Influence of porosity of epitaxial layer on quantity of radiation defects generated during radiation processing in a multilayer structure

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ABSTRACT

In this paper we consider redistribution of radiation defects, which were generated during radiation processing, in a multilayer structure with porous epitaxial layer. It has been shown, that porosity of epitaxial layer gives a possibility to decrease quantity of radiation defects.
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INTRODUCTION

One of actual questions of solid state electronics is increasing of radiation resistance. Several methods are using to increase the radiation resistance of devices of solid state electronics[1-5]. In this paper we consider an approach to decrease quantity of radiation defects, generated during radiation processing of materials. Framework the approach we consider a heterostructure, which consist of a substrate and porous epitaxial layer (see Figure 1). We assume, that the substrate was under influence of radiation processing (ion implantation, effects of cosmic radiation et al) through the epitaxial layer. Radiation processing of materials leads to generation of radiation defects. Main aim of the present paper is analysis of influence of porosity of epitaxial layer on distribution of concentration of radiation defects in the considered heterostructure.

Method of solution

To solve our aim we calculate spatio-temporal distributions of concentrations of radiation defects in considered heterostructure. We determine the above distributions as solutions of the following system of equations[6-11]

Figure 1 : Heterostructure with a substrate and an epitaxial layer. The figure also shows distribution of concentration of implanted dopant
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\[ \frac{\partial I(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial z} \right] - k_{I,v}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) + \]

\[ + \frac{\partial}{\partial x} \left[ \frac{D_{Is}}{V_k T} \frac{\partial \mu_2(x, y, z, T)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{D_{Is}}{V_k T} \frac{\partial \mu_2(x, y, z, T)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{D_{Is}}{V_k T} \frac{\partial \mu_2(x, y, z, T)}{\partial z} \right] \]

\[- k_{I,v}(x, y, z, T) I^2(x, y, z, t) \]

\[ \frac{\partial V(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial z} \right] - k_{V,v}(x, y, z, T) V^2(x, y, z, t) \]

(1)

with boundary and initial conditions

\[ \left. \frac{\partial I(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \]

\[ \left. \frac{\partial I(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \]

\[ \left. \frac{\partial V(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial V(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \]

\[ I(x,y,z,0)=f_i(x,y,z), \quad V(x,y,z,0)=f_v(x,y,z), \]

(2)

Here \( I(x,y,z,t) \) and \( V(x,y,z,t) \) are the spatio-temporal distributions of concentrations of interstitials and vacancies; \( I^* \) and \( V^* \) are the equilibrium distributions of concentrations of interstitials and vacancies; \( D_I(x,y,z,T) \) and \( D_V(x,y,z,T) \) are the diffusion coefficients of interstitials and vacancies; terms \( V^2(x,y,z,t) \) and \( I^2(x,y,z,t) \) correspond to generation divacancies and analogous complexes of interstitials (see, for example,[10] and appropriate references in this work); \( k_{I,v}(x,y,z,T), k_{I,l}(x,y,z,T) \) and \( k_{V,v}(x,y,z,T) \) are the param-
eters of recombination of point defects and generation of their complexes; $k$ is the Boltzmann constant; $V^*$ is the equilibrium distribution of vacancies, $\omega = a^3$, $a$ is the atomic spacing; $\ell$ is the specific surface energy. To take into account porosity we assume, that porous are approximately cylindrical with average dimensions $r = \sqrt{x_i^2 + y_i^2}$ and $z_i$. With time small pores decomposing into vacancies. The vacancies are absorbed by large pores. The large pores takes spherical form during the absorption. Distribution of concentration of vacancies, which was formed due to porosity, could be determined by summing overall pores, i.e.

$$V(x, y, z, t) = \sum_{i=0}^{l} \sum_{j=0}^{m} \sum_{k=0}^{n} V_p (x + i \alpha, y + j \beta, z + k \chi, t), \quad R = \sqrt{x^2 + y^2 + z^2}.$$ 

Here $\alpha$, $\beta$ and $\chi$ are averaged distances between centers of pores in $x$, $y$ and $z$ directions, respectively; $l$, $m$ and $n$ are quantities of pores in the same directions.

