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High-speed railway train timetable optimization model and improved column generation algorithm

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ABSTRACT

Scheduling good train timetable can enhance the quality of the train service of the Railway Company. In China, high-speed railway network has expanded quickly during the last decade, to schedule a feasible train timetable more efficiently or seeking the optimal solution is urgent demand to the technicians on the railway industry. This paper proposes anew optimization mathematical model for train working diagram scheduling and designs an improved column generation method to solve it. Firstly,transfer the mathematical model into a linear problem with Lagrangian relaxation. Then, simplify the original linear problem by modifying the objective function and decision variables, search for an optimum solution by updating restricted master problem and the price problem during the column generation iterations. Branch and bound methods are used to eliminate bad solutions. Finally, take the real world Beijing-Shanghai high-speed railway datumas the case study,the computation results prove that the optimization model and algorithmareeffective andefficient;they have the potential practical usage.

KEYWORDS

Train timetable; Lagrangian relaxation; Column Generation; Railway optimization; Heuristic.

INTRODUCTION

A train timetable defines the planned arrival and departure times of trains to/from yards, terminals and sidings, and train timetable scheduling plays an important role in managing and operating complex railway systems. With the rapid development of high-speed railway and increasing of demand, the level-of-service of train timetables is an essential factor that affects travelers' and freight carriers' decisions in choosing desirable transportation modes. Since the line carrying capacity is scarce rare, the primary problem is how to make sure that the use of the capacity to schedule train timetable effectively. In past four decades, the train timetable-scheduling problem has been investigated in different ways and from various perspectives[1,2]. Many mathematical formulations and solution algorithms playan important role on traintimetabling, such as backtracking search[1], look-ahead search[2], and materialistic algorithms[3]. Brannlund et al.[4] introduced a Lagrangian relaxation method to search for a profit-maximizing schedule, in which track resources are based on reasonable constraints. Alberto Caprara[5] proposed a more faithful to the reality of track occupancy constraint and platform constraints.

Column generation is a superiormethod for large-scale integer programs with a huge number of variables[6]. It is an indispensable tool in computational optimization to solve amathematical program by iteratively adding the variables of the model. Cacchiani[7,8] proposes heuristic and exact algorithms for the periodic and non-periodic train timetabling problem on a corridor that is based on the solution of thelinear program (LP) relaxation of an integral linear programming (ILP) formulation. Min et al[9] proposed a column-generation-based algorithm that exploited the separability of the problem.

TRAIN TIMETABLE SCHEDULING MODEL

The train timetable problem (TTP) is to assign the stations and sections to each train and satisfy the constraints macroscopically. Moreover, the resource usage planning also requires the trains' departure order and stop strategies.

Problem description

In China's railway system, the time unit is 1 minute, as the future usage maybe 15 seconds in high-speed railway lines. When one train starts from the origin station, it has a time span to choose which is expected as the best time for this train defined by train operation scheme, for example, to train T1, the expected best time to depart from original station is from 8:00 A.M. to 8:30 A.M.. We call this time span "the allowing time" for train at stations, after discretized by time unit, the quantity of the allowing time is also the choices of train operated at the station, the longer the allowing time means more choices for train timetable. If train j has an allowing time of β units at each station, and need run through γ stations. Train j departs from the virtual starting node, it has β choices (for β units of allowing time at station 1), for each node at station 1 to station 2, it also has β choices to station 2, and so the maximum number of timetable plan for train j is β^{γ}. We suppose that the running time (RT)for the same class of trains at the section is constant and the travel time (TT) needs add the acceleration and retardation delay or stop time (ST) at the expectation stations.

When only one train in the line, there is no conflicts and it can run depend on its departure time at originate station and stop strategy. When there are many trains in the line, conflicts become more and more. We need to give them an order to untwine. In an operation scheme, we give trains' order and stop strategies. The 1st train runs dependingon its departure time and stop strategy. The2ndruns dependingon itself andthe 1st. And so on. Obviously, each train selects the optimal departure time independent. It means that if getatrains'order and stop strategies, the whole operation scheme can be complete.

Variable description

The notations used for improved train timetable scheduling model are summarized as follows:

Φ A set of time instant of scheduling time span, $\Phi = \{1, \cdots, \phi\}$

S A set of stations, all the stations along the line, $S = \{1, \cdots, s\}$

J A set of trains, $J = \{j\}$

S^j The station set that train j will stop or pass through

o_j Originating station of train j

q_j Terminal station of train j

Γ_{i1} The minimal time interval for adjacent departing trains

Γ_{i2} The minimal time interval for adjacent receiving trains

M_i The number of available tracks of station i

D^i $D^i \in \Phi, i \in S \setminus \{s\}$ Probable departure time at station i

A^i $A^i \in \Phi, \in S \setminus \{1\}$ Probable arrival time at station i

W^i $i \in S$ Probable waiting time at station i

$d_{i,j}$ Thedeparture time of train $j(j \in \mathrm{J})$ at station $i\left(\mathrm{o}_j \le i < q_j\right), d_{i,j} \in D^i, 0 \le d_{i,j} < \Phi$

