ISSN : 0974 - 7435 Volume 8 Issue 8

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 8(8), 2013 [1066-1071]

Mathematical model research of baseball sweet spot based on mechanical analysis and MATLAB simulation

Ziwen Wang

Institute of Physical Education, Huanggang Normal University, Huangzhou 438000, Hubei, (CHINA)

E-mail: tiyuxi@qq.com

ABSTRACT

This paper conducts the dynamics analysis on the baseball's "sweet spot"; on this basis it analyzes the striking effect of adding cork in bat and different bat materials relative to ordinary wooden bat. In the research conduct geometric simplification and abstract description on the shape of the bat, and get the geometric description equations of the bat. Then taking the ball-rod collision system as the study object, use the conservation of momentum theorem, conservation of angular momentum theorem and recovery coefficient to establish rigid body dynamics model, then propose the calculation method of the sweet spot, and draw that the "sweet spot" of ordinary wooden bat is at a distance of 66cm from the bat handle segment. Starting from the variation of bat mass, centroid and rotation moment of inertia by adding the filler, it analyzes the effect of adding cork into ball, and obtains the conclusion that "adding cork can decrease baseball speed".

KEYWORDS

Sweet spot;
Kinetic model;
MATLAB simulation;
Baseball.

INTRODUCTION

Taking ball and rod as a rigid body, you can establish the classical dynamics model for analysis taking "ball-rod" system as the research object. Studies have shown that, at the hitting moment the impact force exerted by the hand on the stick has little effect on the bat; the bat can be approximated as a "free end. Using the "conservation of momentum theorem", "conservation of angular momentum theorem" and "coefficient of restitution" to research the relationship between the hitting position and leaving speed of the ball, and thus find the "sweet spot" and analyze the effect of cork filling and the material changing on the ball hitting effects.

Based on this, this paper takes the bat's best sweet spot as a starting point to research, through the establishment of the model discusses the sweet spot in the case of the bat's reverse speed gets maximum after hitting, the hands' impact force gets minimum after hitting and the hands receive the minimum energy after hitting. And on this basis it analyzes the striking effect of adding cork in bat and different bat materials relative to ordinary wooden bat, thus reducing the damage on the athlete.

MODEL ASSUMPTIONS AND SYMBOL DESCRIPTIONS

A. Ball does not spin during flight.

B. When the bat hits the ball, velocity direction of the bat is orthogonal to the bat axis.
C. Bat shape, size and weight are the same.

TABLE 1 : Symbols description

Symbol	Meaning
e	Coefficient of Restitution
C	Center of gravity position
J	Moment of inertia
M	Mass of Bat

THEORETICAL MECHANICS MODELING AND SOLVING

Abstract description model of bat shape

Literature suggests that the general specifications of wood baseball bat are shown in TABLE 2:

TABLE 2 : The physical parameters of baseball racket

parameter	Value
Bat length L	0.855m
Bat mass M	0.885kg
Wood density ρ	649kg/m^3
Wood Young's modulus E	$1.814 \times 10^{10}\ N/m^2$
Maximum radius r_2	7cm
Minimum radius r_1	2.5cm
Center of gravity position C	Distance from the smaller end face 0.564m

Bat is a rotating body; a cross section along the axis is shown in Figure 1:

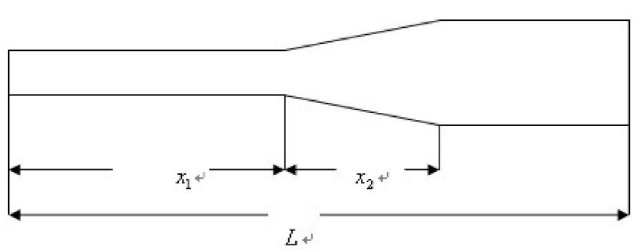

Figure 1 : Section along the axis of the bat

Wherein the length x_1, x_2 of each part can be determined by bat weight and center of gravity position.
Bat mass:

$$M = \rho \cdot V = \rho \cdot [\pi r_1^2 x_1 + \frac{1}{3}\pi r_2^2 H - \frac{1}{3}\pi r_1^2 h + \pi r_2^2 (L - x_2)] \quad \textbf{(1)}$$