We determine spatio-temporal distributions of concentrations of divacancies $\Phi_v (x, y, z, t)$ and diinterstitials $\Phi_i (x, y, z, t)$ by solving the following system of equations:

$$\begin{align*}
\frac{\partial \Phi_i}{\partial t} &= \frac{\partial}{\partial x} \left[ D_{\Phi_i} \frac{\partial \Phi_i}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_i} \frac{\partial \Phi_i}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_{\Phi_i} \frac{\partial \Phi_i}{\partial z} \right] + k_i(x, y, z, T) I(x, y, z, t) + k_{ij}(x, y, z, T) I^2(x, y, z, t) + \\
&+ \frac{\partial}{\partial x} \left[ D_{\Phi_{ij}} \frac{\partial \mu_2}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_{ij}} \frac{\partial \mu_2}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_{\Phi_{ij}} \frac{\partial \mu_2}{\partial z} \right] + \frac{\partial}{\partial x} \left[ \frac{D_{\Phi_{ij}}}{V k T} \frac{\partial \mu_2}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{ij}}}{V k T} \frac{\partial \mu_2}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{D_{\Phi_{ij}}}{V k T} \frac{\partial \mu_2}{\partial z} \right] \\
\frac{\partial \Phi_v}{\partial t} &= \frac{\partial}{\partial x} \left[ D_{\Phi_v} \frac{\partial \Phi_v}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_v} \frac{\partial \Phi_v}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_{\Phi_v} \frac{\partial \Phi_v}{\partial z} \right] + k_v(x, y, z, T) V(x, y, z, t) + k_{vV}(x, y, z, T) V^2(x, y, z, t) + \\
&+ \frac{\partial}{\partial x} \left[ \frac{D_{\Phi_{vV}}}{V k T} \frac{\partial \mu_2}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{vV}}}{V k T} \frac{\partial \mu_2}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{D_{\Phi_{vV}}}{V k T} \frac{\partial \mu_2}{\partial z} \right] \\
\end{align*}$$

with boundary and initial conditions

$$\begin{align*}
\left. \frac{\partial \Phi_i}{\partial x} \right|_{x=0} &= 0, & \left. \frac{\partial \Phi_i}{\partial x} \right|_{x=L_x} &= 0, & \left. \frac{\partial \Phi_i}{\partial y} \right|_{y=0} &= 0, & \left. \frac{\partial \Phi_i}{\partial y} \right|_{y=L_y} &= 0, & \left. \frac{\partial \Phi_i}{\partial z} \right|_{z=0} &= 0, & \left. \frac{\partial \Phi_i}{\partial z} \right|_{z=L_z} &= 0, \\
\left. \frac{\partial \Phi_v}{\partial x} \right|_{x=0} &= 0, & \left. \frac{\partial \Phi_v}{\partial x} \right|_{x=L_x} &= 0, & \left. \frac{\partial \Phi_v}{\partial y} \right|_{y=0} &= 0, & \left. \frac{\partial \Phi_v}{\partial y} \right|_{y=L_y} &= 0, & \left. \frac{\partial \Phi_v}{\partial z} \right|_{z=0} &= 0, & \left. \frac{\partial \Phi_v}{\partial z} \right|_{z=L_z} &= 0.
\end{align*}$$
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\( \Phi_i(x,y,z,0) = f_{\Phi_i}(x,y,z), \Phi_v(x,y,z,0) = f_{\Phi_v}(x,y,z) \). (4)

Here \( D_{\Phi_i}(x,y,z,T) \) and \( D_{\Phi_v}(x,y,z,T) \) are the diffusion coefficients of complexes of radiation defects; \( k_i(x,y,z,T) \) and \( k_v(x,y,z,T) \) are the parameters of decay of the above complexes.

We determine spatio-temporal distributions of concentrations of radiation defects by method of averaging of function corrections\textsuperscript{[13-15]}. To use the approach we write the Eqs. (1) and (3) with account initial distributions of defects, i.e.

\[
\frac{\partial I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_i(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_i(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial y} \right] + \\
+ \frac{\partial}{\partial z} \left[ D_i(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial z} \right] - k_{i,v}(x,y,z,T) I(x,y,z,t) V(x,y,z,t) + f_i(x,y,z) \delta(t) + \\
+ \frac{\partial}{\partial x} \left[ \frac{D_{\Phi_i}}{\sqrt{V} kT} \frac{\partial \Phi_i(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_i}}{\sqrt{V} kT} \frac{\partial \Phi_i(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{D_{\Phi_i}}{\sqrt{V} kT} \frac{\partial \Phi_i(x,y,z,t)}{\partial z} \right] - k_{i,v}(x,y,z,T) I^2(x,y,z,t) (1a)
\]

