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On the question of a dynamic solution in general relativity

Abstract

In 1921, the existence of bounded dynamics solutions was raised by Gullstrand. However, some claimed to have explicit examples. It turns out that the bounded plane-wave of Misner, Thorne and Wheeler is due to calculation errors. Wald claimed the second order term of a wave can be obtained, but failed to have an example. Christodoulou and Klainerman claimed to have constructed a set of bounded dynamic solutions. However, such a construction is actually incomplete. 't Hooft came up with a bounded time-dependent solution, but without an appropriate source. The fact is that bounded dynamic solutions for the Einstein equation actually do not exist. For the dynamic case, the non-linear Einstein equation and its linearization also cannot have compatible solutions. The existence of a dynamic solution requires an additional gravitational energy-momentum tensor with an antigravity coupling. Thus, the space-time singularity theorems, which require the same sign for couplings, are irrelevant to physics. The positive energy theorem of Schoen and Yau means only for stable solutions because no bounded dynamic solutions satisfy the requirement of asymptotically flat. However, such recognition is crucial to identify the charge-mass interaction. Its experimental verification means that Einstein's unification between electromagnetism and gravitation is proven valid.

Keywords

Dynamic solution; Gravitational radiation; Principle of causality; The Wheeler school.

INTRODUCTION

The issue of dynamic solutions in general relativity existed from the beginning of this theory until currently. The question started with the calculation of the perihelion of Mercury. In 1915 Einstein obtained the expected value of the remaining perihelion with his theory, and thus was confident of its correctness. The subsequent confirmation of the bending of light, further boosted his confidence. However, unexpectedly the base of his confidence was questioned by Gullstrand[1], the Chairman of the Nobel Prize for Physics[2]. The perihelion of Mercury is actually a many-body problem, but Einstein had not shown that his calculation could be derived from such a necessary step. Thus, Mathematician D. Hilbert, who approved Einstein's initial calculation, did not come to its defense.

In spite of objections from many physicists, Gullstrand stayed on his position, and Einstein was awarded a Nobel Prize by virtue of his photoelectric effects instead of general relativity as expected. The fact is, however, Gullstrand was right. In 1995, it is proven that Einstein's equation is incompatible with gravitational radiation and also does not have a dynamic solution[3,4]. For space-time metric $g_{\mu\nu}$, the Einstein equation of 1915[5] is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -KT(m)_{\mu\nu} \qquad (1)$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci curvature tensor, $T(m)_{\mu\nu}$ is the energy-stress tensor for massive matter, and K (= $8\pi\kappa c^{-2}$, and κ is the Newtonian coupling constant) is the coupling constant. Thus,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0, \text{ or } R_{\mu\nu} = 0 \qquad (1')$$

at vacuum. However, (1') also implies no gravitational wave to carry away energy-momentum[6].

Nevertheless, there are many erroneous claims for the existence of a dynamic solution. Moreover, such claims are not only accepted by the 1993 Nobel Prize Committee for Physics but also Christodoulou was awarded with honor for his errors against the honorable Gullstrand. As a result, normal progress in physics has been hindered[6].

We shall show that such claims against Gullstrand are incorrect. There are serious consequences in science for the error of the mistaken existence of dynamic solutions for the 1915 Einstein equation. Since the equation is incorrect for the dynamic case; it can lead to only erroneous conclusions. A well-known result is the existence of the so-called space-time singularities due to Penrose and Hawking by implicitly assuming the unique sign for all the coupling constants[7]. Another result is that the correct conjecture of Einstein on unification between gravitation and electromagnetism was not recognized. Many theorists simply failed to recognize the crucial charge-mass interaction whose existence leads to the inescapable conclusion of unification[6]. Thus, the criticism of Gullstrand turns out to be very constructive and beneficial.

Due to inadequacy in mathematics, theorists make serious errors in addition to Einstein's limitation. Thus, generations of physicists are misled into serious errors. This paper starts by identifying the most popular errors of the so-called experts. It is also unfortunate that mathematicians also help in perpetuating the errors because they do not understand the physics[6].

ERRORS OF MISNER, THORNE AND WHEELER AND THE ERRORS OF WALD

The Wheeler School led by Misner, Thorne and Wheeler[8] is probably currently most influential. Unfortunately, this school not only makes the error of claiming the existence of dynamic solutions, but also misinterpreted and distorted Einstein's general relativity. Wald[7] wrote another popular book, but also makes different kinds of errors.

A "wave" form considered by Misner, Thorne, & Wheeler[8] is as follows:

$$\mathbf{ds^2 = c^2dt^2 - dx^2 - L^2\left(e^{2\beta}dy^2 + e^{-2\beta}dz^2\right)} \qquad (2)$$

where $L = L(u)$, $\beta = \beta\ (u)$, $u = ct - x$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2L}{du^2} + L\left(\frac{d\beta}{du}\right)^2 = 0 \qquad (3)$$

Misner et al.[8] claimed that Eq. (3) has a bounded solution, compatible with a linearization of metric (2). It has been shown with mathematics at the undergraduate level[9] that Misner et al. are incorrect and Eq. (3) does not have a physical solution that satisfies Einstein's requirement on weak gravity. In fact, $L(u)$ is unbounded even for a very small $\beta\ (u)$.

On the other hand, from the linearization of the Einstein equation (the Maxwell-Newton approximation) in vacuum, Einstein[10] obtained a solution as follows:

$$\mathbf{ds^2 = c^2dt^2 - dx^2 - (1+2\phi)dy^2 - (1-2\phi)dz^2} \qquad (4)$$

where ϕ is a bounded function of $u\ (= ct - x)$. Note that metric (4) is the linearization of metric (1) if $\phi = \beta\ (u)$. Thus, the problem of waves illustrates that the linearization may not be valid for the dynamic case when gravitational waves are involved since eq. (3) does not have a weak wave solution. Since this crucial calculation can be proven with mathematics at the undergraduate level, it should not be surprising that Misner et al.[8] make other serious errors in mathematic and physics such as on the local time in their eq. (40.14).

The root of the errors of Misner et al. was that they incorrectly[3,4] assumed that a linearization of a nonlinear equation would always produce a valid approximation. Thus, they obtained an incorrect conclusion by adopting invalid assumptions. Linearization of (3) yields $L'' = 0$, and in turn this leads to $\beta'\ (u) = 0$. In turn, this leads to a solution $L = C_1 u + 1$ where C_1 is a constant. Therefore, if $C_1 \neq 0$, it contradicts the requirement $L \approx 1$ unless $|u|$ is very small. Moreover, b' (u) = 0 implies that there is no wave. Thus, one cannot get a weak wave solution through linearization of Eq. (3), which has no bounded solution. This shows also that the assumption of metric form (2)[8], which has a weak form (4), is not valid for the Einstein equation.

Many regard a violation of the Lorentz symmetry also as a violation of general relativity. However, this notion actually comes from the distortion of Einstein's equivalence principle by Misner, Thorne, & Wheeler[8] as follows:

"In any and every local Lorentz frame, anywhere and anytime in the universe, all the (non-gravitational) laws of physics must take on their familiar special-relativistic form. Equivalently, there is no way, by experiments confined to infinitesimally small regions of space-time, to distinguish one local Lorentz frame in one region of space-time frame from any other local Lorentz frame in the same or any other region."

They even claimed the above as Einstein's equivalence principle in its strongest form[8]. However, it actually is closer to Pauli's version, which Einstein regards as a misinterpretation[11], as follows:

"For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either in the motion of particles or any physical process."

Thus, Pauli regards the equivalence principle as merely, at each world point P, the existence of a locally constant space, which may not be a local Minkowski metric, though having an indefinite metric.

Apparently, they do not understand or even were unaware of the related mathematics[12]; otherwise they would not make such serious mistakes. The phrase, "must take on" should be changed to "must take on approximately" Also, the phrase, "experiments confined to infinitesimally small regions of space-time" does not make sense since experiments can be conducted only in a finite region. Moreover, in their eq. (40.14) they got an incorrect local time of the earth (in disagreement with Wald[7], Weinberg[13], etc.) Thus, clearly these three theorists[8] failed to understand Einstein's equivalence principle[5,14].

Furthermore, Thorne[15] criticized Einstein's principle with his own distortion as follows:

"In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you (and your reference frame) are very small."

However, Einstein has already explained these problems in his letter of 1953 to Rehtz[11] as follows:

"The equivalence principle does not assert that every gravitational field (e.g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

Perhaps, Thorne did not know that *the term "Einstein elevator" of Bergmann* is misleading. As Einstein[11] explained to Laue, "What characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the Γ^{l}_{ik} (field strength), not the non-vanishing of the R_{iklm}."

Although Einstein's equivalence principle was clearly illustrated only recently[16,17], the Wheeler School should bear the responsibility of their misinformation on this principle[7] by ignoring both crucial work of Einstein, i.e., references [5] and [14], and related theorems[12]. Moreover, they give *the irrelevant incorrect 1911 assumption*[14] *on the equivalence of Newtonian gravity and acceleration as references* although Newtonian gravity is not equivalent to acceleration.

Wald[7], a well-known author in general relativity is also mistaken on the existence of dynamic solutions. According to Einstein[5], in general relativity weak sources would produce a weak field, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}, \text{ where } |\gamma_{\mu\nu}| << 1 \qquad (5)$$

and $\eta_{\mu\nu}$ is the flat metric when there is no source. For the static case, condition (5) is verified although not for the dynamic case. However, according to the principle of causality, condition (5) should be valid; but this is true only if the equation is valid in physics. Many theorists failed to see this because they failed to see the difference between physics and mathematics clearly[16]. In other words, condition (5) for weak gravity and whether the principle of causality is applicable need a rigorous proof.