$$\frac{h}{r_1} = \frac{h + x_2 - x_1}{r_2}, H = h + x_2 - x_1$$

Based on the definition of centroid, both sides of the centroid of the bat suffer balance center of gravity torque:

$$\boldsymbol{M_{gl} = M_{gr}} \quad \textbf{(2)}$$

$$M_{gr} = \int_0^{x_1} \rho \cdot g \cdot \pi r_1^2 (C - x) dx + \int_{x_1}^{C} \rho \cdot g \cdot \pi r^2 (C - x) dx$$

$$M_{gr} = \int_C^{x_2} \rho \cdot g \cdot \pi r^2 (x - C) dx + \int_{x_2}^{L} \rho \cdot g \cdot \pi r_2^2 (x - C) dx$$

Where r is a function of the variable x of integration, are $\frac{r_2 - r_1}{x_2 - x_1} = \frac{r - r_1}{x - x_1}$:

By the formula (1) and (2), put the data in and obtain: $x_1 = 0.3459$ $x_2 = 0.6423$

Thus, the radius length of each part of the baseball:

$$r(x) = \begin{cases} r_1 \\ r_1 + \dfrac{r_2 - r_1}{x_2 - x_1}(x - x_1) \\ r_2 \end{cases} = \begin{cases} 0.025 \\ 0.025 + 0.152(x - 0.3459) \\ 0.070 \end{cases}$$

The determination of the simplified bat shape model provides convenience for the bat's moment of inertia solution with different materials, uneven material (such as fill cork).

The rigid body dynamics model of the sweet spot

By analyzing the batting action of the hitters, there are two rotating systems in hitting instant: the rotation of arm and bat along the axis of the body center of gravity, the rotation of the bat along the wrist axis.

Take point B of the body center of gravity as the axis coordinate origin, bat axis as x axis, the direction perpendicular to the axis as the y axis to establish Cartesian coordinate system.

Let bat centroid coordinate is C, the body center of gravity coordinate is B, handheld point coordinate is W. Hitting point coordinate is P, bat coordinate that close to the body part is x_0. The distance between BW is R, the distance between WC is H, the distance be-

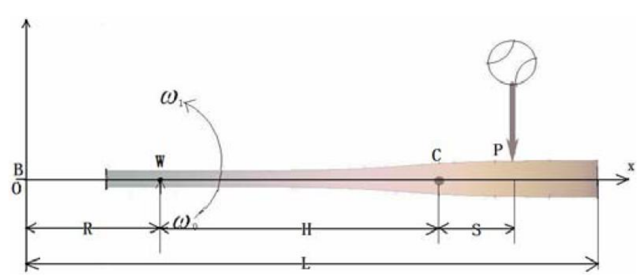

Figure 2 : Abstract illustration of batting process

FULL PAPER

tween CP is S. Schematic is shown in Figure 2:

In the collision instant, the force between the ball and the rod is much larger than the ball gravity, rod gravity, and stamina of the hand; therefore, taking the ball-rod system as the study object, there have the conservation of momentum in y axis direction:

$$m_1v_1 + m_2u_1 = m_1v_2 + m_1u_2 \quad (3)$$

Wherein m_1 is the mass of the ball, m_2 is the mass of the bat, v_1 is the initial velocity of the ball, v_2 is the leaving speed of the ball, u_1 is the centroid speed of the bat before hitting, u_2 is the centroid speed of the bat after hitting.

Let the angular velocity of the bat before and after hitting are respectively ω_1, ω_2, then:

$$u_1 = \omega_1(R + H), u_2 = \omega_2(R + H) \quad (4)$$

Recovery factor e is the relative approach speed of the collision contact point before the collision is divided by the relative speed away after the collision, namely:

$$e = \frac{v_2 - u_2 - \omega_2 S}{-v_1 + u_1 + \omega_1 S} \quad (5)$$

Establish ball-rod system angular momentum conservation equations taking the body center of gravity as shaft axis; taking the bat centroid as axis, ball- rod system has no external torque, so we have conservation of angular momentum, namely:

$$m_1v_1(S + R + H) + J\omega_1 = m_1v_2(S + R + H) + J\omega_2 \quad (6)$$

From (3), (4), (5), and (6) the expression of ball speed after batting can be obtained:

$$v_2 = v_1 - \frac{J(1+e)[v_1 - \omega_1(S+H+R)]}{J + m_1(S+R+H)^2}$$

Where the moment of inertia: $J = \int_{x_0}^{x_0+L} \rho\pi r_{(x)}^2 x^2 dx$

The derivative of v_2 on S:

$$\frac{\partial v_2}{\partial S} = \frac{-J(1+e)\omega_1(J + m_1(S+H+R)^2) - 2m_1 xJ(1+e)(v - m_1(S+H+R))}{(J + m_1(S+H+R)^2)^2},$$

Set $\frac{\partial v_2}{\partial S} = 0$ and obtain the best batting position.

Statistical literature suggests that kinematic parameters of baseball batting instantaneous are as follows in TABLE 3:

The sweet spot position was70cm from the end of the bat handle.

Changing the position of the batting, leaving velocity curve after baseball hitting is shown in Figure 3:

TABLE 3 : Kinematic parameters of batting instant

Kinematic parameter	Values
v_1	27.7m/s
ω_1	17.288rad/s

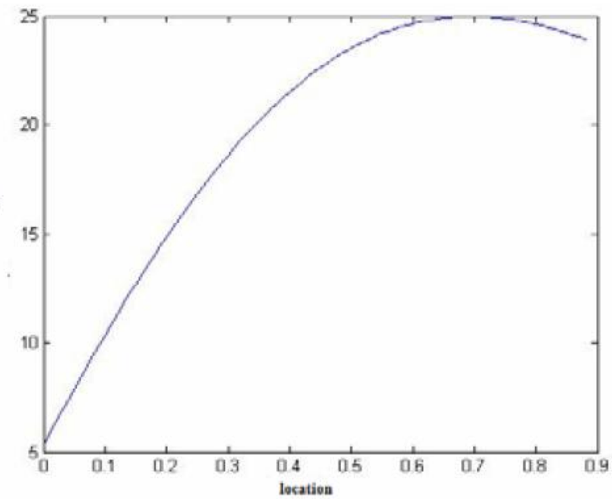

Figure 3 : The relationship between the batting positions with the ball leaving speed

Influence analysis of bat filler

Baseball player adds cork in the sweet spot. The cork is generally a cylinder 1 inch in diameter (2.54cm), 10 inch (25.4cm) deep. Cork density is 260 ~ 320 kg/m^3; density range is large, which is determined by the density of regional bark, usually no more than 340 kg/m^3, less than the density of the wood bat kg/m^3. Therefore, filling the bat with the cork will bring the following changes: (1) Bat mass becomes small, inertia decreases, which can increase the controllability of the bat (bat control). (2) The bat centroid becomes close to holding point, which can reduce the bat's angular momentum when hitting the ball, thereby affecting the batting results. (3) The rotation moment of inertia becomes smaller, thereby affecting the sweet spot and maximum ball speed.

The following conducts quantitative analysis of the effect of filler on the ball from the above three aspects.

Quality reduction amount: $\Delta M = (\rho - \rho_s)V_s$

Centroid changing distance: $\Delta C = C - C'$

In the established coordinate system, the original centroid abscissa C = 56.4. Let the centroid abscissa after filling is C'; according to the centroid definition, gravity moment balance in the left and right sides of the center of mass, we have:

$$\int_0^{C'} \rho g \pi r_{(x)}^2 (C' - x)dx = \int_{C'}^{L-d} \rho g \pi r_{(x)}^2 (x - C')dx$$

$$+ \int_{L-d}^{L} [\rho g \pi (r_{(x)}^2 - r_s^2) + \rho_s g \pi r_s^2]dx$$

By this formula the centroid coordinates C' after filling can be calculated. When layers hit the ball, the rotational inertia of the swing bat taking the body center of gravity as rotation axis is:

$$J' = \int_{x_0}^{x_0+L-d} \rho \pi r_{(x)}^2 x^2 dx + \int_{x_0+L-d}^{x_0+L} [\rho \pi (r_{(x)}^2 - r_s^2) + \rho_s \pi r_s^2]dx$$