\[
\frac{\partial V(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_v(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_v(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial y} \right] + \\
+ \frac{\partial}{\partial z} \left[ D_v(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial z} \right] - k_{i,v}(x,y,z,T) I(x,y,z,t) V(x,y,z,t) + f_v(x,y,z) \delta(t) + \\
+ \frac{\partial}{\partial x} \left[ \frac{D_{\Phi_v}}{\sqrt{V} kT} \frac{\partial \Phi_v(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_v}}{\sqrt{V} kT} \frac{\partial \Phi_v(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{D_{\Phi_v}}{\sqrt{V} kT} \frac{\partial \Phi_v(x,y,z,t)}{\partial z} \right] - k_{v,v}(x,y,z,T) V^2(x,y,z,t) (3a)
\]

\[
\frac{\partial \Phi_i(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\Phi_i}(x,y,z,T) \frac{\partial \Phi_i(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_i}(x,y,z,T) \frac{\partial \Phi_i(x,y,z,t)}{\partial y} \right] + \\
+ \frac{\partial}{\partial z} \left[ D_{\Phi_i}(x,y,z,T) \frac{\partial \Phi_i(x,y,z,t)}{\partial z} \right] + k_i(x,y,z,T) I(x,y,z,t) + k_{i,i}(x,y,z,T) I^2(x,y,z,t) + \\
+ \frac{\partial}{\partial x} \left[ \frac{D_{\Phi_v}}{\sqrt{V} kT} \frac{\partial \Phi_v(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_v}}{\sqrt{V} kT} \frac{\partial \Phi_v(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{D_{\Phi_v}}{\sqrt{V} kT} \frac{\partial \Phi_v(x,y,z,t)}{\partial z} \right] +
\]

\[
+ f_{\Phi_i}(x,y,z) \delta(t) (3a)
\]

\[
\frac{\partial \Phi_v(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\Phi_v}(x,y,z,T) \frac{\partial \Phi_v(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_v}(x,y,z,T) \frac{\partial \Phi_v(x,y,z,t)}{\partial y} \right] +
\]

\[
+ f_{\Phi_v}(x,y,z) \delta(t)
\]
Further we replace the required concentrations in right sides of the Eqs. (1a) and (3a) on their not yet known average values $\alpha_{i_p}$. The replacement gives us possibility to obtain following equations for determination the first-order approximations of concentrations of radiation defects in the following form

$$
\frac{\partial I_1(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x}\left[\frac{D_{\Phi_1}}{V k T} \frac{\partial \mu_1(x, y, z, t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{D_{\Phi_1}}{V k T} \frac{\partial \mu_1(x, y, z, t)}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{D_{\Phi_1}}{V k T} \frac{\partial \mu_1(x, y, z, t)}{\partial z}\right] + f_1(x, y, z)\delta(t) +
$$

$$
\frac{\partial V_1(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x}\left[\frac{D_{\Phi_2}}{V k T} \frac{\partial \mu_2(x, y, z, t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{D_{\Phi_2}}{V k T} \frac{\partial \mu_2(x, y, z, t)}{\partial y}\right] + f_v(x, y, z)\delta(t) +
$$

$$
\frac{\partial \Phi_1(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x}\left[\frac{D_{\Phi_1}}{V k T} \frac{\partial \mu_1(x, y, z, t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{D_{\Phi_1}}{V k T} \frac{\partial \mu_1(x, y, z, t)}{\partial y}\right] + f_{\Phi_1}(x, y, z)\delta(t) +
$$

$$
\frac{\partial \Phi_2(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x}\left[\frac{D_{\Phi_2}}{V k T} \frac{\partial \mu_2(x, y, z, t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{D_{\Phi_2}}{V k T} \frac{\partial \mu_2(x, y, z, t)}{\partial y}\right] + f_{\Phi_2}(x, y, z)\delta(t) +
$$

Integration of the left and right sides of Eqs. (1b) and (3b) on time gives a possibility to obtain first-order approximations of concentrations of radiation defects in the final form

$$
I_1(x, y, z, t) = \frac{\partial}{\partial x}\int_0^\infty \frac{D_{\Phi_1}}{V k T} \frac{\partial \mu_1(x, y, z, t)}{\partial x} d\tau + \frac{\partial}{\partial y}\int_0^\infty \frac{D_{\Phi_1}}{V k T} \frac{\partial \mu_1(x, y, z, t)}{\partial y} d\tau + f_1(x, y, z) +
$$
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\[ + \frac{\partial}{\partial z} \int_0^1 D_{1s} \frac{\partial \mu_z}{\partial z} (x, y, z, t) d \tau - \alpha_{1l}^2 \int_0^1 k_{1l} (x, y, z, T) d \tau - \alpha_{1l} \alpha_{1w} \int_0^1 k_{1w} (x, y, z, T) d \tau (1c) \]