Unfortunately, many believe that condition (5) for weak gravity is always valid because of accurate predictions for the static case. When the Einstein equation has a weak solution, an approximate weak solution can be derived through the approach of the field equation being linearized. The linearized Einstein equation with the linearized harmonic gauge $\partial^{\mu}\bar{\gamma}_{\mu\nu} = 0$ is

$$\frac{1}{2}\partial^{\alpha}\partial_{\alpha}\bar{\gamma}_{\mu\nu} = \kappa T_{\mu\nu} \text{ where } \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\gamma$$

$$\text{and } \gamma = \eta^{\alpha\beta}\gamma_{\alpha\beta} \qquad (6)$$

Note that we have

$$G_{\mu\nu} = G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)} \text{ and } G^{(1)}_{\mu\nu} = \frac{1}{2}\partial^{\alpha}\partial_{\alpha}\bar{\gamma}_{\mu\nu} -$$

$$\partial^{\alpha}\partial_{\mu}\bar{\gamma}_{\nu\alpha} - \partial^{\alpha}\partial_{\nu}\bar{\gamma}_{\mu\alpha} + \frac{1}{2}\eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{\gamma}_{\alpha\beta} \qquad (7)$$

The linearized vacuum Einstein equation means

$$G^{(1)}_{\mu\nu}[\gamma^{(1)}_{\alpha\beta}] = 0 \qquad (8)$$

Thus, as pointed out by Wald, in order to maintain a solution of the vacuum Einstein equation to second order we must correct $\gamma^{(1)}{}_{\mu\nu}$ by adding to it the term $\gamma^{(2)}{}_{\mu\nu}$, where $\gamma^{(2)}{}_{\mu\nu}$ satisfies

$$G^{(1)}_{\mu\nu}[\gamma^{(2)}{}_{\alpha\beta}] + G^{(2)}_{\mu\nu}[\gamma_{\alpha\beta}] = 0, \text{ where } \gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu} \qquad (9)$$

which is the correct form of eq. (4.4.52) in[7] (Wald did not distinguish $\gamma_{\mu\nu}$ from $\gamma^{(1)}{}_{\mu\nu}$). This equation does have a solution for the static case. However, detailed calculation shows that this equation does not have a solution for the dynamic case[3,4].

If there is no solution for eq. (9), then the Einstein equation does not have a bounded dynamic solution. For instance, metric (4) is the linearization of metric (2), but eq. (3) does not have a bounded wave solution. In conclusion, due to confusion between mathematics and physics, Wald[7] also made errors in mathematics at the undergraduate level. The principle of causality requires the existence of a dynamic solution, but Wald did not see that the Einstein equation can fail this requirement.

Now, consider another well-known metric obtained by Bondi, Pirani, & Robinson[18] as follows:

$$ds^2 = e^{2\varphi}\left(d\tau^2 - d\xi^2\right) - u^2 \begin{bmatrix} \cosh 2\beta\left(d\eta^2 + d\varsigma^2\right) \\ +\sinh 2\beta \cos 2\theta\left(d\eta^2 - d\varsigma^2\right) \\ -2\sinh 2\beta \sin 2\theta \, d\eta \, d\varsigma \end{bmatrix} \quad \textbf{(10a)}$$

where ϕ, β and θ are functions of u (= τ - ξ). It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u\left(\beta'^2 + \theta'^2 \sinh^2 2\beta\right) \quad \textbf{(10b)}$$

They claimed this is a wave from a distant source. (10b) implies ϕ *cannot be a periodic function.* The metric is irreducibly unbounded because of the factor u^2. Both eq. (3) and eq. (10b) are special cases of $G_{\mu\nu} = 0$. However, linearization of (10b) does not make sense since variable u is not bounded. Thus, they claim Einstein's notion of weak gravity invalid because they do not understand the principle of causality adequately.

Moreover, when gravity is absent, it is necessary to have $\phi = \sinh 2\beta = \sin 2\theta = 0$. These would reduce (10a) to

$$ds^2 = \left(d\tau^2 - d\xi^2\right) - u^2\left(d\eta^2 - d\zeta^2\right) \quad \textbf{(10c)}$$

However, this metric is not equivalent to the flat metric. Thus, metric (10c) violates the principle of causality. Also it is impossible to adjust metric (10a) to become equivalent to the flat metric.

This challenges the view that both Einstein's notion of weak gravity and his covariance principle are valid. These conflicting views are supported respectively by the editorials of the "Royal Society Proceedings A" and the "Physical Review D"; thus there is no general consensus. As the Royal Society correctly pointed out[19,20], Einstein's notion of weak gravity is inconsistent with his covariance principle. However, Einstein's covariance principle has been proven invalid since counter examples have been found[21,22]. Moreover, Einstein's notion of weak gravity is supported by the principle of causality.

A major problem is that there are theorists who also ignore the principle of causality. For example, another "plane wave", which is intrinsically non-physical, is the metric accepted by Penrose[23] as follows:

$$ds^2 = du\,dv + Hdu^2 - dx_i dx_i, \textbf{ where } H = h_{ij}(u) x_i x_j \quad \textbf{(11)}$$

where $u = ct - z$, $v = ct + z$. However, there are non-physical parameters (the choice of origin) that are unrelated to any physical causes. Being a mathematician, Penrose[23] over-looked the principle of causality.

Another good example is the plane-wave solution of Liu & Zhou[24], which satisfies the harmonic gauge, is as follows:

$$\mathbf{ds^2 = dt^2 - dx^2 + 2\,F(dt - dx)^2 - \cosh 2\psi(e^{2\phi}\,dy^2 + e^{-2\phi}dz^2)}$$
$$\mathbf{-\,2\sinh 2\psi\; dy\; dz} \quad \textbf{(12)}$$

where $\phi = \phi(u)$ and $\psi = \psi(u)$. Moreover, $F = F_p + H$, where

$$\mathbf{F_P = \frac{1}{2}\,(\dot\psi^2 + \dot\phi^2 \cosh^2 2\psi)\;[\cosh 2\psi\;(e^{2\phi}\,y^2 + e^{-2\phi}z^2)}$$
$$\mathbf{+2\sinh 2\phi\; yz]} \quad \textbf{(13)}$$

and H satisfies the equation,

$$\mathbf{\cosh 2\psi\;(e^{-2\phi}H,_{22} + e^{2\phi}\,H,_{33}) - 2\sinh 2\psi\; H,_{23} = 0} \quad \textbf{(14)}$$

For the weak fields one has $1 >> |\phi|$, $1 >> |\psi|$, but there is no weak approximation as claimed to be

$$\mathbf{ds^2 = dt^2 - dx^2 - (1 + 2\phi)\,dy^2 - (1 - 2\phi)dz^2 - 4\psi dydz} \quad \textbf{(15)}$$

because F_p is not bounded unless $\dot\phi$ and $\dot\psi$ are zero (i.e., no wave).

The linearized equation for a dynamic case has been illustrated as incompatible with the non-linear Einstein equation, which has no bounded dynamic solutions. Thus, Eq. (3), Eq. (10b), and Eq. (12) serve as good simple examples that can be shown through explicit calculation that linearization of the Einstein equation is not valid. Also, metric (11) suggests that the cause of having no physical solution would be due to inadequate source terms[3,25].

An independent supplementary convincing evidence for the absence of a bounded dynamic solution is, as shown by Hu, Zhang & Ting[26], that gravitational radiation calculated would depend on the approach used. This is also a manifestation that there is no bounded solution. A similar problem in approximation schemes such as post-Newtonian approximation[13] is that their validity is also only assumed.

In the pretext of a "modern view", Wald[7] implicitly rejected Einstein's equivalence principle. Wald[7] incorrectly claimed the equivalence of inert mass and the gravitational mass due to Galileo and Newton as "the equivalence principle". However, the 1993 Nobel Prize Committee for Physics adopted this view because of inadequate understanding of the equivalence principle that Einstein emphasizes in his life time. In so doing, Wald also avoided criticizing Misner et al.[8] because they have misidentified Einstein's equivalence principle of 1916 as the invalid 1911 assumption of equivalence between acceleration and Newtonian gravity. However, this also exposed that Wald does not understand Einstein's equivalence principle, which plays a crucial role in establishing the validity of the Maxwell-Newton Approximation independently[27]. Apparently, he did not see that Einstein's covariance principle is invalid in physics.

THE ERRORS OF CHRISTODOULOU

The fact that Christodoulou received honors for his errors related to the Einstein equation testified, "Unthinking respect for authority is the greatest enemy of truth" as Einstein asserted. The strategy of the Nobel Prize based on the recognition time lag failed because mathematical and logical errors can be subtle. Many theorists just do not have the caution, and patience and/or the mathematical background to find out the subtle errors involved as shown in the press release of the Nobel Committee[28].

Due to errors in undergraduate mathematics[29], Christodoulou & Klainerman[30] claimed that they have constructed dynamic solutions. However, one should not be too surprised because Christodoulou obtained his Ph. D. under Wheeler, who also has similar problems in mathematics (see Section 2). Because of the support of the Princeton University, progress in physics did suffer not only from their errors, but also wasting the resources. Fortunately such a struggle comes to an end when their errors can be illustrated with mathematics at the undergraduate level[9,29,31,32]. Moreover, only after the non-existence of a dynamic solution for the Einstein equation was recognized, Einstein's conjecture of the unification between electromagnetism and gravitation is proven correct[3,4,9,12,31-34].

