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Mechanical model of hollow basket shot angle and Matlab simulation of the basketball movement trajectory

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ABSTRACT

The impact of basketball shot angle on the hitting rate is very large; the paper first builds mathematical model of the hollow basket effective area, mathematical model of basketball trajectory, mathematical model of angle and optimization mathematical model of shot angle, selects appropriate numerical value to test the model based on the above model and under reasonable assumptions, and obtains the corresponding angle of incidence $\theta = 46.9°$ and the effective area $S=146.186cm^2$ of the best shot angle. On this basis, it uses Matlab software simulation to achieve the basketball centroid movement trajectory of the best shot angle at three-pointer position, explores the best shot angle with different horizontal distances using Matlab software simulation, and obtains the trend figure of the best shot angle changing with the horizontal distances.

KEYWORDS

Effective area;
Angle of incident;
Optimum shot angle;
MATLAB simulation.

INTRODUCTION

The key to basketball score is the hit rate, the level of which is caused by many factors, including the athletes shooting level, physical fitness, natural environment, rule changes, psychological quality and other uncertainties. The above factors will produce errors on shooting, leading to low hitting rate; but if we can increase the relative basket effective area when the basketball enter into the basket, it can increase the permissible errors of shooting, that is, increasing the hit rate of shooting. The projection of basketball entering into the basket in the velocity direction is an ellipse, which is the projected area. When the projected area reaches the maximum or a suitable size it will produce the best shooting form, the paper discusses it based on this content.

Many people have made efforts on the research of basketball sports biomechanics, Basketball trajectory, basketball hitting rate and shot angle, and the results provide a good theoretical base and prospect prospects for the development of basketball cause and basketball robot. Based on the previous study this paper puts forward the basket effective area model of basketball hitting, conducts research and discussion on the reasonable maximization of the effective area, reaches the best shot angle of different distances and provides theoretical guidance for basketball training and related research.

PROBLEM ANALYSIS

Firstly, this paper studies the basket effective area of shooting successfully, finds the best angle of incidence under the goal of for improving the effective region of shooting successfully, explores the relationship between the angle of incidence and the shot angle, obtains the mathematical equations of the shot angle and the incident angle through analyzing the relationship between the angle of incidence and shot angle, and finally finds the best shot angle according to the relationship between the angle of incidence and the maximum hitting basket region.

The basket effective area based on the hollow shooting

When the basketball falling into the hollow basket, the velocity direction of basketball is not perpendicular to the surface of basket, but there is an acute angle; the shape of the basket becomes the ellipse viewed from the moving direction of the basketball. Assuming basketball radius is r, the radius of the basket is R and the incident angle is θ; the projection of the basket in the projecting direction is shown in Figure 1:

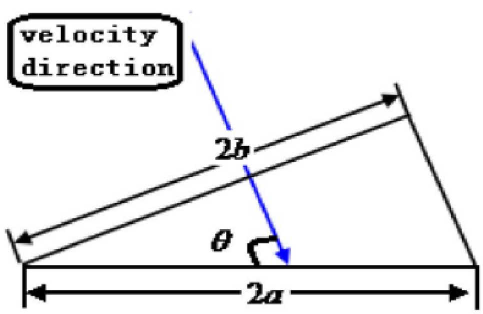

Figure 1 : Schematic diagram of basket projection

In Figure 1, the incident angle of basketball is θ and there is $a = R$; then the projection pattern of the basketry in the projection direction is an ellipse with the $2a$ major axis and the $2b$ minor axis, suppose the ellipse is T, and we have the relation in the formula (1):

$$\begin{cases} a = R \\ b = R\sin\theta \end{cases} \qquad (1)$$

Based on the above analysis, it shows that the effective area entering box when basketball enters into a hollow is shown in Figure 2:

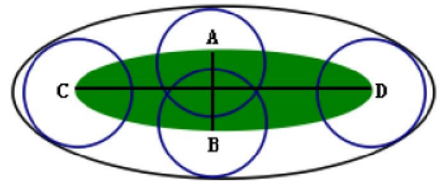

Figure 2 : Schematic diagram of the effective area entering into box

In Figure 2 the green shaded area is the effective area entering into box, the shape of this region is an ellipse denoted by T_0, the length of $|AB|$ is the minor axis $2b_0$ of that ellipse, the length of $|CD|$ is the major axis $2a_0$ of that ellipse, $a_0 \le a - r$, $b_0 = b - r$ as shown in Figure 2, then the effective area T_0 entering into the box can be drawn by the formula (2):

$$S_0 = \pi a_0 b_0 \le \pi(a - r)(b - r) = \pi(R - r)(R\sin\theta - r) \qquad (2)$$

From the above analysis when the area T_0 is larger, the hit rate of entering the hollow box is higher.

Model assumption

1) The basketball after releasing receives only gravity, and acceleration of gravity takes 9.8m/s2;
2) Only study the effective hollow sphere region when basketball centroid trajectory passes through the basket;
3) The initial position of basketball shooting does not change, i.e. the basketball shooting position and basketball hoop position are relatively unchanged;
4) The position releasing basketball in the coordinate system is the origin of coordinates.

MODEL BUILDING

Basketball trajectory model

When the basketball only receives gravity, it makes an oblique up-throwing movement, and the movement trajectory is a parabola, as shown in Figure 3 schematically:

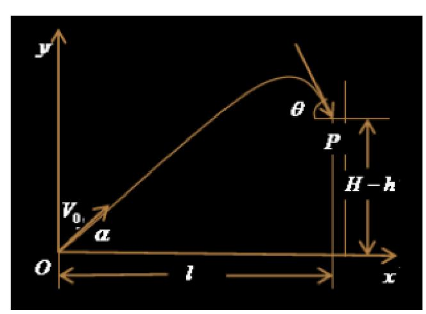

Figure 3 : Schematic diagram of basketball moving trajectory

The sub-displacement parameter equation of basketball in the horizontal direction and the vertical direction is in the formula (3) below:

$$\begin{cases} x(t) = V_0 \cos\alpha \cdot t \\ y(t) = V_0 \sin\alpha \cdot t - \frac{1}{2} g t^2 \end{cases} \quad (3)$$

Eliminate parameter *t* and obtain formula (4):

$$y = \tan\alpha \cdot x - \frac{g}{2V_0^2}\left(1 + \tan^2\alpha\right) \cdot x^2 \quad (4)$$

The formula (4) shows that factors associated with basketball trajectory are shot angle α and shot speed V_0.

Relationship analysis of shot angle and angle entering into the box

In the assumption 2), we only study the situation the basketball centroid trajectory passes through the center of the basket, so in Figure 1, basketball trajectory must passes through point $P(l, H-h)$; substituting point P into the formula (4) can obtain the trajectory equation as shown in formula (5):

$$y = \tan\alpha \cdot x - \frac{l \cdot \tan\alpha - H + h}{l^2} \cdot x^2 \quad (5)$$

Formula (5) represents the parabolic that the basketball centroid trajectory passes through the center of the basket; for the formula (5) finds the negative inverse when $x = l$, the tangent value of the incidence angle θ can be obtained, as shown in formula (6) below:

$$\tan\theta = -\frac{dy}{dx}\Big|_{x=l} = \tan\alpha - \frac{2(H-h)}{l} \quad (6)$$

By the formula (6), when the shot angle α increases, the angle θ of incidence will also increase, which have the same increase-decrease characteristics, so as shown in the formula (2) the upper bound $(R-r)(R\sin\theta - r)$ of S_0 will increase as the shot angle α increases.