\[ V_l (x, y, z, t) = \frac{\partial}{\partial x} \int_0^1 D_{vs} \frac{\partial \mu_z}{\partial x} (x, y, z, t) d \tau + \frac{\partial}{\partial y} \int_0^1 D_{vs} \frac{\partial \mu_z}{\partial y} (x, y, z, t) d \tau + f_v (x, y, z) + \frac{\partial}{\partial z} \int_0^1 D_{vs} \frac{\partial \mu_z}{\partial z} (x, y, z, t) d \tau - \alpha_{1w} \int_0^1 k_{1w} (x, y, z, T) d \tau (3c) \]

\[ \Phi_{1l} (x, y, z, t) = f_{\phi_l} (x, y, z) + \int_0^1 k_{l} (x, y, z, T) I_l (x, y, z, t) d \tau + \int_0^1 k_{l, v} (x, y, z, T) I_l^2 (x, y, z, t) d \tau + \frac{\partial}{\partial x} \int_0^1 D_{vs} \frac{\partial \mu_z}{\partial x} (x, y, z, t) d \tau + \frac{\partial}{\partial y} \int_0^1 D_{vs} \frac{\partial \mu_z}{\partial y} (x, y, z, t) d \tau + \frac{\partial}{\partial z} \int_0^1 D_{vs} \frac{\partial \mu_z}{\partial z} (x, y, z, t) d \tau . \]

Average values of the first-order approximations of the required approximations could be calculated by the following relation\[13-15\]

\[ \alpha_{1l} \rho = \frac{1}{\Theta L_x L_y L_z} \int \int \int \int \rho_l (x, y, z, t) d z d y d x d t . \tag{5} \]

Substitution of the relations (1c) and (3c) into the relation (5) gives a possibility to calculate the required average values in the following form

\[ \alpha_{1l} = \sqrt{ \left( \frac{a_3 + A}{4 a_4} \right)^2 - 4 \left( \frac{B + \Theta a_3 B + \Theta^2 L_x L_y L_z a_1}{a_4} \right) } - \frac{a_3 + A}{4 a_4} , \quad \alpha_{1w} = \frac{1}{S_{y00}} \Theta \times \]

\[ \times \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_l (x, y, z) d z d y d x \left| \alpha_{1l} \right| + \Theta L_x L_y L_z \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_{\phi_l} (x, y, z) d z d y d x , \quad \alpha_{1v} = \frac{1}{S_{y00}} \Theta \times \]

\[ \times \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_{\phi_v} (x, y, z) d z d y d x \left| \alpha_{1v} \right| \]

Here

\[ S_{\rho ij} = \int \Theta \left( \Theta - t \right) \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} k_{\rho ij} (x, y, z, T) I_i^i (x, y, z, t) V_l^i (x, y, z, t) d z d y d x d t , \quad a_4 = S_{h00} \times \]

\[ \times \left( S_{\nu 00} - S_{\nu 00} S_{y00} \right) , \quad a_3 = S_{\nu 00} S_{h00} + S_{\nu 00} - S_{h00} S_{y00} , \quad a_2 = \int \int \int f_v (x, y, z) d z d y d x \times \]
\( \times S_{IV,00}S_{IV,00}^2 + 2S_{IV,00}S_{IV,00}^2 [\int_0^{L_z} \int_0^{L_y} \int_0^{L_z} f_i (x, y, z) d z d x - \int_0^{L_z} \int_0^{L_y} \int_0^{L_z} f_i (x, y, z) d z d x + S_{IV,00} \times \]

\( \times \Theta L_y^2 L_z^2 - \Theta L_y^2 L_z^2 S_{IV,00} \),

\[ a_i = S_{IV,00} \int_0^{L_y} \int_0^{L_y} \int_0^{L_z} f_i (x, y, z) d z d y d x, \quad A = \sqrt{8y + \Theta \frac{a_2^2}{a_4} - 4\Theta \frac{a_2}{a_4}}, \]

\[ a_0 = S_{IV,00} \left[ \int_0^{L_y} \int_0^{L_y} \int_0^{L_z} f_i (x, y, z) d z d y d x \right]^2, \quad B = \frac{\Theta a_2}{6a_4} + \frac{3}{\sqrt{q^2 + p^3}} - q - \frac{3}{\sqrt{q^2 + p^3}} + q, \]