The book of Christodoulou & Klanerman[30] is confusing (see Appendix A). Their main Theorem 1.0.3 states that any strongly asymptotically flat (S.A.F.) initial data set that satisfies the global smallness assumption leads to a unique globally hyperbolic asymptotically flat development. However, because the global smallness assumption has no dynamic requirements in their proofs, there is no assurance for the existence of a dynamic S.A.F. initial data set[29]. Thus, the existence of a bounded dynamic initial set is assumed only, and their proof is at least incomplete.

Mathematician Perlik[35] commented, "What makes the proof involved and difficult to follow is that the authors introduce many special mathematical constructions, involving long calculations, without giving a clear idea of how these building-blocks will go together to eventually prove the theorem. The introduction, almost 30 pages long, is of little help in this respect. Whereas giving a good idea of the problems to be faced and of the basic tools necessary to overcome each problem, the introduction sheds no light on the line of thought along which the proof will proceed for mathematical details without seeing the thread of the story. This is exactly what happened to the reviewer." Essentially, they assume the existence of a bounded initial set to prove the existence of a bounded solution. *Moreover, his initial condition has not been proven as compatible with the Maxwell-Newton approximation which is known to be valid for weak gravity*[29].

This book review originally appeared in ZfM[35] in 1996; and republished in the journal, GRG[36] again with the editorial note, "One may extract two messages: On the one hand, (by seeing e.g. how often this book has been cited), the result is in fact interesting even today, and on the other hand: There exists, up to now no generally understandable proof of it."

The review actually suggests that problems would be adequately identified in the introduction. As shown in [29], the possible nonexistence of their dynamic solutions and its incompatibility with Einstein's radiation formula can be discovered in their introduction. From this review, what the Shaw Prize claimed as "for their highly innovative works on nonlinear partial differential equations in Lorentzian and Riemannian geometry and their applications to general relativity and topology.", in the case of Christodoulou, seems to be just a euphemism for a highly confusing and incomprehensible presentation. These manifest that the authors have not grasped the essence of the problem. Moreover, they seem to try to create enough confusion to gain the acceptance from the readers, with the support of the Princeton University.

Their style of claim is similar to what Misner et al.[8] did. They claimed their plane-wave equation (3), has a bounded plane-wave solution with some confusing invalid calculations. However, careful calculation with undergraduate mathematics shows that this is impossible[9]. Thus, many others like Chistodoulou made or accepted an invalid claim.

Many theorists assume a physical requirement would be unconditionally satisfied by the Einstein equation[16], and such a view was adapted by Christodoulou. According to the principle of causality, a bounded dynamic solution should exist, but this does not necessarily mean that the Einstein equation has such a solution. As shown, the mathematical analysis of Christodoulou is also not reliable at the undergraduate level although he claimed to have such a strong interest in his autobiography.

Gullstrand was not the only theorist who questioned the existence of the bounded dynamic solution for the Einstein equation. As shown by Fock[37], any attempt to extend the Maxwell-Newton approximation (the same as the linearized equation with mass sources[3]) to higher approximations leads to divergent terms. In 1995, it has been proven[3,4] that for a dynamic case the linearized equation as a first order approximation, is incompatible with the nonlinear Einstein field equation. Moreover, the Einstein equation does not have a dynamic solution for weak gravity unless the gravitational energy tensor with an anti-gravity coupling is added to the source (see also eq. [1]). The necessity of an anti-gravity coupling term manifests why a bounded wave solution is impossible for Einstein's equation.

Their book[30] was accepted because it supports and is consistent with existing errors as follows:

(1) It supports errors that created a faith on the existence of dynamical solutions of physicists including Einstein etc.

(2) Due to the inadequacy of the mathematics used, the book was cited before 1996 without referring to the details.

(3) Nobody suspected that professors in mathematics and/or physics could make mistakes at the undergraduate level.

(4) Because physical requirements were not understood, unphysical solutions were accepted as valid[23,38-40].

Thus, in the field of general relativity, strangely there is no expert almost 100 years after its creation.

In physics, a dynamic solution must be related to dynamic sources, but a "time-dependent" solution may not necessarily be a physical solution[18,41,42,93]. To begin with, their solutions are based on dubious physical validity[29]. For instance, their "initial data sets" can be incompatible with the field equation for weak gravity. Second, the only known cases are static solutions. Third, they have not been able to relate any of their constructed solutions to a dynamic source. In pure mathematics, if no example can be given, such abstract mathematics is likely wrong[43].

In fact, there is no time-dependent example to illustrate the claimed dynamics[29]. In 1953 Hogarth[44] conjectured that a dynamic solution for the Einstein equation does not exist. Moreover, in 1995 it is proven impossible to have a bounded dynamic solution because the principle of causality is violated[3].

Nevertheless, the mistakes of the 1993 Nobel Committee probably show that the level of misunderstanding in general relativity then that had led to a number of awards and honors for the errors of D. Christodoulou (Wikipedia) as follows:

MacArthur Fellows Award (1993);

Bôcher Memorial Prize (1999);

Member of American Academy of Arts and Sciences (2001);

Tomalla Foundation Prize (2008);

Shaw Prize (2011);

Member of U.S. National Academy of Sciences (2012).

Note that there are many explicit examples that show the claims of Christodoulou are incorrect[9,31,32,94]. However, due to the practice of biased authority worship, many theorists just ignored them. Physically, a bounded dynamic solution should exist, but Einstein's field equation just does not have such a solution. Now, in view of the facts that Christodoulou's contributions to general relativity are essentially just errors, it is up to the U.S. National Academy of Sciences to handle such a special case.

Note that their book[30] has been criticized by Volker Perlick[35,36] as "incomprehensible". Moreover, S. T. Yau has politely lost his earlier interests on their claims[30]. A correct evaluation of this book should be as an example to show what went wrong in general relativity. The awards and honors to Christodoulou clearly manifested an unpleasant fact that most of the physicists do not understand pure mathematics adequately and many mathematicians do not understand physics.

Note that Damour and Taylor[45,46] were not certain on their Post-Keplerian parameters in generic gravity theories. It will be shown that this is impossible since Einstein equation of 1915 does not have a bounded dynamic solution.

THE GRAVITATIONAL WAVE AND NONEXISTENCE OF DYNAMIC SOLUTIONS FOR EINSTEIN'S EQUATION

First, a major problem is a mathematical error on the relationship between Einstein equation (1) and its "linearization". It was incorrectly believed that the linear Maxwell-Newton Approximation[3]

$$\frac{1}{2}\partial^{c}\partial_{c}\bar{\gamma}_{\mu\nu} = -K\,T(m)_{\mu\nu}, \text{ where}$$

$$\bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\eta^{cd}\gamma_{cd}) \tag{16a}$$

and

$$\bar{\gamma}_{\mu\nu}(x^{i}, t) = -\frac{K}{2\pi}\int \frac{1}{R}\,T_{\mu\nu}[y^{i}, (t-R)]d^{3}y, \text{ where}$$

$$R^{2} = \sum_{i=1}^{3}(x^{i} - y^{i})^{2} \tag{16b}$$

provides the first-order approximation for equation (1). This belief was verified for the static case only.

For a dynamic case, however, this is no longer valid. Note that the Cauchy data cannot be arbitrary for (1). The Cauchy data of (1) must satisfy four constraint equations, $G_{\mu t} = -KT(m)_{\mu t}$ (μ = x, y, z, t) since $G_{\mu t}$ contains only first-order time derivatives[13]. This shows that (16a) would be dynamically incompatible[94] with equation (1)[47]. Further analysis shows that, in terms of both theory[27] and experiments[3], this mathematical incompatibility is in favor of (16), instead of (1).

In 1957, Fock[37] pointed out that, in harmonic coordinates, there are divergent logarithmic deviations from expected linearized behavior of the radiation. This was interpreted to mean merely that the contribution of the complicated nonlinear terms in the Einstein equation cannot be dealt with satisfactorily following this method and that another approach is needed. Subsequently, vacuum solutions that do not involve logarithmic deviation were founded by Bondi, Pirani & Robinson[18] in 1959. Thus, the incorrect interpretation appears to be justified and the faith on the dynamic solutions maintained. It was not recognized until 1995[3] that such a symptom of divergence actually shows the absence of bounded physical dynamic solutions.

In physics, the amplitude of a wave is related to its energy density and its source. Equation (16) shows that a gravitational wave is bounded and is related to the dynamic of the source. These are useful to prove that (16), as the first-order approximation for a dynamic problem, is incompatible with equation (1). Its existing "wave" solutions are unbounded and therefore cannot be associated with a dynamic source[27]. In other words, there is no evidence for the existence of a physical dynamic solution.

With the Hulse-Taylor binary pulsar experiment[48], it became easier to identify that the problem is in (1). Subsequently, it has been shown that (16), as a first-order approximation, can be derived from physical requirements which lead to general relativity[27]. Thus, (16) is on solid theoretical ground and general relativity remains a viable theory. Note, however, that the proof of the nonexistence of bounded dynamic solutions for (1) is essentially independent of the experimental supports for (16).