Analysis shows that the area T_0 of the ellipse is the effective area of the basketball entering into the basket hollow, and because the radius of the basketball is r, the semi-minor axis b_0 of the ellipse T_0 should be greater than r; with such constraints the lower bound of the shot angle α can be obtained, and the formula (7) shows constraint conditions:

$$\left.\begin{array}{l} b = R\sin\theta \\ b > r \end{array}\right\} \Rightarrow \sin\theta > \frac{r}{R} \quad (7)$$

According to formula (7), formula (8) can be derived:

$$\tan\alpha = \tan\theta + \frac{2(H-h)}{l} > \frac{r}{\sqrt{R^2 - r^2}} + \frac{2(H-h)}{l}$$
$$\Downarrow$$
$$\alpha > \arctan\left[\frac{r}{\sqrt{R^2 - r^2}} + \frac{2(H-h)}{l}\right] \quad (8)$$

In the Formula (8) we obtain the lower bound of shot angle α.

Optimal shot angle model

Unite formula (4) and (5) the relationship between shot velocity and shot angle can be drawn, as shown in formula (9) below:

$$V_0 = l \cdot \sqrt{\frac{g\left(1 + \tan^2\alpha\right)}{2(l\tan\alpha - H + h)}} \quad (9)$$

The formula (9) is deformed using substitution method, suppose tan $\alpha = X$ and $V_0^2 = Y$ formula (10) can be obtained:

$$Y(X) = \frac{A(X)}{B(X)} = \frac{gl^2\left(1 + X^2\right)}{2(l \cdot X - H + h)} \quad (10)$$

Conduct increase-decrease analysis of formula (10), the solution on the first-order derivative of X is shown in the formula (11) below:

$$\begin{cases} \frac{dY}{dX} = \frac{A(X)' B(X) - A(X) B(X)'}{B(X)^2} \\ A(X)' = 2gl^2 X, B(X)' = 2l \end{cases} \quad (11)$$

By the formula (11) when $X \geq \frac{H-h}{l} + \sqrt{\frac{(H-h)^2}{l^2} + 1}$, Y will increase rapidly with the increase of α; when Y rapidly increases, then the shot speed will be very large; when the shot speed is very big, the basketball players need to exert greater force to the ball, which is of detriment to athletes shooting stability, thereby reducing the shooting rate; so the shot angle paper α that can make the effective shooting basketball area S_0 reach the corresponding upper bound is the best shot angle.

The formula (2) shows that the condition that the effective shooting basketball area S_0 reaches the upper boundary is $a_0 = a - r$, i.e. when the basketball enters into the box, the edge section is tangent to the two end

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points of the long axis in the incident sectional area, as shown in Figure 4:

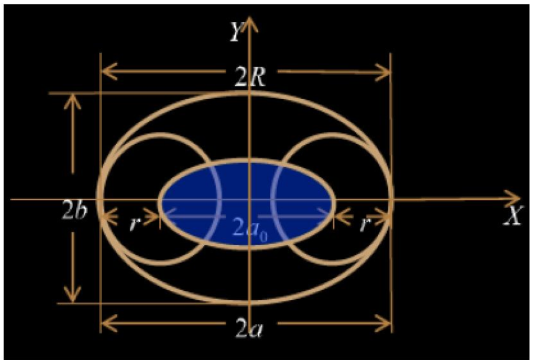

Figure 4 : Schematic diagram of maximum effective area

Figure 4 shows the equation of the incidence cross-section ellipse is shown in formula (12); the equation when the basketball is tangent to the edge point of cross-sectional major axis is shown in formula (13):

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (12)$$

$$\begin{cases}(x-a_0)^2+y^2=r^2\\(x+a_0)^2+y^2=r^2\end{cases} \quad (13)$$

As the ellipse is the axis symmetry graph and the circular also is the axis symmetry graph, so combine one of the circle in equation (12) and equation (13) and make the equations have one and can only have one solution (–*a*,0), as shown in formula (14):

$$\begin{cases}\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\\(x+a_0)^2+y^2=r^2\end{cases} \quad (14)$$