$a_{i,j}$ Thearrival time of train $j(j \in \mathrm{J})$ at station $i\left(o_j < i \le q_j\right), a_{i,j} \in A^i,\ 0 < a_{i,j} \le \Phi$

$w_{i,j}$ The waiting time span of train $j(j \in \mathrm{J})$ at station $i\left(\mathrm{o}_j \le i \le \mathrm{q}_j\right), w_{i,j} \in \mathrm{W}^i, 0 \le w_{i,j} < \Phi$

$v_{i,j}$ The waiting time of train $j(j \in \mathrm{J})$ at station $i\left(\mathrm{o}_j \le i \le \mathrm{q}_j\right), w_{i,j} \in \mathrm{W}^i, 0 \le w_{i,j} < \Phi$

$R^j_{i,i+1}$ Run time of train j between station i and station $i+1$

$P_{i,j}$ Stop time standard of train j at station i

G_i The minimum number of trains' stop at station i depending on operation plans

$H_{i1,i2}$ The minimum number of trains' stop at station $i1$ and $i2$ depending on operation plans

The decision variables are as follows:

$$\eta_j = \begin{cases} 1 & \text{if train } j \text{ is exist in final operation scheme} \\ 0 & \text{otherwise} \end{cases}$$

$$x^{\phi}_{i,j} = \begin{cases} 1 & \text{if train } j \text{ depart at station } i \text{ at time unit } \phi \\ 0 & \text{otherwise} \end{cases}$$

$$y^{\phi}_{i,j} = \begin{cases} 1 & \text{if train } j \text{ arrive at station } i \text{ at time unit } \phi \\ 0 & \text{otherwise} \end{cases}$$

$$z^{\phi}_{i,j} = \begin{cases} 1 & \text{if train } j \text{ wait at station } i \text{ at time unit } \phi \\ 0 & \text{otherwise} \end{cases}$$

Mathematical model

Considering the actual situation of Chinese Railway Industry's Regulation System,the optimization objective function in this paper is the maximum of trains.

The objective function is:

$$\max \sum_j \eta_j \tag{1}$$

The constraints include:

$$\sum_j \sum_\phi x^{\phi}_{i,j} \le 1 (i \in S / \{s\}) \tag{2}$$

$$\sum_j \sum_\phi y^{\phi}_{i,j} \le 1 (i \in S / \{1\}) \tag{3}$$

$$\sum_i \sum_\phi x^{\phi}_{i,j} = \sum_i \sum_\phi y^{\phi}_{i,j} (j \in \mathrm{J}, \phi \in \Phi) \tag{4}$$

$$\sum_j \sum_{\phi+\Gamma_{i1}}^{\phi-\Gamma_{i1}} \mathrm{x}^{\phi}_{i,j} \le 1 (i \in S / \{s\}) \tag{5}$$

$$\sum_j \sum_{\phi+\Gamma_{i2}}^{\phi-\Gamma_{i2}} y^{\phi}_{i,j} \le 1 (i \in S / \{1\}) \tag{6}$$

$$\sum_j z^{\phi}_{i,j} \le M_i (i \in S, \phi \in \Phi) \tag{7}$$

$$w_{i,j} \le \kappa^j_i \tag{8}$$

$$\underset{d_{i,j1} < \phi_1}{x^{\phi}_{i,j1}} + \underset{d_{i,j2} \ge \phi_1}{x^{\phi}_{i,j2}} + \underset{a_{i+1,j1} \ge \phi_2}{y^{\phi}_{i+1,j1}} + \underset{d_{i+1,j2} < \phi_2}{y^{\phi}_{i+1,j1}} \le 3$$
$$(i \in S / \{s\}, j_1, j_2 \in \mathrm{J}, 0 \le \phi_1 < \phi_2 < \Phi) \tag{9}$$

$$\sum_i \sum_\phi z_{i,j}^{\phi} \geq G_i \quad \textbf{(10)}$$

$$\sum_j \left(\sum_\phi z_{i1,j}^{\phi} \times \sum_\phi z_{i2,j}^{\phi} \right) \geq \mathrm{H}_{i1,i2} \quad \textbf{(11)}$$

Formula (2) and (3) meanstation workconstraints. Formula (4)represents flow conservation constraints. Formula (5)represents departing time interval constraint. Formula (6)represents arriving time interval constraint. Formula (7)represents available station tracks constraint. Formula(8)represents train's stop time constraint. Formula (9) is train overtaking constraint.[10] Formula (10) and (11) represent constraints of operation scheme.

Take Beijing-Shanghai high-speed railway for example, the number of variables is almost 19,872,000 and the number of constraints is 1,616,640. Since the TTP' scale is very large, it is difficult to find theoptimal solutionin polynomial time.

THE HEURISTIC

The column generation algorithm is anefficient method for solvinginteger programming and combinatorial optimization problems, especially forlarge-scale integer linear programming problems. It is suitable for the problem whose numbers of decision variables are much greater than number of constraints.

The overarching idea is that many linear programs are too large to consider all the variables explicitly. Since most of the variables will be non-basic and assume a value of zero in the optimal solution, only a subset of variables need to be considered in theory when solving the problem. Column generation leverages this idea to generate only the variables, which have the potential to improve the objective function—that is, to find variables with negative reduced cost. Thus, it is very suitable to use column generation algorithm to solve TTP.

