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Left Γ-Filters on Γ-Semigroups

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Abstract

In this article we define left Γ-filters, right Γ-filters and prime left Γ-ideal in Γ-semigroup and characterize Γ-semigroups in terms of these notions. Finally, we give the relation between the left Γ-filters and the prime right Γ-ideals.

Keywords: Nano powder; Hexagonal wurtzite structure; Chemical precipitation; X-ray diffraction

Introduction

Anjaneyulu [1] initiated the study of ideals in semigroups Petrich [2] made a study on filters in general semigroups. Lee and Lee [3] introduced the notion of a left filter in a PO semigroup. Kehayopulu [4-6] gave the characterization of the filters of S in terms of prime ideals in ordered semigroups [7-9]. Sen [10] introduced Γ-semigroups in 1981. Saha [11] introduced Γ -semigroups different from the first definition of Γ -semigroups in the sense of sen.

Let S and Γ be two nonempty sets. Then S is said to be a Γ -semigroup if there exist a mapping from $SX\Gamma XS \rightarrow S$ which maps $(a,\alpha,b) \rightarrow a\alpha b$ satisfying the condition $(a\gamma b)\mu c = a\gamma(b\mu c)$ $\forall$ $a,b,c \in S$ and $\mu,\gamma \in \Gamma$ [8].

Let *S* be a Γ -semigroup. If *A* and *B* are two subsets of *S*, we shall denote the set $\{a\gamma b : a \in A, b \in B and \gamma \in \Gamma\}$ by $A\Gamma B$.

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Let S be a Γ- semigroup. A non-empty subset *A* of S is called a right Γ-ideal of *S* if $A\Gamma S \subseteq A$. A non empty subset A of a Γ-semigroup S is a right Γ-ideal of S if $s \in S,\ a \in A,\ \alpha \in \Gamma$ implies $a\alpha s \in A$. [8].

Let S be a Γ -semigroup. A non empty A of S is called a left Γ -ideal of S if $S\Gamma A \subseteq A$. A nonempty subset A of a Γ -semigroup S is a right Γ-ideal of S if $s \in S, a \in A,\ \alpha \in \Gamma$ implies $s\alpha a \in A$. A is called an Γ -ideal of S if it is a right and left Γ -ideal of S.

A subset T of S is called a prime if $A\Gamma B \subseteq T \Rightarrow A \subseteq T$ or $B \subseteq T$ for subsets A, B of S. T is called a prime right ideal if T is prime as a right ideal. T is called a prime left ideal if T is a prime as a left ideal. T is called a prime ideal if T is prime as an ideal [11].

We now introduce the left Γ -filter, right Γ -filter and Γ -filter.

A Γ -sub semigroup F of a Γ -semigroup S is called a left Γ-filter of S if $a\alpha b \in F$ for $\alpha \in \Gamma, a, b \in S \Rightarrow a \in F$. A Γ-semigroup F of a Γ -semigroup S is called a right Γ-filter of S if $a\alpha b \in F$ for $a, b \in S;\ \alpha \in \Gamma \Rightarrow b \in F$. [13].

Theorem (1)

Let *S* be a Γ-semigroup and *F* a non-empty subset of *S*. The following are equivalent.

1. *F* is a left Γ -filter of *S*.
2. $S \setminus F = \phi$ or $S \setminus F$ is a prime right Γ -ideal.

Proof: $(1) \Rightarrow (2)$: Suppose that $S \setminus F \neq \phi$. Let $x \in S \setminus F; \alpha \in \Gamma$ and $y \in S$. Then $x\alpha y \in S \setminus F$. Indeed: If $x\alpha y \notin S \setminus F$; then $x\alpha y \in F$. Since *F* is a left Γ-filter, $x \in F$. It is impossible. Thus $x\alpha y \in S \setminus F$, and so $(S \setminus F)\Gamma S \subseteq S \setminus F$. Therefore $S \setminus F$ is a Γ right ideal.

Next, we shall prove that $S \setminus F$ is a prime.

Let $x\alpha y \in S \setminus F$ for $x, y \in S$ and $\alpha \in \Gamma$. Suppose that $x \notin S \setminus F$ and $y \notin S \setminus F$. Then $x \in F$ and $y \in F$. Since *F* is a sub semigroup of S , $x\alpha y \in F$. It is impossible. Thus $x \in S \setminus F$ or $y \in S \setminus F$. Hence $S \setminus F$ is a prime, and so $S \setminus F$ is a prime right $\Gamma -$ideal.