\[ q = \frac{\Theta^3 a_2}{24a_4^2} \left( 4a_0 - \Theta L_y L_z \frac{a a_3}{a_4} \right) - \Theta^2 \frac{A_0}{8a_4^2} \left( 4\Theta a_2 - \Theta^2 \frac{a a_3}{a_4} \right) - \frac{\Theta^3 a_4^2}{54a_4^2} - \frac{L_y^2 L_z^2 \Theta^4 a_2^2}{8a_4^2}, \]

\[ R_{p_i} = \Theta \left[ (\Theta - t) \int_0^{L_y} \int_0^{L_y} \int_0^{L_z} k_i (x, y, z, T) I_i (x, y, z, t) d z d y d x d t \right], \quad p = \Theta \frac{4a a_4}{12a_4^2} - \Theta a_2 - \Theta a a_4 \times \]

\( \times L_y L_z \).

We determine approximations with the second and higher orders of concentrations of radiation defects framework standard iterative procedure of method of averaging of function corrections\(^{[13-15]}\). Framework the procedure we determine the approximation of the n-th order by replacement of the concentrations of radiation defects \( I(x, y, z, t), V(x, y, z, t), \Phi_i(x, y, z, t) \) and \( \Phi_i(x, y, z, t) \) in right sides of the Eqs.(1b) and (3b) on the following sums \( \alpha_{mp} + \rho_{mp}(x, y, z, t) \). The replacement gives a possibility to obtain the second-order approximations of concentrations of radiation defects

\[
\frac{\partial}{\partial t} I_2 (x, y, z, t) = \frac{\partial}{\partial x} \left[ D_c (x, y, z, T) \frac{\partial I_1 (x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_c (x, y, z, T) \frac{\partial I_1 (x, y, z, t)}{\partial y} \right] + \\
+ \frac{\partial}{\partial z} \left[ D_c (x, y, z, T) \frac{\partial I_1 (x, y, z, t)}{\partial z} \right] - k_{1,v} (x, y, z, T) [\alpha_{1v} + I_1 (x, y, z, t)] [\alpha_{1v} + V_1 (x, y, z, t)] + \\
+ \frac{\partial}{\partial x} \left[ D_{IS} \frac{\partial \mu_c (x, y, z, t)}{\partial x} \right] d \tau + \frac{\partial}{\partial y} \left[ D_{IS} \frac{\partial \mu_c (x, y, z, t)}{\partial y} \right] d \tau + \frac{\partial}{\partial z} \left[ D_{IS} \frac{\partial \mu_c (x, y, z, t)}{\partial z} \right] d \tau - \\
- k_{1,v} (x, y, z, T) [\alpha_{1v} + I_1 (x, y, z, t)] \\

\]
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\[-k_{v,y}(x,y,z,T)[\alpha_{v} + V_{1}(x,y,z,t)]^{3}\]

\[\frac{\partial \Phi_{2\nu}(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \Phi_{1\nu}(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \Phi_{1\nu}(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \Phi_{1\nu}(x,y,z,t)}{\partial z} \right] +\]

\[+ \frac{\partial}{\partial x}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \mu_{1\nu}(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \mu_{1\nu}(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \mu_{1\nu}(x,y,z,t)}{\partial z} \right] + f_{\phi\nu}(x,y,z)\delta(t)\]

\[\frac{\partial \Phi_{2\nu}(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \Phi_{1\nu}(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \Phi_{1\nu}(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \Phi_{1\nu}(x,y,z,t)}{\partial z} \right] +\]

\[+ \frac{\partial}{\partial x}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \mu_{1\nu}(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \mu_{1\nu}(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z}\left[ D_{\phi\nu}(x,y,z,T) \frac{\partial \mu_{1\nu}(x,y,z,t)}{\partial z} \right] + f_{\phi\nu}(x,y,z)\delta(t)\] (3d)

Integration of left and right sides of Eqs. (1d) and (3d) gives a possibility to obtain relations for the second-order approximations of the required concentrations of radiation defects in the following form

\[I_{2}(x,y,z,t) = \int_{0}^{\dagger} \frac{D_{I}(x,y,z,T)}{D_{I}(x,y,z,T)} \frac{\partial I_{1}(x,y,z,\tau)}{\partial x} d\tau + \int_{0}^{\dagger} \frac{D_{I}(x,y,z,T)}{D_{I}(x,y,z,T)} \frac{\partial I_{1}(x,y,z,\tau)}{\partial y} d\tau +\]