To prove this, it is sufficient to consider weak gravity since a physical solution must be compatible with Einstein's[5] notion of weak gravity (i.e., if there were a dynamic solution for a field equation, it should have a dynamic solution for a related weak gravity[27]). To calculate the radiation, consider,

$G_{\mu\nu} \equiv G^{(1)}{}_{\mu\nu} + G^{(2)}{}_{\mu\nu}$, where

$$G^{(1)}{}_{\mu\nu} = \frac{1}{2}\partial^c\partial_c\bar{\gamma}_{\mu\nu} + H^{(1)}{}_{\mu\nu} \tag{17a}$$

$$H^{(1)}{}_{\mu\nu} \equiv -\frac{1}{2}\partial^c[\partial_\nu\bar{\gamma}_{\mu c} + \partial_\mu\bar{\gamma}_{\nu c}] + \frac{1}{2}\eta_{\mu\nu}\partial^c\partial^d\bar{\gamma}_{cd}, \text{ and } 1 >> |\gamma_{\mu\nu}| \tag{17b}$$

$G^{(2)}{}_{\mu\nu}$ is at least of second order in terms of the metric elements. For an isolated system located near the origin of the space coordinate system, $G^{(2)}{}_{\mu t}$ at large r (= $[x^2 + y^2 + z^2]^{1/2}$) is of $O(K^2/r^2)$[7,8,13].

One may obtain some general characteristics of a dynamic solution for an isolated system as follows:

(1) The characteristics of some physical quantities of an isolated system:

For an isolated system consisting of particles with typical mass $\bar{M}$, separation $\bar{r}$, and velocities $\bar{v}$, Weinberg[13] estimated, the power radiated at a frequency ω of order $\bar{v}/\bar{r}$ will be of order

$$P \approx \kappa(\bar{v}/\bar{r})^6\bar{M}^2\bar{r}^4 \text{ or } P \approx \bar{M}\,\bar{v}^8/\bar{r} \tag{18}$$

since $\kappa\bar{M}/\bar{r}$ is of order $\bar{v}^2$. The typical deceleration $\bar{a}_{rad}$ of particles in the system owing this energy loss is given by the power P divided by the momentum $\bar{M}\,\bar{v}$, or $\bar{a}_{rad} \approx \bar{v}^7/\bar{r}$. This may be compared with the accelerations computed in Newtonian mechanics, which are of order $\bar{v}^2/\bar{r}$, and with the post-Newtonian correction of $\bar{v}^4/\bar{r}$. Since radiation reaction is smaller than the post-Newtonian effects by a factor $\bar{v}^3$, if $\bar{v} << c$, the velocity of light, the neglect of radiation reaction is perfectly justified. This allows us to consider the motion of a particle in an isolated system as almost periodic.

Consider two particles of equal mass m with an almost circular orbit in the x-y plane whose origin is the center of the circle (i.e., the orbit of a particle is a circle if radiation is neglected). Thus, the principle of causality[25,47] implies that the metric $g_{\mu\nu}$ is weak and very close to the flat metric at distance far from the source and that $g_{\mu\nu}$ (x, y, z, t') is an almost periodic function of t' (= t - r/c).

(2) The expansion of a bounded dynamic solution $g_{\mu\nu}$ for an isolated weak gravitational source:

According (16), a first-order approximation of metric $g_{\mu\nu}$ (x, y, z, t') is bounded and almost periodic since $T_{\mu\nu}$ is. Physically, the equivalence principle requires $g_{\mu\nu}$ to be bounded[27], and the principle of causality requires $g_{\mu\nu}$ to be almost periodic in time since the motion of a source particle is. Such a metric $g_{\mu\nu}$ is asymptotically flat for a large distance r, and the expansion of a bounded dynamic solution is:

$$g_{\mu\nu}(n^x, n^y, n^z, r, t') = \eta_{\mu\nu} + \sum_{k=1}^{\infty} f_{\mu\nu}{}^{(k)}(n^x, n^y, n^z, t')/r^k, \text{ where } n^\nu = x^\nu/r \tag{19a}$$

(3) The non-existence of dynamic solutions:

It follows expansion (19a) that the non-zero time average of $G^{(1)}{}_{\mu t}$ would be of $O(1/r^3)$ due to

$$\partial_\mu n^\nu = (\delta^\nu{}_\mu + n^\nu n_\mu)/r \tag{19b}$$

since the term of $O(1/r^2)$, being a sum of derivatives with respect to t', can have a zero time-average. If $G^{(2)}{}_{\mu t}$ is of $O(K^2/r^2)$ and has a nonzero time-average, consistency can be achieved only if another term of time-average $O(K^2/r^2)$ at vacuum be added to the source of (1). Note that there is no plane-wave solution for (1')[25,47].

It will be shown that there is no dynamic solution for (1) with a massive source. Let us define

$\gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu}$; $\bar{\gamma}^{(i)}{}_{\mu\nu} = \gamma^{(i)}{}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\gamma^{(i)}{}_{cd}\,\eta^{cd})$

where $i = 1, 2$;

and

$$\frac{1}{2}\partial^{\alpha}\partial_{\alpha}\bar{\gamma}^{(1)}{}_{\mu\nu} = -K\,T(m)_{\mu\nu} \quad (20)$$

Then $\bar{\gamma}^{(1)}{}_{\mu\nu}$ is of a first-order; and $\gamma^{(2)}{}_{\mu\nu}$ is finite. On the other hand, from (1), one has

$$\frac{1}{2}\partial^{\alpha}\partial_{\alpha}\bar{\gamma}^{(2)}{}_{\mu\nu} + H^{(1)}{}_{\mu\nu} + G^{(2)}{}_{\mu\nu} = 0 \quad (21)$$

Note that, for a dynamic case, equation (21) may not be satisfied. If (20) is a first-order approximation, $G^{(2)}{}_{\mu\nu}$ has a nonzero time-average of $O(K^2/r^2)$[8]; and thus $\bar{\gamma}^{(2)}{}_{\mu\nu}$ cannot have a solution.

However, if $\bar{\gamma}^{(2)}{}_{\mu\nu}$ is also of the first-order of K, one cannot estimate $G^{(2)}{}_{\mu\nu}$ by assuming that $\bar{\gamma}^{(1)}{}_{\mu\nu}$ provides a first-order approximation. For example, (6) does not provide the first approximation for the static Schwarzschild solution, although it can be transformed to a form such that (6) provides a first-order approximation[27]. According to (7), $\bar{\gamma}^{(2)}{}_{\mu\nu}$ will be a second order term if the sum $H^{(1)}{}_{\mu\nu}$ is of second order. From (17b), this would require $\partial^{\mu}\bar{\gamma}_{\mu\nu}$ being of second order. For weak gravity, it is known that a coordinate transformation would turn $\partial^{\mu}\bar{\gamma}_{\mu\nu}$ to a second order term (can be zero)[13,37,49]. (Eq. [21] implies that $\partial^{c}\partial_{c}\gamma^{(2)}{}_{\mu\nu} - \partial^{c}[\partial_{\nu}\bar{\gamma}_{\mu c} + \partial_{\mu}\bar{\gamma}_{\nu c}]$ would be of second order.) Thus, it is always possible to turn (20) to become an equation for a first-order approximation for weak gravity.

From the viewpoint of physics, since it has been proven that (16) necessarily gives a first-order approximation[27], a failure of such a coordinate transformation means only that such a solution is not valid in physics. Moreover, for the dynamic of massive matter, experiment[48] supports the fact that Maxwell-Newton Approximation (16) is related to a dynamic solution of weak gravity[3]. Otherwise, not only is Einstein's radiation formula not valid, but the theoretical framework of general relativity, including the notion of the plane-wave as an idealization, should be re-examined. In other words, theoretical considerations in physics as well as experiments eliminate other unverified speculations thought to be possible since 1957.

As shown, the difficulty comes from the assumption of boundedness, which allows the existence of a bounded first-order approximation, which in turn implies that a time-average of the radiative part of $G^{(2)}{}_{\mu\nu}$ is non-zero. The present method has an advantage over Fock's approach to obtaining logarithmic divergence[3,37] for being simple and clear.

In short, according to Einstein's radiation formula, a time average of $G^{(2)}{}_{\mu t}$ is non-zero and of $O(K^2/r^2)$[3]. Although (16) implies $G^{(1)}{}_{\mu t}$ is of order K^2, its terms of $O(1/r^2)$ can have a zero time average because $G^{(1)}{}_{\mu t}$ is linear on the metric elements. Thus, (1') cannot be satisfied. Nevertheless, a static metric can satisfy (1), since both $G^{(1)}{}_{\mu\nu}$ and $G^{(2)}{}_{\mu\nu}$ are of $O(K^2/r^4)$ in vacuum. Thus, that a gravitational wave carries energy-momentum does not follow from the fact that $G^{(2)}{}_{\mu\nu}$ can be identified with a gravitational energy-stress[8,13]. Just as $G_{\mu\nu}$, $G^{(2)}{}_{\mu\nu}$ should be considered only as a geometric part. Note that $G_{\mu t} = -KT(m)_{\mu t}$ are constraints on the initial data.

In conclusion, in disagreement with the physical requirement, assuming the existence of dynamic solutions of weak gravity for (1)[18,37,50-55] is invalid. This means that the calculations[45,46] on the binary pulsar experiments should, in principle, be re-addressed[56,95]. This explains also that an attempt by Christodoulou and Klainerman[30] to construct bounded "dynamic" solutions for $G_{\mu\nu} = 0$ fails to relate to a dynamic source and to be compatible with (16)[29] although their solutions do not imply that a gravitational wave carries energy-momentum.

For a problem such as scattering, although the motion of the particles is not periodic, the problem remains. This will be explained (see Section 6) in terms of the 1995 update of the Einstein equation, due to the necessary existence of the gravitational energy-momentum tensor term with an antigravity coupling in the source. To establish the 1995 update equation, the supports of binary pulsar experiments for (3) are needed[3].