Moment of inertia reduction: $\Delta J = J - J'$

Utilization the proposed model of problem (1) it is easy to know, the calculation of sweet spot after filling cork in bat and the maximum leaving speed of the ball just need the rotation moment of inertia in the corrected formula, namely:

$$v_2' = v_1 - \frac{J'(1+e)[v_1 - \omega_1 x]}{J' + m_1 x^2}$$

Conduct derivation we can easily get the ball's maximum leaving speed and the corresponding sweet spot. The change of maximum leaving speed: $\Delta v = v_2 - v_2'$

Substituting density lower limit of cork $\rho_{sl} = 260$ kg/m^3 and density upper limit of cork $\rho_{su} = 320$ kg/m^3, we can obtain the effect range that inserts the cork:

TABLE 4 : The affect results of bat filler

Item	ΔM	ΔC	ΔJ	Δv
ρ_{su} = 320 kg/m^3	40 g	0.01cm	$0.02 kg \cdot m^2$	$1.1 m/s$
ρ_{sl} = 260 kg/m^3	46 g	0.01cm	$0.03 kg \cdot m^2$	$1.6 m/s$

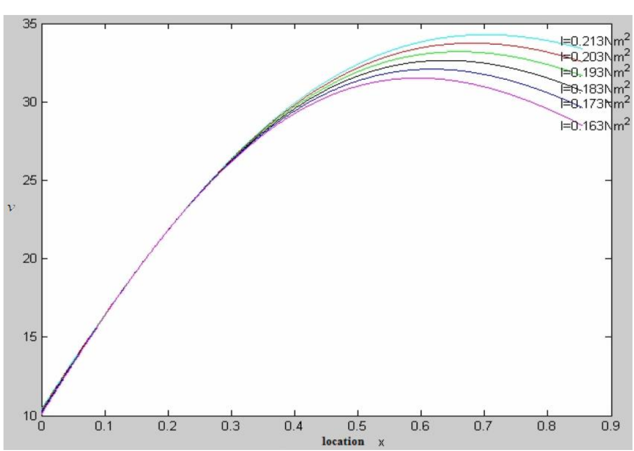

Figure 4 : The ball leaving speed changes with the hitting point under different moment of inertia

TABLE 5 : The maximum leaving speed of the ball with different moment of inertia

J	0.163	0.173	0.183	0.193	0.203	0.213
Maximum batting speed point	0.596	0.617	0.638	0.660	0.682	0.702
Maximum Speed	31.517	32.085	32.645	33.198	33.744	34.283

Results analysis: through the above results, adding filler into the bat does not have a positive impact on hitting effect, mainly due to lighter bat, bat's moment of inertia decreases, and the transmission efficiency of collision energy becomes low. However, the main advantage of this approach mainly reflects in the ability to control the bat better, and faster acceleration can extend the reaction time.

The batting effect of baseball with different materials

Different density and different mass result in different moment of inertia; different materials can result in different recovery coefficients and different elasticity coefficients. Using the classical theoretical mechanics model, suppose two bats' shape are the same with the front abstract model; from the perspective of the moment of inertia and the recovery coefficient, study the bat performance of two materials, it is easy to get the ball's maximum leaving speed: $v_2 = v_1 - \frac{J(1+e)[v_1 - \omega_1 x]}{J + m_1 x^2}$

Consulting relevant information to get the recovery coefficient of the aluminum rod, integral to calculate the aluminum rod's moment of inertia; substituting the representative values of the wooden stick and metal bar into the above equation, we obtain the relationship between the sweet spot and the ball speed shown in Figure 5:

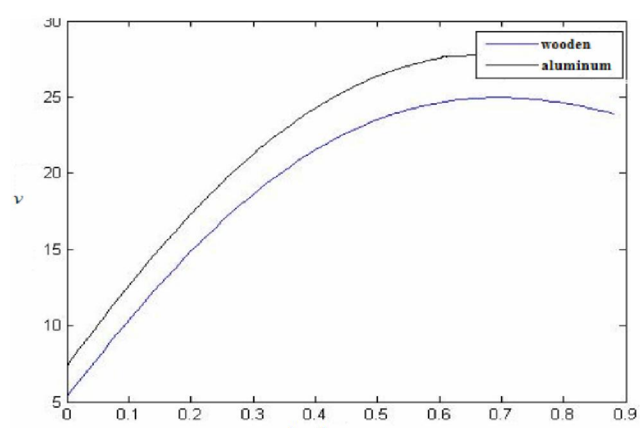

Figure 5 : The ball speed comparison of wooden bat and aluminum bat

Rough modeling results show that compared with the wooden bat hitting using the aluminum bat is easy to play a high and far fly ball. The purpose of the competitive sports is to improve baseball players' physical quality and skills, rather than engaged in sports equipment race, so the regular game is not allowed to use the metal rod.