\[+ \int_{0}^{\dagger} \frac{D_{I}(x,y,z,T)}{D_{I}(x,y,z,T)} \frac{\partial I_{1}(x,y,z,\tau)}{\partial z} d\tau - \int_{0}^{\dagger} k_{I,1}(x,y,z,\tau) [\alpha_{2I} + I_{1}(x,y,z,\tau)] d\tau -\]

\[\int_{0}^{\dagger} k_{I,V}(x,y,z,\tau) [\alpha_{2V} + I_{1}(x,y,z,\tau)] d\tau + f_{I}(x,y,z) +\]

\[+ \int_{0}^{\dagger} \frac{D_{I}(x,y,z,T)}{D_{I}(x,y,z,T)} \frac{\partial \mu_{2}(x,y,z,\tau)}{\partial x} d\tau + \int_{0}^{\dagger} \frac{D_{I}(x,y,z,T)}{D_{I}(x,y,z,T)} \frac{\partial \mu_{2}(x,y,z,\tau)}{\partial y} d\tau +\]

\[+ \int_{0}^{\dagger} \frac{D_{I}(x,y,z,T)}{D_{I}(x,y,z,T)} \frac{\partial \mu_{2}(x,y,z,\tau)}{\partial z} d\tau +\]

\[V_{2}(x,y,z,t) = \int_{0}^{\dagger} \frac{D_{V}(x,y,z,T)}{D_{V}(x,y,z,T)} \frac{\partial V_{1}(x,y,z,\tau)}{\partial x} d\tau + \int_{0}^{\dagger} \frac{D_{V}(x,y,z,T)}{D_{V}(x,y,z,T)} \frac{\partial V_{1}(x,y,z,\tau)}{\partial y} d\tau +\]

\[+ \int_{0}^{\dagger} \frac{D_{V}(x,y,z,T)}{D_{V}(x,y,z,T)} \frac{\partial V_{1}(x,y,z,\tau)}{\partial z} d\tau - \int_{0}^{\dagger} k_{V,V}(x,y,z,\tau) [\alpha_{2V} + V_{1}(x,y,z,\tau)] d\tau -\]
\[- \int_{0}^{t} k_{i,j} \left( x, y, z, T \right) \left[ \alpha_{i\ell} + I_{1} \left( x, y, z, \tau \right) \right] \left[ \alpha_{2\ell} + V_{1} \left( x, y, z, \tau \right) \right] d \tau + f_{i} \left( x, y, z \right) + \]

\[\frac{\partial}{\partial x} \left[ \frac{D_{V,S}}{\sqrt{V \kappa T}} \frac{\partial \mu_{x} \left( x, y, z, t \right)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{D_{V,S}}{\sqrt{V \kappa T}} \frac{\partial \mu_{y} \left( x, y, z, t \right)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{D_{V,S}}{\sqrt{V \kappa T}} \frac{\partial \mu_{z} \left( x, y, z, t \right)}{\partial z} \right] \]

\[\Phi_{x \ell} \left( x, y, z, t \right) = \frac{\partial}{\partial x} \int_{0}^{t} D_{\Phi_{x \ell}} \left( x, y, z, T \right) \frac{\partial \Phi_{x \ell} \left( x, y, z, \tau \right)}{\partial x} d \tau + f_{\Phi_{x \ell}} \left( x, y, z \right) + \]

\[+ \frac{\partial}{\partial y} \int_{0}^{t} D_{\Phi_{x \ell}} \left( x, y, z, T \right) \frac{\partial \Phi_{x \ell} \left( x, y, z, \tau \right)}{\partial y} d \tau + \frac{\partial}{\partial z} \int_{0}^{t} D_{\Phi_{x \ell}} \left( x, y, z, T \right) \frac{\partial \Phi_{x \ell} \left( x, y, z, \tau \right)}{\partial z} d \tau + \]

\[+ \frac{\partial}{\partial x} \left[ \frac{D_{V,S}}{\sqrt{V \kappa T}} \frac{\partial \mu_{x} \left( x, y, z, t \right)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{D_{V,S}}{\sqrt{V \kappa T}} \frac{\partial \mu_{y} \left( x, y, z, t \right)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{D_{V,S}}{\sqrt{V \kappa T}} \frac{\partial \mu_{z} \left( x, y, z, t \right)}{\partial z} \right] \]

\[\Phi_{2\ell} \left( x, y, z, t \right) = \frac{\partial}{\partial x} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \Phi_{2\ell} \left( x, y, z, \tau \right)}{\partial x} d \tau + f_{\Phi_{2\ell}} \left( x, y, z \right) + \frac{\partial}{\partial y} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \Phi_{2\ell} \left( x, y, z, \tau \right)}{\partial y} d \tau + \]