GRAVITATIONAL RADIATIONS, BOUNDEDNESS OF PLANE-WAVES, AND THE MAXWELL-NEWTON APPROXIMATION

An additional piece of evidence is that there is no plane-wave solution for (1). A plane-wave is a spatial-local idealization of a weak wave from a distant source. The plane-wave propagating in the z-direction is a physical model although its total energy is infinite[13,47]. According to (16), one can substitute (t - R) with (t - z) and the other dependence on r can be neglected because r is very large. This results in $\bar{\gamma}_{\mu\nu}(x^i, t)$ becoming a bounded periodic function of (t - z). Since the Maxwell-Newton Approximation provides the first-order, the exact plane-wave as an idealization is a bounded periodic function. Since the dependence of 1/r is neglected, one considers essentially terms of $O(1/r^2)$ in

$G^{(2)}{}_{\mu\nu}$. In fact, the non-existence of bounded plane-wave for $G_{\mu\nu} = 0$, was proven directly in 1991[25,49]. In short, Einstein & Rosen[57,58] is essentially right, i.e., there are no wave solutions for $R_{\mu\nu} = 0$. The fact that the existing "wave" solutions are unbounded also confirms the nonexistence of dynamic solutions. The failure to extend from the linearized behavior of the radiation is due to the fact that there is no bounded physical wave solution for (1) and thus this failure is independent of the method used.

Note that the Einstein radiation formula depends on (16) as a first-order approximation. Thus, metric $g_{\mu\nu}$ must be bounded. Otherwise $G_{\mu\nu} = 0$ can be satisfied. For example, the metric of Bondi et al.[18] (see eq. 10) is not bounded, because this would require the impossibility of $u^2 <$ constant. An unbounded function of u, f(u) grows anomaly large as time τ goes by. It should be noted also that metric (10) is only a plane, but not a periodic function because a smooth periodic function must be bounded. This unboundedness is a symptom of unphysical solutions because they cannot be related to a dynamic source (see also [25, 27]). Note that solution (10) can be used to construct a smooth one-parameter family of solutions[27] although solution (10) is incompatible with Einstein's notion of weak gravity[5].

In 1953, questions were raised by Schiedigger[59] as to whether gravitational radiation has any well-defined existence. The failure of recognizing $G_{\mu\nu} = 0$ as invalid for gravitational waves is due to mistaking (16) as a first-order approximation of (1). Thus, in spite of Einstein's discovery[60] and Hogarth's conjecture[44,96] on the need of modification, the incompatibility between (1) and (16) was not proven until 1993[3] after the non-existence of the plane-waves for $G_{\mu\nu} = 0$, had been proven.

THE INVALIDITY OF SPACE-TIME SINGULARITY THEOREMS AND THE UPDATE OF THE EINSTEIN EQUATION

In general, (16) is actually an approximation of the 1995 update of the Einstein equation[3],

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R = - K\,[T(m)_{\mu\nu} - t(g)_{\mu\nu}] \qquad (22)$$

where $t(g)_{\mu\nu}$ is the energy-stress tensors for gravity. Then,

$$\nabla^{\mu}T(m)_{\mu\nu} = 0, \text{ and } \nabla^{\mu}t(g)_{\mu\nu} = 0 \qquad (23)$$

Equation (22) implies that the equivalence principle would be satisfied. From (22), the equation in vacuum is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R = K\, t(g)_{\mu\nu} \qquad (22')$$

Note that $t(g)_{\mu\nu}$ is equivalent to $G^{(2)}{}_{\mu\nu}$ (and Einstein's gravitational pseudotensor) in terms of his radiation formula. The fact that $t(g)_{\mu\nu}$ and $G^{(2)}{}_{\mu\nu}$ are related under some circumstances does not cause $G^{(2)}{}_{\mu\nu}$ to be an energy-stress nor $t(g)_{\mu\nu}$ a geometric part, just as $G_{\mu\nu}$ and $T_{\mu\nu}$ must be considered as distinct in (1).

When gravitational wave is present, the gravitational energy-stress tensor $t(g)_{\mu\nu}$ is non-zero. Thus, a radiation does carry energy-momentum as physics requires. This explains also that the absence of an anti-gravity coupling which is determined by Einstein's radiation formula, is the physical reason that the 1915 Einstein equation (1) is incompatible with radiation.

Note that the radiation of the binary pulsar can be calculated without detailed knowledge of $t(g)_{\mu\nu}$. From (22'), the approximate value of $t(g)_{\mu\nu}$ at vacuum can be calculated through $G_{\mu\nu}/K$ as before since the first-order approximation of $g_{\mu\nu}$ can be calculated through (16). In view of the facts that $Kt(g)_{\mu\nu}$ is of the fifth order in a post-Newtonian approximation, that the deceleration due to radiation is of the three and a half order in a post-Newtonian approximation[13] and that the perihelion of Mercury was successfully calculated with the second-order approximation from (1), the orbits of the binary pulsar can be calculated with the second-order post-Newtonian approximation of (22) by using (1). Thus, the calculation approaches of Damour and Taylor[45,46] would be essentially valid except that they did not realize the crucial fact that (16) is actually an approximation of the updated equation (22)[16].

In light of the above, the Hulse-Taylor experiments support the anti-gravity coupling being crucial to the existence of the gravitational wave[3,47], and (16) being an approximation of weak waves generated by massive matter. Thus, it has been experimentally verified that (1) is incompatible with radiation.

It should be noted that the existence of an anti-gravity coupling[97] means the energy conditions in the singularity theorems[8,61] are not valid for a dynamic situation. Thus, the existence of singularity is not certain, and the claim of inevitably breaking of general relativity is actually baseless since these singularity theorems have been proven to be unrealistic in physics. As pointed out by Einstein[5], his equation may not be valid for very high density of field and matter. In short, the singularity theorems show only the breaking down of theories of the Wheeler-Hawking school, which are actually different[98]. The theories of this school, in addition to making crucial mistakes in mathematics as shown in this paper (see also [27, 29]), differ from general relativity in at least the following important aspects:

(1) They reject an anti-gravity coupling[97], which is considered as highly probable by Einstein himself.
(2) They implicitly replaced Einstein's equivalence principle in physics[97] with merely the mathematical requirement of the existence of local Minkowski spaces[7,61].
(3) They, do not consider physical principles[25,27,29,47] (see also Section 5), such as the principle of causality, the coordinate relativistic causality, the correspondence principle, etc. of which the satisfaction is vital for a physical space, which models reality, such that Einstein's equivalence principle can be applicable.

Thus, in spite of declaring their theories as the development of general relativity[7,8], these theories actually contradict crucial features that are indispensable in Einstein's theory of general relativity. More importantly, in the development of their so-called "orthodox theory," they violate physical principles that took generations to establish. As a result, Einstein's theory has been unfairly considered as irrelevant in the eyes of many physicists. They also support their accusations with false evidence[8,15].

Of course, the exact form of $t(g)_{\mu\nu}$ is important for the investigation of high density of field. However, the physics of very high density of field and matter seems to be not yet mature enough at present to allow a definitive conclusion. For instance, it is unclear what influence the discovery of quarks and gluons in particle physics would have on the evolution of stars. It is known that atomic physics supports the notion of white-dwarf stars, and that nuclear physics leads to the notion of neutron stars.

UNBOUNDED "GRAVITATIONAL WAVES", THE PRINCIPLE OF CAUSALITY, AND THE ERRORS OF 't Hooft

"To my mind there must be at the bottom of it all, not an equation, but an utterly simple idea. And to me that idea, when we finally discover it, will be so compelling, so inevitable, that we will say to one another, 'Oh, how beautiful. How could it have been otherwise?' " — J. A. Wheeler[62].

It seems, the principle of causality[99] (i.e., phenomena can be explained in terms of identifiable causes)[25,47] would be qualified as Wheeler's utterly simple idea. Being a physicist, his notion of beauty should be based on compelling and inevitability, but would not be based on some perceived mathematical ideas. It will be shown that the principle of causality is useful in examining validity of accepted "waves".

According to the principle of causality, a wave solution must be related to a dynamic source, and therefore is not just a time-dependent metric. A time-dependent solution, which can be obtained simply by a coordinate transformation, may not be related to a dynamic source[63,100]. Even in electrodynamics, satisfying the vacuum equation can be insufficient. For instance, the electromagnetic potential solution $A_0[\exp(t - z)^2]$ (A_0 is a constant), is not valid in physics because one cannot relate such a solution to a dynamic source. Thus, a solution free of singularities may not be physically valid.

A major problem in relativity is that the equivalence principle has not been understood adequately[27,64]. Since a Lorentz manifold was mistaken as always valid, physical principles were often not considered. For instance, the principle of causality was neglected such that a gravitational wave was not considered as related to a dynamic source, which may not be just the source term in the field equation[13,65].

Since the principle of causality was not understood adequately, solutions with arbitrary nonphysical parameters were accepted as valid[64]. Among the existing "wave" solutions, not only Einstein's equivalence principle but also the principle of causality is not satisfied because they cannot be related to a dynamic source. (However, a source term may not necessarily represent the physical cause[25,64].)

Here, examples of accepted "gravitational waves" are shown as invalid.