MODEL EXTENSIONS

Let the mass of the ball is m, the radius is R_0, initial velocity is V_1, the mass of the bat is M, and the length is L. Bat suffers the grip force by the hand, and assuming the contact surface of bat and the hand is narrow shape. Suppose a_1 is the acceleration of the ball after hitting a_2 is the rod's tangential acceleration of collision end, a_c is the centroid acceleration of the rod, and β is the angular acceleration of the rod. Suppose the impact force of the ball on the rod is F_1, the compression deformation is δ_1; the contact force of the hand on the rod is F_2, the compression deformation is δ_2.

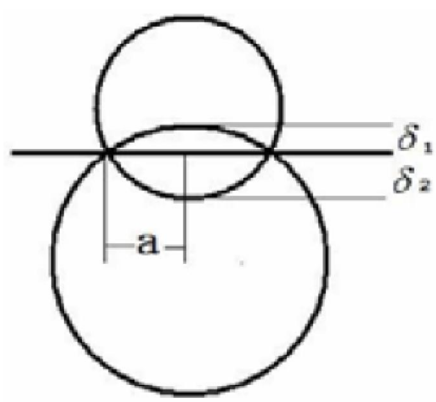

Figure 6 : Contact surface of the rotating body

Throughout the collision process, there are kinematic relations: $\begin{cases} a_1 - a_2 = \delta''_{1(t)} \\ a_2 = a_c + \beta S \\ a_1 = a_c - H\beta = -\delta''_{2(t)} \end{cases}$

According to Newton's law, centroid motion theorem and centroid angular momentum, we have:

$$\begin{cases} -F_1 = ma_1 \\ F_1 + F_2 = Ma_c \\ F_1(R+H+S) - F_2 R = J\beta \end{cases} \Rightarrow$$

$$\begin{cases} \delta''_{1(t)} = F_1\left(-\frac{1}{m} - \frac{1}{M} - \frac{(R+H+S)S}{J}\right) + F_2\left(\frac{RS}{J} - \frac{1}{M}\right) \\ \delta''_{2(t)} = F_1\left(\frac{H(R+H+S)}{J} - \frac{1}{M}\right) - F_2\left(\frac{RH}{J} + \frac{1}{M}\right) \end{cases} \quad (7)$$

The function relationship of F_1 and δ_1, F_2 and δ_2 can be determined by the formula in Hertz contact mechanics:

$$\begin{cases} \delta_{1(t)} = \sqrt[3]{\frac{9}{4}\left(\frac{1-\mu^2}{E}\right)^2 \frac{P_1^2}{R_0}}, \qquad a = \sqrt[3]{\frac{3}{2}\left(\frac{1-\mu^2}{E}\right) R_0 P_1} \\ \delta_{2(t)} = \frac{2(1-\mu^2)P_2}{\pi E e}\left(\ln\frac{\pi E e}{2(1-\mu^2)P_2} + \ln(r'_2 - r'_2) + 0.814\right) \end{cases} \quad (8)$$

μ: Poisson ratio, here it is 0.25.

