between gravitational and inertial mass. Within what has been said up to here, these two notions may become directly translated into the Newtonian mass m (as seen from C_{Mm}) and the Galilean reduced mass m_i (as seen from C_M) respectively. The following equation contains a per se correct equation of two transcriptions of the gravitational interacting force

$$F_{mM} = m_i \mathring{a}_{mM} = m a_{mM} \qquad (13)$$

However, a fourfold sloppy interpretation of this correct formula has been the starting point to misleading conclusions on the duality of "gravitational mass" and "inertial mass" as follows:

- To equate the composite Galilean translational acceleration figure $a^j mM$ with the Newtonian translational acceleration figure a_{mM}
- To equate the reduced Galilean mass figure m_i with the Newtonian mass figure m
- To consider m_i as a "passive only" mass
- To consider m as "active only" mass

But if substitute in formula (13) the composite translational mass value $a^j mM$ by its components measured in a Newtonian frame of reference, we get:

$$m_i a_{mM} \left(M + m\right) / M = m a_{mM} \qquad (14)$$

$$m_i \left(M + m\right) / M = m \qquad (15)$$

and

$$m_i / m = M/(M + m) \quad (15) \qquad (16)$$

This means that the Galilean mass has always a reduced value with respect to the genuine Newtonian mass, by a mass proportionality factor as in formula (3) which defines the offset of C_M from C_{Mm}. It is a "reduced" mass (not the "simplest form") because it must compensate the excess of translational acceleration measured in the Galilean setup [6].

Conclusions of Galileo's law review

- The composite Galilean mass considers within the mass correction fac- tor, beyond the mass of the interaction partner as in the Newtonian frame of reference, also the own genuine

- Newtonian, to compensate the offset between C_{Mm} and C_M (which always depends from both masses)

- For handsome bodies (e.g. with m =1 kg) falling to Earth (M 10^{24} kg), (i.e. $m << M$) the reducing factor deviates from 1 by only about $\sim 10^{-24}$. Therefore, the measured acceleration is practically equal to its true Newtonian value with respect to C_{Mm} for all practical applications on Earth

- If a body of the same mass of the Earth would fall on Earth ($m = M$) its acceleration measured with respect to Earth would have the double value of its true Newtonian value related to C_{Mm}

- The most precise measurements to detect the hypothetical mass du-ality (the E¨otv¨os-Dick experiments with a measuring accuracy of ~10-13) had obviously till now negative results [7,8]

Basically, the theory of Aristoteles that heavier masses fall faster to Earth, has not been confuted by Newton's laws and formula (whose falling-mass independent acceleration refers to C_{Mm} but not to Earth) but erroneously corrected by Galileo's "law".

What has contributed to the status of Galileo's approximation as universal law?

The persistence of the status of the Galilean "law" of Free Fall confirms first of all the theory of N. Cartwright [9] that physical laws generally only deliver a patch-wise mapping of physical reality. It also confirms the observation made by Th. S. Kuhn [10], that scientific paradigms have a special persistence, if they are consonant with other paradigms and/or with collateral realms of perception and thought. In our case there are several such consonances.

The empirical environment on Earth

Everyday experience induces to conceive the Earth as the sole "active cause" of gravitation and any falling object as its "passive victim". The reason is the enormous disproportion between the two masses the impossibility for an observer on the ground to perceive that while an apple is falling from a tree, also the Earth is accelerating towards it. This unilateral perception has also entered into colloquial speech, which in turn fossilizes this concept. For instance, we generally speak only about "Earth's gravity" but never about the "apple's gravity".

The convincing novel scientific approach of Galileo

The novel approach applied by Galileo for his fall experiments, namely, to lay bare a physical law by neglecting collateral physical effects of other laws, made him to a founder of modern science. His move to explicitly discount the collateral effects of the aerodynamic resistance, which has nothing to do with the Earth attraction, was so convincing, that his postulate "all bodies fall to Earth with the same acceleration" has been categorially accepted with no further analysis of its frame conditions.