Let us examine the cylindrical waves of Einstein & Rosen[58]. In cylindrical coordinates, ρ, φ, and z,

$$ds^2 = \exp(2\gamma - 2\Psi)(dT^2 - d\rho^2) - \rho^2\exp(-2\Psi)d\varphi^2 - \exp(2\Psi)dz^2 \qquad (24)$$

where T is the time coordinate, and γ and ψ are functions of ρ and T. They satisfy the equations

$$\Psi_{\rho\rho} + (1/\rho)\Psi_\rho - \Psi_{TT} = 0,\ \gamma_\rho = \rho[\Psi_\rho{}^2 + \Psi_T{}^2],\ \text{and } \gamma_T = 2\rho\Psi_\rho\Psi_T \qquad (25)$$

Rosen[66] consider the energy-stress tensor $T_{\mu\nu}$ that has cylindrical symmetry. He found that

$$T_4{}^4 + t_4{}^4 = 0, \text{ and } T_4{}^l + t_4{}^l = 0 \qquad (26)$$

where $t_{\mu\nu}$ is Einstein's gravitational pseudotensor, $t_4{}^l$ is momentum in the radial direction.

However, Weber & Wheeler[67] argued that these results are meaningless since $t_{\mu\nu}$ is not a tensor. They further pointed out that the wave is unbounded and therefore the energy is undefined. Moreover, they claimed metric (24) satisfying the equivalence principle and speculated that the energy flux is non-zero.

Their claim shows an inadequate understanding of the equivalence principle. To satisfy this principle requires that a time-like geodesic must represent a physical free fall. This means that all (not just some) physical requirements are necessarily satisfied. Thus, the equivalence principle may not be satisfied in a Lorentz Manifold[27,65], which implies only the necessary condition of the mathematical existence of a co-moving local Minkowski space along a time-like geodesic. It will be shown that manifold (24) cannot satisfy coordinate relativistic causality. Moreover, as pointed out earlier, an unbounded wave is unphysical.

Weber and Wheeler's arguments for unboundedness are complicated, and they agreed with Fierz's analysis, based on (25), that γ is a strictly positive where $\Psi = 0$[67]. However, it is possible to see that there is no physical wave solution in a simpler way. Gravitational red shifts imply that $g_{tt} \leq 1$[5]; and

$$-g_{\rho\rho} \geq g_{tt},\ -g_{\varphi\varphi}/\rho^2 \geq g_{tt}, \text{ and } -g_{zz} \geq g_{tt} \tag{27a}$$

are implied by coordinate relativistic causality. According to these constraints, from metric (24) one has

$$\exp(2\gamma) \leq 1 \text{ and } \exp(2\gamma) \leq \exp(2\Psi) \tag{27b}$$

Equation (27) implies that $g_{tt} \leq 1$ and that $\gamma \leq 0$. However, this also means that the condition $\gamma > 0$ cannot be met. Thus, this shows again that there is no physical wave solution for $G_{\mu\nu} = 0$.

Weber and Wheeler are probably the earliest to show the unboundedness of a wave solution for $G_{\mu\nu} = 0$. Nevertheless, due to their inadequate understanding of the equivalence principle, they did not reach a valid conclusion. It is ironic that they therefore criticized Rosen who came to a valid conclusion, though with dubious reasoning.

Robinson and Trautman[68] dealt with a metric of spherical "gravitational waves" for $G_{\mu\nu} = 0$. However, their metric has the same problem of unboundedness and having no dynamic source connection. Their metric has the following form:

$$ds^2 = 2d\rho d\sigma + (K - 2H\rho - 2m/\rho)d\sigma^2 - \rho^2 p^{-2}\{[d\xi + (\partial q/\partial\eta)d\sigma]^2 + [d\eta + (\partial q/\partial\xi)d\sigma]^2\} \tag{28a}$$

where m is a function of σ only, p and q are functions of σ, ξ, and η,

$$H = p^{-1}\partial p/\partial\sigma + p\partial^2 p^{-1} q/\partial\xi\partial\eta - pq\,\partial^2 p^{-1}/\partial\xi\partial\eta \tag{28b}$$

and K is the Gaussian curvature of the surface $\rho = 1$, σ = constant,

$$K = p^2(\partial^2/\partial\xi^2 + \partial^2/\partial\eta^2)\ln p \tag{28c}$$

For this metric, the empty-space condition $G_{\mu\nu} = 0$ reduces to

$$\partial^2 q/\partial\xi^2 + \partial^2 q/\partial\eta^2 = 0, \text{ and } \partial^2 K/\partial\xi^2 + \partial^2 K/\partial\eta^2 = 4p^{-2}(\partial/\partial\sigma - 3H)m \tag{29}$$

To see this metric has no dynamic connection, let us examine their special case as follows:

$$ds^2 = 2d\rho d\sigma - 2Hd\sigma^2 - d\xi^2 - d\eta^2, \text{ and } \partial H/\partial\rho = \partial^2 H/\partial\xi^2 + \partial^2 H/\partial\eta^2 = 0 \tag{30}$$

This is a plane-fronted "wave"[69] derived from metric (28) by specializing

$$p = 1 + (\xi^2 + \eta^2)K(\sigma)/4 \tag{31a}$$

substituting

$$\rho = \lambda^{-2} + \lambda^{-1}\bar{\rho},\ \sigma = \lambda\bar{\sigma},\ \xi = \lambda^2\bar{\xi},\ \eta = \lambda^2\bar{\eta},\ q = \lambda^4\bar{q} \tag{31b}$$

where λ is constant, and taking the limit as λ tends to zero[66]. Although (30) is a Lorentz metric, there is a singularity on every wave front where the homogeneity conditions

$$\partial^3 H/\partial\xi^3 = \partial^3 H/\partial\eta^3 = 0 \tag{32}$$

are violated[68]. Obviously, this is also incompatible with Einstein's notion of weak gravity[5]. A problem in current theory is its rather insensitivity toward theoretical self-consistency[1,3,25,65,70].

To illustrate an invalid source and an intrinsic non-physical space, consider the following metric,

$$ds^2 = du\,dv + Hdu^2 - dx_i\,dx_i, \text{ where } H = h_{ij}(u)x_i x_j \tag{11}$$

where $u = ct - z$, $v = ct + z$, $x = x_1$ and $y = x_2$, $h_{ii}(u) \geq 0$, and $h_{ij} = h_{ji}$[23]. This metric satisfies the harmonic gauge. The cause of metric (11) can be an electromagnetic plane wave and satisfies the Einstein equation. However, this does not mean that causality is satisfied although metric (11) is related to a dynamic source. It violates the principle of causality because it involves unphysical parameters, the choice of origin of the coordinates.

Apparently, Penrose[23] over-looked the physical requirements, in particular the principle of causality. Being unbounded, metric (11) is also incompatible with the calculation of light bending and classical electrodynamics. These examples confirm that there is no bounded wave solution for (1) although a "time-dependent" solution need not be logarithmic divergent[37]. However, in defense of the errors of the 1993 Nobel Committee for Physics, 't Hooft[71,72] comes up with a bounded time-dependent cylindrical symmetric solution as follows:

$$\Psi(r,t) = A\int_0^{2\pi} d\varphi\, e^{-\alpha(t - r\cos\varphi)^2} \tag{33}$$

where A and α (> 0) are free parameters. For simplicity, take them to be one. $|\Psi|$ is everywhere bounded. He claimed that, at large values for t and r, the stationary points of the cosine dominate, so that there are peaks at $r = |t|$. And this is a packet coming from $r = \infty$ at $t = -\infty$ bouncing against the origin at $t \sim 0$, and moving to $r = \infty$ again at $t \to \infty$.

Now, consider that $x = r\cos\varphi$ in a coordinate system (r, φ, z). Then, 't Hooft[71] claimed that his solution, Ψ is obtained by superimposing plane wave packets of the form $exp[-\alpha(x - t)^2]$ rotating them along the z axis over angle φ, so as to obtain a cylindrical solution. Note that since the integrand $exp[-\alpha(t - r\cos\varphi)^2] = exp[-\alpha(t - x)^2]$, there is no rotation along the z axis. The function $exp[-\alpha(t - x)^2]$ is propagating from $x = -\infty$ to $x = \infty$ as time t increases.

Note, however, that in a superimposition the integration is over a parameter of frequency ω unrelated to the x-axis; whereas the solution (33) is integrated over $\varphi(x, y)$. Since, (33) is a combination that involves the coordinate φ (x, y), it is not a superimposition of plane waves propagating along the x-axis. Furthermore, the integration over all angles φ is a problem that would violate the requirement of the idealization because it requires that the plane wave is valid over the whole x-y plane. Thus, function (33) is not valid as an idealization in physics.

Therefore, in solution (33), two errors have been made, namely: 1) the plane wave has been implicitly extended beyond its physical validity, and 2) the integration over $d\varphi$ is a process without a valid physical justification. Moreover, it has been shown that there are no valid sources that can be related to solution (33)[72]. Thus, the principle of causality is also violated.

ERRORS OF THE MATHEMATICIANS, SUCH AS ATIYAH, PENROSE, WITTEN, AND YAU

While physicists can make errors because of inadequate background in mathematics, one may expect that professional mathematician would help improve the situation. However, this expectation may not always be fulfilled. The reason is that a mathematician may not understand the physics involved and thus not only he may not be helpful, but also could make the situation worse. A good example is the cooperation between Einstein and Grossman. They even wrote a joint paper, but from which it is clear that they did not understand each other. However, the situation could be worse if they had misled each other.