\[+ \frac{\partial}{\partial z} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \Phi_{2\ell} \left( x, y, z, \tau \right)}{\partial z} d \tau \times \]

\[I^{2} \left( x, y, z, \tau \right) d \tau + \frac{\partial}{\partial z} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \mu_{z} \left( x, y, z, \tau \right)}{\partial z} d \tau + \int_{0}^{t} k_{x \ell} \left( x, y, z, T \right) I \left( x, y, z, \tau \right) d \tau \quad (3e) \]

\[\Phi_{2\ell} \left( x, y, z, t \right) = \frac{\partial}{\partial x} \left[ 1 \frac{\partial}{\partial x} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \Phi_{2\ell} \left( x, y, z, \tau \right)}{\partial x} d \tau + f_{\Phi_{2\ell}} \left( x, y, z \right) + \frac{\partial}{\partial y} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \Phi_{2\ell} \left( x, y, z, \tau \right)}{\partial y} d \tau + \]

\[+ \frac{\partial}{\partial z} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \Phi_{2\ell} \left( x, y, z, \tau \right)}{\partial z} d \tau \times \]

\[I^{2} \left( x, y, z, \tau \right) d \tau + \frac{\partial}{\partial z} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \mu_{z} \left( x, y, z, \tau \right)}{\partial z} d \tau + \int_{0}^{t} k_{x \ell} \left( x, y, z, T \right) V \left( x, y, z, \tau \right) d \tau \quad + \]

\[+ \frac{\partial}{\partial x} \left[ 1 \frac{\partial}{\partial x} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \mu_{x} \left( x, y, z, \tau \right)}{\partial x} d \tau + \frac{\partial}{\partial y} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \mu_{y} \left( x, y, z, \tau \right)}{\partial y} d \tau + \frac{\partial}{\partial z} \int_{0}^{t} D_{\Phi_{2\ell}} \left( x, y, z, T \right) \frac{\partial \mu_{z} \left( x, y, z, \tau \right)}{\partial z} d \tau \]

\[\int_{0}^{t} k_{v \ell} \left( x, y, z, T \right) V^{2} \left( x, y, z, \tau \right) d \tau \quad + \]

We determine average values of the second-order approximations by the standard relation\(^{[13-15]}\)

\[\alpha_{2\rho} = \frac{1}{\Theta L_{x} L_{y} L_{z}} \int \int \int \left[ \rho_{2} \left( x, y, z, t \right) - \rho_{1} \left( x, y, z, t \right) \right] d z d y d x d t \quad (6)\]

Substitution of the relations (1e) and (3e) in the relation (6) gives a possibility to obtain relations for the required values \(\alpha_{2\rho}\)

\[\alpha_{2c} = 0, \quad \alpha_{2c} = 0, \quad \alpha_{2c} = 0, \quad \alpha_{2c} = 0, \quad \alpha_{2v} = \sqrt{\frac{\left( b_{3} + E \right)^{2}}{4 b_{4}^{2}}} - 4 \left( \frac{\Theta a_{2} F + \Theta^{2} L_{x} L_{y} L_{z} b_{1}}{b_{4}} \right) - \frac{b_{3} + E}{4 b_{4}}, \quad \]

\[\alpha_{2v} = C_{v} - \alpha_{2v} S_{v v 0 0} - \alpha_{2v} \left( 2 S_{v v 0 1} + S_{v v 1 0} + \Theta L_{x} L_{y} L_{z} \right) - S_{v v 0 2} - S_{v v 1 1} \]

\[S_{v v 0 1} + \alpha_{2v} S_{v v 0 0} \]

where \(b_{4} = \frac{1}{\Theta L_{x} L_{y} L_{z}} S_{v v 0 0}^{2} S_{v v 0 0} - \frac{1}{\Theta L_{x} L_{y} L_{z}} S_{v v 0 0}^{2} S_{v v 0 0} \), \(b_{3} = - \left( 2 S_{v v 0 1} + S_{v v 1 0} + \Theta L_{x} L_{y} L_{z} \right) \times \)