$(2) \Rightarrow (1)$: If $S \setminus F = \phi$ then $F = S$. Thus F is a left Γ-filter of S. Next assume that $S \setminus F$ is a prime right Γ - ideal of S. Then F is a Γ -sub semigroup of S. Indeed: Suppose that $x\alpha y \notin F$ for $x, y \in F$ and $\alpha \in \Gamma$. Then

$x\alpha y \in S \setminus F$ for $x, y \in F$ and $\alpha \in \Gamma.$ Since $S \setminus F$ is prime, $x, y \in S \setminus F.$ It is impossible. Thus $x\alpha y \in F$ and so F is a sub semigroup of $S.$

Let $x\alpha y \in F$ for $x, y \in S$ and $\alpha \in \Gamma.$ Then $x \in F.$ Indeed: If $x \notin F,$ then $x \in S \setminus F.$ Since $S \setminus F$ is a prime right Γ-ideal of $S,$ $x\alpha y \in (S \setminus F)\Gamma S \subseteq S \setminus F.$ It is impossible. Thus $x \in F.$ Therefore F is a left filter of $S.$

Theorem (2)

Let S be a Γ -semigroup and F be a non-empty subset of $S.$ The following are equivalent.

(1) F is a right Γ filter of $S.$

(2) $S \setminus F = \phi$ or $S \setminus F$ is a prime left Γ-ideal.

Proof: $(1) \Rightarrow (2)$: Suppose that $S \setminus F \neq \phi.$ Let $y \in S \setminus F;$ $\alpha \in \Gamma$ and $y \in S.$ Then $x\alpha y \in S \setminus F.$ Indeed: If $x\alpha y \notin S \setminus F;$ then $x\alpha y \in F.$ Since F is a right Γ-filter, $y \in F.$ It is impossible. Thus $x\alpha y \in S \setminus F,$ and so $S\Gamma(S \setminus F) \subseteq S \setminus F.$ Therefore $S \setminus F$ is a left Γ-ideal.

Next, we shall prove that $S \setminus F$ is a prime.

Let $x\alpha y \in S \setminus F$ for $x, y \in S$ and $\alpha \in \Gamma.$ Suppose that $x \notin S \setminus F$ and $y \notin S \setminus F.$ Then $x \in F$ and $y \in F.$ Since F is a sub semigroup of $S,$ $x\alpha y \in F.$ It is impossible. Thus $x \in S \setminus F$ or $y \in S \setminus F.$ Hence $S \setminus F$ is a prime and so that $S \setminus F$ is a prime left Γ -ideal.

$(2) \Rightarrow (1)$: If $S \setminus F = \phi$ then $S = F$. Thus F is a right Γ-filter of $S.$ Next assume that $S \setminus F$ is a prime left Γ-ideal of $S.$ Then F is a Γ-sub semigroup of $S.$ Indeed: Suppose that for $x, y \in F$ and $\alpha \in \Gamma.$ Then $x\alpha y \in S \setminus F$ for $x, y \in F$ and $\alpha \in \Gamma.$ Since $S \setminus F$ is a prime, $x, y \in S \setminus F.$ It is impossible. Thus $x\alpha y \in F;$ $\alpha \in \Gamma$ and so F is a Γ sub semigroup of $S.$

Let $x\alpha y \in F$ for $x, y \in S$ and $\alpha \in \Gamma$. Then $y \in F.$ Indeed: If $y \notin F,$ then $y \in S \setminus F.$ Since $S \setminus F$ is a prime right Γ ideal of $S,$ $x\alpha y \in S\Gamma(S \setminus F) \subseteq S \setminus F.$ It is impossible. Thus $y \in F.$ Therefore F is a right Γ filter of $S.$ From theorem 2.6 and 2.7, we get the following.

Corollary: Let S be a Γ-semigroup and F be a non-empty subset of $S.$ The following are equivalent.

(1) F is a Γ filter of $S.$

(2) $S \setminus F = \phi$ or $S \setminus F$ is a prime Γ-ideal of $S.$

Proof: $(1) \Rightarrow (2)$: Assume that $S \setminus F \neq \phi$.

By theorem (1), $S \setminus F$ is a right Γ ideal.

By theorem (2), $S \setminus F$ is a left Γ ideal.

By theorem (1) and (2), $S \setminus F$ is a Γ ideal.

By theorem (2) and (2), $S \setminus F$ is a prime Γ ideal of S.

$(2) \Rightarrow (1)$: If $S \setminus F = \phi$ then $F = S$. Thus F is a Γ-filter of S. Next assume that $S \setminus F$ is a prime Γ-ideal of S. By theorem (1) and (2). F is a Γ-subsemigroup of S. Let $x\alpha y \in F$ for $x, y \in S$ and $\alpha \in \Gamma$. By theorem (1); F is a left Γ-filter of S. By theorem (2); F is a right Γ-filter of S. Therefore F is a Γ-filter of S.

Conclusion

This concept is used in filters of chemistry, physical chemistry, electronics.

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