A bad example is the participation in physics by mathematician Roger Penrose. He was misled by imperfect understanding of the physical situation that $E= mc^2$ is always valid and thus he believed that all the coupling of energy-momentum tensors must have the same sign. Then his mathematical talent comes up with the space-time singularity theorems without realizing such an assumption is invalid in physics. Then, he and Hawking convince the physicists that such singularities must exist and general relativity is not valid in the microscopic scale. Thus, an accompanying error is that physicists failed to see that the photons must include gravitational components since photons can be converted into mass[73,74]. This is supported by the experimental fact that the meson π_0 can decay into two γ rays. However, the electromagnetic energy alone cannot be equivalent to mass because the trace of the massive energy-momentum tensor is non-zero.

Another example is the positive energy theorem of Schoen and Yau[75]. From the free encyclopedia Wikipedia, the contributions of Professor Yau were summarized as follows:

"Yau's contributions have had a significant impact on both physics and mathematics. Calabi–Yau manifolds are among the 'standard tool kit' for string theorists today. He has been active at the interface between geometry and theoretical physics. His proof of the positive energy theorem in general relativity demonstrated—sixty years after its discovery—that Einstein's theory is consistent and stable. His proof of the Calabi conjecture allowed physicists—using Calabi-Yau compactification—to show that string theory is a viable candidate for a unified theory of nature."

Thus, it was claimed that Yau's "proof " of the positive energy theorem[75,76] in general relativity would have profound influence that leads to even the large research efforts on string theory. Based on his proof, it was claimed that Einstein's theory is consistent and stable. This would be in a direct conflict with the fact the there is no dynamic solution for the Einstein equation.

Now, let us examine their theorem that would imply that flat space-time is stable, a fundamental issue for the theory of general relativity. Briefly, the positive mass conjecture says that if a three-dimensional manifold has positive scalar curvature and is asymptotically flat, then a constant that appears in the asymptotic expansion of the metric is positive. A crucial assumption in the theorem of Schoen and Yau is that the solution is asymptotically flat. However, since the Einstein equation has no dynamic solution, which is bounded, the assumption of asymptotically flat implies that the solution is a stable solution such as the Schwarzschild solution, the harmonic solution, the Kerr solution, etc.

Therefore, Schoen and Yau actually prove a trivial result that the total mass of a stable solution is positive. Note that since the dynamic case is actually excluded from the consideration in the positive energy theorem, this explains why it was found from such a theorem that Einstein's theory is consistent and stable. This is, of course, misleading.

Due to inadequacy in pure mathematics among physicists, the non-existence of dynamic solutions for the Einstein equation was not recognized. So, Yau could only assume the existence of a bounded dynamic solution. Thus, the positive energy theorem of Schoen and Yau also continues such an error. Although Yau may not have made errors in mathematics, their positive energy theorem produced not only just useless but also misleading results in physics. Yau failed to see this problem of misleading since he has not attempted to find explicit examples to illustrate their theorem. Nevertheless, in awarding him a Fields Medal in 1982, this theorem is cited as an achievement. Moreover, E. Witten made the same mistake in his alternative proof[76], but that was also cited as an achievement for his Fields Medal in 1990. Apparently, Atiyah also failed to understand this issue[101].

In fact, Yau[75] and Christodoulou[30] make essentially the same error of defining a set of solutions that actually includes no dynamic solutions. Their fatal error is that they neglected to find explicit examples to support their claims. Had they tried, they should have discovered their errors. Note that Yau has wisely avoided committing himself to the errors of Christodoulou & Klainerman, by claiming that his earlier interest has changed[30]. However, he was unable to see that the binary pulsars experiment of Hulse & Taylor not only confirms that there is no dynamic solution but also that the signs of coupling constants are not unique[3]. In fact, Yau has made the same errors of Penrose and Hawking[9], and implicitly uses the invalid assumption of unique sign in his positive energy theorem of 1981. Nevertheless, Prof. Yau is a good mathematician as shown by his other works although he does not understand physics well. Since the Einstein equation must be modified for a dynamic case, their positive energy theorem is also irrelevant to physics just as the space-time singularity theorem of Penrose and Hawking.

The facts that Atiyah[31], Hilbert[2], Witten, and Yau were unable (or neglected) to identify their errors, would misleadingly created a false impression that Einstein, the Wheeler School, and their associates did not make errors in mathematics.

CONCLUSIONS AND DISCUSSIONS

In general relativity, the existence of gravitational wave is a crucial test of the field equation. Thus, an important question is: what does the gravitational field of a radiating asymptotically Minkowskian system look like? Without experimental inputs, to answer this question would be very difficult.

Bondi[55] commented, "it is never entirely clear whether solutions derived by the usual method of linear approximation necessarily correspond in every case to exact solutions, or whether there might be spurious linear solutions which are not in any sense approximations to exact ones." Thus, in calculating gravitational waves from the Einstein equation, problems are considered as due to the method rather than inherent in the equations.

Physically, it is natural to continue assuming Einstein's notion of weak gravity is valid. (Boundedness, though a physical requirement, may not be mathematically compatible to a nonlinear field equation. But, no one except perhaps Gullstrand[1], expected the nonexistence of dynamic solutions.) The complexity of the Einstein equation makes it very difficult to have a closed form. Thus, it is necessary that a method of expansion should be used to examine the problem of weak gravity.

A factor which contributes to this faith is that $\nabla^{\mu}G_{\mu\nu} \equiv 0$ implies $\nabla^{\mu}T(m)_{\mu\nu} = 0$, the energy-momentum conservation law. However, this is only necessary but not sufficient for a dynamic solution. Although the 1915 equation gives an excellent description of planetary motion, including the advance of the perihelion of Mercury, this is essentially a test-particle theory, in which the reaction of the test particle is neglected. Thus, the so obtained solutions are not dynamic solutions. As pointed out by Gullstrand[1] such a solution may not be obtainable as a limit of a dynamic solution. Nevertheless, Einstein, Infeld, and Hoffmann[53] incorrectly assumed the existence of a bounded dynamic solution and deduced the geodesic equation from the 1915 equation. Recently, Feymann[54] made the same incorrect assumption that a physical requirement would be unconditionally applicable.

The nonlinear nature of Einstein equation certainly gives surprises. In 1959, Fock[37] pointed out that, in harmonic coordinates, there are divergent logarithmic deviations from expected linearized behavior of the radiation. After the discovery that some vacuum solutions are not logarithmic divergent[18], the inadequacy of Einstein's equation was not recognized. Instead, the method of calculation was mistaken as the problem.

To avoid the appearance of logarithms, Bondi et al.[55] and Sachs[77] introduced a new approach to gravitational radiation theory. They used a special type of coordinate system, and instead of assuming an asymptotic expansion in the gravitational coupling constant κ, they assume the existence of an asymptotic expansion in inverse power of the distance r (from the origin where the isolated source is located in $r \leq a$, which is a positive constant). The approach of Bondi-Sachs was clarified by the geometrical 'conformal' reformulation of Penrose[78].

However, this approach is unsatisfactory, "because it rests on a set of assumptions that have not been shown to be satisfied by a sufficiently general solution of the inhomogeneous Einstein field equation[79]." In other words, this approach provides only a definition of a class of space-times that one would like to associate to radiative isolated systems, neither the global consistency nor the physical appropriateness of this definition has been proven. Moreover, perturbation calculations have given some hints of inconsistency between the Bondi-Sachs-Penrose definition and some approximate solution of the field equation.

There are two other main classes of approach: 1) the post-Newtonian approaches (1/c expansions) and the post-Minkowskian approaches (K expansions). The post-Newtonian approaches are fraught with serious internal consistency problems[79] because they often lead, in higher approximations, to divergent integrals. The post-Minkowskian approach is an extension of the linearization, one may expect that there are some problems related to divergent logarithmic deviations[37]. Moreover, it has unexpectedly been found that perturbative calculations on radiation actually depend on the approach chosen[26].

Mathematically, this non-uniqueness shows, in disagreement with (3), that a dynamic solution of (1) is unbounded. Also, based on the binary pulsar experiments, it is proven that the Einstein equation does not have any dynamic solution even for weak gravity[3]. This long process is, in part, due to theoretical consistency that was inadequately considered[3,25,47,65]. Moreover, it was not recognized that boundedness of a wave is crucial for its association with a dynamic source. These inadequacies allowed acceptance of unphysical "time-dependent" solutions as physical waves (Sections 5 & 7).

In view of impressive observational confirmations, it seemed natural to assume that gravitational waves would be produced. Moreover, gravitational radiation is often considered as due to the acceleration in a geodesic alone[80-82]. It is remarkable that in 1936 Einstein and Rosen[57] are the first to discover this problem of excluding the gravitational wave. However, without clear experimental evidence, it was difficult to make an appropriate modification.

From studying the gravity of electromagnetic waves, it was also clear that Einstein equation must be modified[27,49]. However, the Hulse and Taylor binary pulsar experiments, which confirm Hogarth's 1953 conjecture[61,65], are indispensable for verifying the necessity of the anti-gravity coupling in general relativity[3,47]. In addition to experimental supports, the Maxwell-Newton Approximation can be derived from physical principles, and the equivalence principle also implies boundedness of a normalized metric in general relativity[27]. A perturbative approach cannot be fully established for (1) simply because there are no bounded dynamic solutions[102], which must, owing to radiation, be associated with an anti-gravity coupling.

Nevertheless, Christodoulou and Klainerman[30] claimed to have constructed bounded gravitational (unverified) waves. Obviously, their claim is incompatible with the findings of others. Furthermore, their presumed solutions are incompatible with Einstein's radiation formula and are unrelated to dynamic sources[27,47]. Thus, they simply have mistaken an unphysical assumption (which does not satisfy physical requirements) as a wave[29,103]. Thus, their book serves as evidence that the Princeton University can be wrong just as any human institute[104].