When controlling the formula (14) to have only one solution, the range of short axis *b* in incidence section can be drawn; then according to the range of *b*, obtain the range of incidence angle θ; and then according to the relationship between the shot angle and incidence angle, the best shot angle can be obtained shown in formula (15) below:

$$\alpha=\max\left\{\arctan\left[\sqrt{\frac{r}{R-r}}+\frac{2(H-h)}{l}\right],\arctan\left[\frac{r}{\sqrt{R^2-r^2}}+\frac{2(H-h)}{l}\right]\right\} \quad (15)$$

From Figure 4, the best shot angle α is shown as formula (16) below:

$$\alpha=\arctan[\sqrt{r/(R-r)}+2(H-h)/l] \quad (16)$$

MATLAB SIMULATION

Trajectory simulation of basketball centroid

Data Selection: Basketball radius r = 0.12m, basket radius R = 0.225m, the height of basket from the ground H = 3.05m, the shot height of basketball h = 2.00m, the horizontal distance of basketball shot location and the basket center l = 6.25m. Can be drawn from the above data that the tangent of the basketball best shot angle is tan α = 1.405; according to equation (9) basketball shot speed V_0 = 8.581 m/s2 can be drawn; by the equation (4), the simulation trajectory of the basketball centroid trajectory equation is obtained shown in Figure 5:

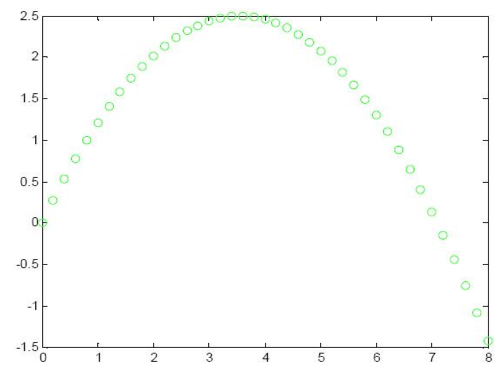

Figure 5 : Basketball centroid trajectory simulation diagram of optimal angle

The best shot angle with different horizontal distances

According to equation (16), establish the equation of best shot angle on the horizontal distance, conduct Matlab simulation in the horizontal distance of 1-8 meters and explore the change situation simulation results of the best shot angle under different horizontal distances, as shown in Figure 6 below:

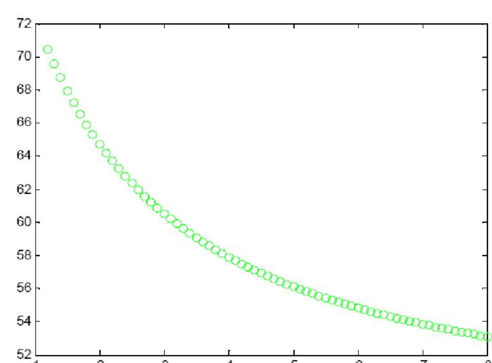

Figure 6 : The change trend chart of best shot angle with the horizontal distance

According to Figure 6 the best shot angle decreases with the increasing of the horizontal distance, which is in line with the actual situation.

To sum up: In the premise to ensure the best shot angle invariable, calculate the best shot angle and shot speed. In this article the best angle of incidence θ= 46.9^0 can be obtained according to equation (6), and the effective regional area of hollow shooting is S_0 = 146.186cm2.

CONCLUSIONS

1) According to the position relationship between the ball and the basket of hollow shooting, this paper builds the effective regional model of basketry; 2) This paper explores the relationship between the shot angle and the incident angle, and then optimization designs the best shot angle when the effective shooting area is the largest; 3) rationally use Matlab software simulation to achieve the best movement trajectory of basketball centroid of three-pointer hollow shooting; 4) This paper explores the relationship between the horizontal distance and the best shot angle; 5) obtains the best incident angle 46.9^0, and the corresponding basketry effective regional area of hollow shooting is $146.186cm^2$.

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