The lapidary formulations of Newton's laws and Formula

As already mentioned above, Newton formulated his three laws as polemic statements against the (mostly Aristotelian) mainstream of his time, and not as a balanced coverage of all related aspects. His laws are lapidary statements without frame specifications. Especially the formulations of the 1st and 2nd law have inadvertently induced the interpretation that there is a unilateral impression of grav- itational force upon a purely passive and a lazy victim (later called "inertial mass") of gravitational impression. The proportionality of the Gravitational Interaction Force to the product (and not the sum) of the two interacting masses, which is funda- mental to the argumentation of this paper, became evident only after the compactation of Newtons three proportionality assessments to the unique, so called Newton Gravitational Formula, which has been per- formed by Alfred Cornu and Baptistin Baille almost 200 years later [11].

If we restrict us to a literal and isolate interpretation of each law and the formula, their implicit and holistic content remains unveiled.

Euler's incomplete reformulation of the 2nd Newto- nian law

With his reformulation of the 2nd Newtonian law he published in 1750 (force) = (mass) × (acceleration)

Euler has contributed unintentionally to the fossilization of the identification of the Galilean and the Newtonian masses, by omitting the boundary condition “related to the common mass centre”. Because out of the myriad of possible mass values and their complementary acceleration values that de- liver a given force value, only that mass value obtained in the inertial frame of reference C_{Mm} is directly comparable with another Newtonian mass value and only with such one.

Einstein’s derivation of General Relativity

Einstein’s original derivation of his General Relativity Theory in his presenation in 1916 has been based on Galileo’s “law” rather than the Newtonian concept of the mutuality of any gravitational attraction between masses by equating as follows:

(Gravitational force)=(Galilean mass m_i) × (Galilean acceleration) equivalent to

(Gravitational force)=(Newtonian mass m) × (intensity of gravitational field)

This has de facto further fossilized the misinterpretation of Galileo’s “law” of Free Fall, to consider the Earth as an immovable, acting only mass and the falling apple as moving, passive only mass [12,13].

Conclusions and final remarks

Reviewing the Galilean setup and the “law” of Free Fall with Newtonian glasses the following results have emerged which may help to circumvent misunderstandings in future.

The mass-independence of acceleration is unconditionally valid only in the inertial Newtonian frame related to the common centre of mass C_{Mm}.

The presently dominant reformulation of Newtons 2nd law made by Euler is incomplete and therefore misleading; it has contributed to the misinterpretation of Galileo’s Free Fall “law”. It should be comple- mented as follows: “Force = Mass times the Translational Acceleration related to the common mass centre of the interacting masses”.

The offset of the Galilean frame of reference from the inertial Newto- nian frame has as consequence that the mass obtained by dividing the force by the acceleration measured in the Galilean frame of reference, loses its “simplest form”: this Galilean mass value m_i differs from the Newtonian mass by a mass reducing factor

$$m_i / m = M / \left(M + m\right)$$

which is in the order of only 10^{24} in terrestrial fall situations, making all bodies fall to Earth approximately with the same acceleration. But if another Earth mass falls on Earth ($m \simeq M$) its Newtonian acceleration (referred to C_{Mm}) is equal to that of an apple, but the composite Galilean acceleration (referred to C_M) is about two times larger than that of an apple.

The statement of Galileo’s Free Fall “law” (all masses m fall with equal acceleration to Earth”) is not valid for all proportions of M / m but is only approximately valid for the huge mass disproportions (in the order of $\sim 10^{24}$) as present in terrestrial fall situations.

The Galilean Law of Free Fall should therefore be renamed “Galileo’s approximative law of Free Fall” (the attribute “empirical” would not exclude a universal validity).

Another issue of Galileo’s “law” being not discussed so far is the fact that the acceleration of falling body is not constant in time as it increases quadrati- cally with the distance to the earth according to Newton’s Gravitational law. This is true independently of the chosen frame of reference (as given either in C_M or in C_{Mm}) and the considered ratio of M / m.

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