The impact force of the ball and the rod: $F_1 = \pi a^2 \bar{p} = \frac{2}{3}\pi a^2 p_0$

Combine formula (P) the maximum elastic deformation of the contact point of the ball and the rod can be obtained:

$$\delta_{\max} = \pi^2\left(\frac{1-\mu^2}{E}\right)^2 R_0 {p_0}^2 \quad (9)$$

Substituting formula (9) into formula (7) and substituting parameters can obtain:

$$\delta''_{1(t)} = -K_1[\delta_{1(t)}]^{\frac{3}{2}} + K_2\delta_{2(t)}; \delta''_{2(t)} = K_3[\delta_{1(t)}]^{\frac{3}{2}} - 2K_2\delta_{2(t)}$$

$$K_1 = 3.46\times10^{12} m^{-\frac{1}{2}}\cdot s^{-2}; K_2 = 7.69\times10^{11} m^{-\frac{1}{2}}\cdot s^{-2};$$

$$K_3 = 2.18\times10^{10} s^{-2}; \delta_{1\max} = 6.20\times10^{-5} m$$

After the collision, the system angular momentum conservation:

$$lmV_0 = -lmV_m \int_0^l y\frac{M}{l}dy\left[\omega y - \delta'_{2(t)}\right] = -lmV_m - \frac{l}{2}M\delta'_{2(t)} + \frac{1}{3}Ml^2\omega$$

$$\omega = \left[-\delta'_{1(t)} - V_m + \delta'_{2(t)}\right]/l$$

Where V_m indicates the final speed of the ball.

Therefore, the energy loss before and after the collision is: $\Delta E = \frac{1}{8}M[\delta'_{2(t)}]^2$

Energy of the system loss before and after the collision is in direct proportion to the mass of the rod, the square of the first derivative of the compression deformation. Adding the filler in the bat can increase the first derivative of the compression deformations, so it does not produce the desired good hitting ball effect. Using aluminum rod will cause the bat mass reduction, and reduce the system energy loss, so as to have better batting results.

CONCLUSIONS

This paper used theoretical mechanics, contact mechanics, wave mechanics and so many theories to conduct analysis on the ball-rod collision from the outside performance, the internal mechanism process and multi-angles. Conduct abstract simplification on bat shape to make it easy to describe and modeling the study object. This paper takes the bat's best sweet spot as a starting point to research, through the establishment of the model discusses the sweet spot in the case of the bat's reverse speed gets maximum after hitting, the hands' impact force gets minimum after hitting and the hands receive the minimum energy after hitting. And on this basis it analyzes the striking effect of adding cork in bat and different bat materials relative to ordinary wooden bat, thus reduces the damage on the athlete and plays a guiding role for the material selection and design of the baseball bat.

REFERENCES

[1] Bing Zhang, Yan Feng; The special quality evaluation of the triple jump and the differential equation model of long jump mechanics based on gray correlation analysis. International Journal of Applied Mathematics and Statistics, **40(10)**, 136-143 **(2013)**.

[2] Cai Cui; Application of mathematical model for simulation of 100-meter race. International Journal of Applied Mathematics and Statistics, **42(12)**, 309-316 **(2013)**.

[3] Dai Qun, Ji Cheng-Su, Li Zhong-Hao; The influence of personality on sports injuries among baseball athletes. Journal of Tianjin Institute of Physical Education, **20(2)**, 11-13 **(2005)**.

[4] Haibin Wang, Shuye Yang; An analysis of hurdle performance prediction based on mechanical analysis and gray prediction model. International Journal of Applied Mathematics and Statistics, **39(9)**, 243-250 **(2013)**.

[5] Hongwei Yang; Evaluation model of physical fitness of young tennis athletes based on AHP-TOPSIS comprehensive evaluation. International Journal of Applied Mathematics and Statistics, **39(9)**, 188-195 **(2013)**.

[6] Wang Xiang-Mao; Discussion on the development countermeasures of baseball vocational at home. Journal of Guangzhou Physical Education Institute, **22(3)**, 84-85 **(2002)**.

[7] Wu Jun-Chang, Li Ming-Xian, Xiang Zi-Yuan; Comparison analysis research between striking results by an aluminum bat and a wooden bat. Taipei Journal, **29**, 252-271 **(2005)**.

[8] Xue Qing-Yun, Katsuya Nobuhara; The biomechanics of baseball pitching of shoulder. Journal of Medical Biomechanics, **10(1)**, 35-39 **(1993)**.

[9] Zeng Ling-Jie, Wei Zhi-Hui, Liu Wei-Guo, Biomechanical analysis of shoulder joint injury in baseball game. Journal of Clinical Rehabilitative Tissue Engineering Research, **13(11)**, 2154-2157 **(2009)**.