\[\frac{1}{\Theta L_{x} L_{y} L_{z}} S_{v v 0 0}^{2} S_{v v 0 0} - \frac{1}{\Theta L_{x} L_{y} L_{z}} S_{v v 0 0}^{2} S_{v v 0 0} \]
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\[ \frac{S_{V}^{00}}{\Theta L_{x} L_{y} L_{z}} + \frac{S_{V}^{00}S_{V}^{00}}{\Theta L_{x} L_{y} L_{z}} \left( S_{V_{01}} + 2S_{V_{10}} + S_{V_{01}} + \Theta L_{x} L_{y} L_{z} \right) \left( 2S_{V_{01}} + S_{V_{10}} + \Theta L_{x} L_{y} L_{z} \right) \frac{S_{V}^{00}}{\Theta L_{x}} \times \]

\[ \frac{1}{L_{x} L_{z}} \cdot \frac{S_{V}^{10}S_{V}^{10}}{\Theta L_{x} L_{y} L_{z}} \cdot b_{2} = \frac{S_{V}^{00}S_{V}^{00}}{\Theta L_{x} L_{y} L_{z}} \left( S_{V_{02}} + S_{V_{11}} + C_{V} \right) - \left( \Theta L_{x} L_{y} L_{z} - 2S_{V_{02}} + S_{V_{11}} \right)^{2} + \]

\[ + \frac{S_{V}^{01}S_{V}^{00}}{\Theta L_{x} L_{y} L_{z}} \left( \Theta L_{x} L_{y} L_{z} + S_{V_{01}} + 2S_{V_{10}} + 2S_{V_{01}} \right) \left( \Theta L_{x} L_{y} L_{z} \times \right. \]

\[ = \frac{S_{V}^{00} + S_{V_{00}}}{\Theta L_{x} L_{y} L_{z}} \left( 2S_{V_{01}} + S_{V_{10}} + \Theta L_{x} L_{y} L_{z} \right) + \frac{S_{V}^{01}}{\Theta L_{x} L_{y} L_{z}} \left( \Theta L_{x} L_{y} L_{z} + 2S_{V_{10}} + S_{V_{01}} \right) \left( 2S_{V_{02}} + \right. \]

\[ \frac{S_{V}^{10} + 3S_{V_{01}}}{\Theta L_{x} L_{y} L_{z}} \cdot b_{0} = \frac{S_{V}^{00} + S_{V_{02}}}{\Theta L_{x} L_{y} L_{z}} \left( \Theta L_{x} L_{y} L_{z} + 2S_{V_{10}} + S_{V_{11}} \right) \left( C_{V} - S_{V_{02}} - S_{V_{11}} \right) + \frac{S_{V}^{01}}{\Theta L_{x} L_{y} L_{z}} \left( \Theta L_{x} L_{y} L_{z} + 2S_{V_{10}} + S_{V_{01}} \right) \times \]

\[ \left( C_{V} - S_{V_{02}} - S_{V_{11}} \right) + 2C_{V}^{2} \frac{S_{V}^{01}}{\Theta L_{x} L_{y} L_{z}} \left( \Theta L_{x} L_{y} L_{z} + 2S_{V_{10}} + S_{V_{01}} \right) \right), \]

\[ C_{V} = \alpha_{1i}^{2} S_{V_{00}} + \alpha_{V}^{2} S_{V_{00}} - S_{V_{02}} - S_{V_{11}}, \]

\[ E = \frac{\Theta a_{2}}{6a_{4}} + 3\sqrt{r^{2} + s^{2}} - r - \frac{\Theta b_{2}}{24b_{4}} \times \]

\[ \left( 4b_{0} - \Theta L_{x} L_{y} L_{z} \frac{b_{3}}{b_{4}} \right) \frac{3b_{2}^{2}}{54b_{4}^{2}} - L_{x} L_{y} L_{z} \frac{3b_{2}^{2}}{8b_{4}^{2}}, \]

\[ \times \frac{1}{12b_{4}}. \]

Framework this paper the required spatio-temporal distributions of concentrations of radiation defects have been determined by using the second-order approximations by using method of averaging of function corrections. The approximations is usually enough good approximations to make qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with results of numerical simulations.

**DISCUSSION**

In this section we compare concentrations of radiation defects in porous and nonporous materials. Figs. 2 and 3 shows spatial distributions of radiation defects and their simplest complexes. The figures
show, that porosity of materials of epitaxial layer gives a possibility to decrease quantity of radiation defects. In this situation porous overlayer over device area gives a possibility to increase safety during radiation processing. Analogous situation could be find during using nonporous overlayer over device area. However pores probably became drains of radiation defects. In this situation porosity of overlayer leads to higher safety of device area.

CONCLUSION

In the present paper we analyzed redistributions of radiations defects in material with porous and nonporous overlayer after radiation processing. It has been shown, that presents of porous overlayer gives a possibility to decrease quantity of radiation defects.

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