Within the theoretical framework of general relativity, however, the gravitational field of a radiating asymptotically Minkowskian system is given by the Maxwell-Newton Approximation[3]. Note that, for the dynamic case, the Maxwell-Newton Approximation is a linearization of the up-dated modified Einstein equation of 1995, but not the Einstein equation, which has no bounded dynamic solution. With the need of rectifying the 1915 Einstein equation established, the exact form of $t(g)_{\mu\nu}$ in the equation of 1995 update[3] is an important problem since a dynamic solution that gives an approximation for the perihelion of Mercury remains unsolved[1]. Moreover, the updated equation shows that the singularity theorems prove only the breaking down of theories of the Wheeler-Hawking school, but not general relativity (see Section 6). This analysis suggests that further confirmation of this approximation is expected.

However, the Wheeler School still has strong influence; and even the MIT open course Phys, 8,033 and Phys, 8.962 is currently filled with their errors[99]. Due to the influences of the Wheeler School, general relativity is believed as effective only for large scale problems. Thus, the study for the applications of general relativity on earth and understanding material structure is neglected or ignored[6,16]. For example, there are numerous experiments on the weight reduction of a charged capacitor[83-85]. However, due to the biased view and ignorance of editors of journals such as the Physical Review, and Nature, these experiments are unfairly regarded as due to errors. These experiments are important because they support the charge-mass interaction that is a crucial for the unification of electromagnetism and gravitation[86,105]. It shows also that the speculation of unconditional E = mc^2 is invalid[34]. In conclusion, general relativity remains to be completed.

A basic problem is that just as in Maxwell's classical electromagnetism, there is also no radiation reaction force in general relativity. Although an accelerated massive particle would create radiation, the metric elements in the geodesic equation are created by particles other than the test particle. In other words, not only the field equation, but also the equation of motion must be modified[106]. Now, it should be clear that gravitation is not a problem of geometry as the Wheeler School advocated. Moreover, general relativity is not yet complete, independent of the need of unification.

APPENDIX A: PERLICK'S BOOK REVIEW ON "THE GLOBAL NONLINEAR STABILITY OF THE MINKOWSKI SPACE"

After it has been shown that there is no bounded dynamic solution for the Einstein equation[3], in 1996 Perlick published a book review in ZFM, pointing out that Christodoulou and Klanerman have made some unexpected mistakes, and their proof is difficult to follow, and suggested their main conclusion may be unreliable. However, to many readers, a suggestion of going through more than 500 pages of mathematics is not a very practical proposal. The review is as follows:

"For Einstein's vacuum field equation, it is a difficult task to investigate the existence of solutions with prescribed global properties. A very interesting result on that score is the topic of the book under review. The authors prove the existence of globally hyperbolic, geodesically complete, and asymptotically flat solutions that are close to (but different from) Minkowski space. These solutions are constructed by solving the initial value problem associated with Einstein's vacuum field equation. More precisely, the main theorem of the book says that any initial data, given on R^3, that is asymptotically flat and sufficiently close to the data for Minkowski space give rise to a solution with the desired properties. In physical terms, these solutions can be interpreted as space-times filled with source-free gravitational radiation. Geodesic completeness means that there are no singularities. At first sight, this theorem might appear intuitively obvious and the enormous amount of work necessary for the proof might come as a surprise. The following two facts, however, should caution everyone against such an attitude. First, it is known that there are nonlinear hyperbolic partial differential equations (e.g., the equation of motion for waves in non-linear elastic media) for which even arbitrarily small localized initial data lead to singularities. Second, all earlier attempts to find geodesically complete and asymptotically flat solutions of Einstein's vacuum equation other than Minkowski space had failed. In the class of spherically symmetric space-time and in the class of static space-times the existence of such solutions is even excluded by classical theorems. These facts indicate that the theorem is, indeed, highly non-trivial. Yet even in the light of these facts it is still amazing that the proof of the theorem fills a book of about 500 pages. To a large part, the methods needed for the proof are rather elementary; abstract methods from functional analysis are used only in so far as a lot of L^2 norms have to be estimated. What makes the proof involved and difficult to follow is that the authors introduce many special mathematical constructions, involving long calculations, without giving a clear idea of how these building-blocks will go together to eventually prove the theorem. The introduction, almost 30 pages long, is of little help in this respect. Whereas giving a good idea of the problems to be faced and of the basic tools necessary to overcome each problem, the introduction sheds no light on the line of thought along which the proof will proceed for mathematical details without seeing the thread of the story. This is exactly what happened to the reviewer."

"To give at least a vague idea of how the desired solutions of Einstein's vacuum equation are constructed, let us mention that each solution comes with the following: (a) a maximal space-like foliation generalizing the standard foliation into surfaces t = const. in Minkowski space; (b) a so-called optical function u, i.e. a solution u of the eikonal equation that generalizes the outgoing null function u = r - t on Minkowski space; (c) a family of "almost conformal killing vector fields on Minkowski space. The construction of these objects and the study of their properties require a lot of technicalities. Another important tool is the study of "Bianchi equations" for "Weyl tensor fields". By definition, a Weyl tensor field is a fourth rank tensor field that satisfies the algebraic identities of the conformal curvature tensor, and Bianchi equations are generalizations of the differential Bianchi identities."

"In addition to the difficulties that are in the nature of the matter the reader has to struggle with a lot of unnecessary problems caused by inaccurate formulations and misprints. E.g., "Theorem 1.0.2" is not a theorem but rather an inaccurately phrased definition. The principle of conservation of signature" presented on p. 148 looks like a mathematical theorem that should be proved; instead, it is advertised as an "heuristic principle which is essentially self-evident." For all these reasons, reading this book is not exactly great fun. Probably only very few readers are willing to struggle through these 500 pages to verify the proof of just one single theorem, however interesting."

"Before this book appeared in 1993 its content was already circulating in the relativity community in form of a preprint that gained some notoriety for being extremely voluminous and extremely hard to read. Unfortunately, any hope that the final version would be easier to digest is now disappointed. Nonetheless, it is to be emphasized that the result presented in this book is very important. Therefore, any one interested in relativity and/or in nonlinear partial differential equations is recommended to read at least the introduction."

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lated with (1). However, as suspected by Gullstrand[1] and conjectured by Hogarth, the truth is the opposite.

[94] K.Kuchar[87] claimed to have proved that the initial condition of Einstein's equation (1) can be approximated by the initial condition of the linear equation (16) by using a power series expansion. Note, however, that the only valid case of such a power series expansion is a non-dynamic solution. Thus, he has proven only that the properties are true in an unintended void set. Such a basic mistake is essentially repeated 20 years later by Christodoulou and Klainerman for claiming the existence of bounded radiative solutions.

[95] After discussions with me for about a month, P. Morrison of MIT went to Princeton University to see J. H. Taylor to discuss their calculations on the binary pulsars. As expected, Taylor failed to give a valid justification[6].

[96] Hogarth[44] conjectured that, for an exact solution of the two-particle problem, the energy-momentum tensor did not vanish in the surrounding space and would represent the energy of gravitational radiation.

[97] The possibility of having an anti-gravity coupling was formally mentioned by Pauli[88]. In a different way, such a possibility was actually first mentioned by Einstein in 1921. On the other hand, Hawking and Penrose had implicitly assumed, in their singularity theorems, the impossibility of an anti-gravity coupling[7].

[98] This explicit reinterpretation of Einstein's equivalence principle (based on Pauli's misinterpretation that Einstein objected) as just the signature of Lorentz metric was advocated by Synge[12] earlier and Friedman[101] later.

[99] A traditional viewpoint of the Physics Department of MIT is that general relativity must be understood in terms of physics. However, after P. Morrison past away, the Wheeler School started to take over in 2006, and thus many errors of the Wheeler School appear in MIT open courses Phys. 8.033 and Phys. 8.962[89]. However, the instructors failed to see these.

[100] The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the principle of causality. This is the basis of relevance for all scientific investigations[3,4].

[101] Michael Francis Atiyah was in the 2011 Selection Committee for the Shaw Prize in Mathematics Sciences that awarded Christodoulou a half prize[31]. Atiyah has been president of the Royal Society (1990-1995). Since 1997, he has been an honorary professor at the University of Edinburgh (Wikipedia). Understandably, some journals avoid criticizing him.

[102] John L.Friedman; Divisional Associate Editor of Phys.Rev.Letts., officially claims 'The existence of local Minkowski space has replaced the equivalence principle that initially motivated it.' 17 Feb. **(2000)**.

[103] Max Planck once remarked; 'A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.' Fortunately, it seems, mathematics is an exception to his rule.

[104] Many of my teachers were graduates of Princeton University; such as Prof. A.J.Coleman, who pointed out errors of Einstein, and Prof. I.Halperin, who was my advisor for my M.Sc. & Ph.D. in mathematics.

[105] The charge-mass interaction has the potential to explain the NASA's Space-Probes Pioneer Anomaly satisfactorily[90]. So far, this problem has not gotten a good valid explanation yet. Thus, this charge-mass interaction would be very important for astrophysics in the future[91,92].

[106] Ludwig D.Faddeev; the Chairman of the Fields Medal Committee, wrote ('On the work of Edward Witten'): 'Now I turn to another beautiful result of Witten – proof of positivity of energy in Einstein's theory of gravitation. … – a formidable problem solved by Yau and Schoen in the late seventy as Atiyah mentions, 'leading in part to Yau's Fields Medal at the Warsaw Congress'.'