In this paper, we propose an improved column generation algorithm to solve. First,transfer the mathematical model into a linear problem with Lagrangian relaxation. Then, simplify the original linear problem by modifying the objective function and decision variables, search for an optimum solution by updating restricted master problem and the price problem during the column generation iterations. In the meantime, branch and bound methods are used to eliminate bad solutions.

Relaxation

Not considering the interaction between trains by relaxing constraints (5) (6) (7) (8), we get anew ILP:

$$P_L = \max L$$

$$L = \sum_j \eta_j - \sum_i \lambda_{i1}(1 - \sum_j \sum_\phi x_{i,j}^{\phi}) + \sum_i \lambda_{i2}(1 - \sum_j \sum_\phi y_{i+1,j}^{\phi}) + \sum_i \lambda_{i3}(M_i - \sum_j \sum_\phi z_{i,j}^{\phi}) + \sum_{\mathrm{i}} \lambda_{i4}(3 - x_{i,j1}^{\phi} - x_{i,j2}^{\phi} - y_{i+1,j1}^{\phi} - y_{i+1,j2}^{\phi}) + \sum_i \lambda_{i5} \sum_j (\kappa_i^j - w_{i,j}) \quad \textbf{(12)}$$

$i \in S / \{\mathrm{s}\}, j, j_1, j_2 \in \mathrm{J}$

s.t. (2),(3),(4),(10),(11)

$\lambda_{i,\phi}$ and λ_{i5} are Lagrange multipliers. $\lambda_{i,\theta}, \lambda_{i5}$ is defined by train conflicts, $w_{i,j}, x_{i,j}^{\phi}, y_{i,j}^{\phi}, z_{i,j}^{\phi}$ is defined by operation scheme.

Column generation

We will simplify TTP model and give the iteration step of column generation algorithm. In order to make the problem simpler and intuitive, replacevariables of the objective function first.

The train arrival time is determined by the departure time at the last station and the run time in the section, station stop time is computed by the arrival and departure time.

$$x_{i,j}^{\phi 1} = y_{i+1,j}^{\phi 2} \ (\phi_1 = d_{i,j}, \phi_2 = a_{i+1,j}, i\left(\mathrm{o}_j \leq \ i < q_j\right), d_{i,j} \in D^i, a_{i+1,j} \in A^{i+1}, \ 0 \leq d_{i,j} < a_{i+1,j} < \Phi) \quad \textbf{(13)}$$

$$z_{i,j}^{\phi 3} = x_{i,j}^{\phi 1} \ (\phi 1 + P_{i,j} \leq \phi 3 < \phi 2, \phi_1 = d_{i,j}, \phi_2 = a_{i,j}, i\left(\mathrm{o}_j < i < q_j\right), d_{i,j} \in D^i, a_{i,j} \in A^i, \ 0 \leq a_{i,j} < d_{i,j} < \Phi) \quad \textbf{(14)}$$

Based on formula (13), (14), we replace $y_{i,j}^{\phi}, z_{i,j}^{\phi}$ by. $x_{i,j}^{\phi}$:

$$L = [\sum_j \left(\eta_j - \sum_i (\lambda_{i,\phi} \sum_\phi x_{i,j}^{\phi}) - \sum_i \lambda_{i5} w_{i,j} \right) + C_0] \quad \textbf{(15)}$$

$i \in S / \{s\}, j, j_1, j_2 \in \mathrm{J}$

s.t. (2),(3),(4),(10),(11)

We can simplify formula 15 into a new one.

$$L = \sum_j C_j + C_0 \tag{16}$$

$i \in S / \{s\}, j, j_1, j_2 \in \mathrm{J}$

s.t. (4),(5),(6),(12),(13)

In formula (16), C_0 is computed by $\lambda_{i\phi}$.

$$C_j = \left(\eta_j - \sum_i (\lambda_{i,\phi} \sum_\phi x^\phi_{i,j}) - \sum_i \lambda_{i5} w_{i,j}\right) \tag{17}$$

This, we propose two new parameters: $\xi_j, \alpha_{i,j}$.

$\alpha_{i,j}$ represent the operation scheme.

$$\alpha_{i,j} = \begin{cases} 1 & \text{train } j \text{ starts, stops or ends at station } i \\ 0 & \text{other} \end{cases}$$

ξ_j meansthe profit of train j. It is computed by $\eta_j, x^\phi_{i,j}, w_{i,j}, \lambda_{i5}, \lambda_{i,\theta}$.

$$\xi_j = \eta_j - \frac{\lambda_{i5} \times \sum_i w_{i,j} - \sum_i \lambda_{i,\theta}}{\sum_i x^\phi_{i,j}} \tag{18}$$

Thus, we rewrite formula (17) and obtain a new one.

$$C_j = \xi_j \sum_i \alpha_{i,j} \tag{19}$$

At last, the problem become into this:

$$\max \mathrm{L} = \sum_j \xi_j \sum_i \alpha_{i,j} \tag{20}$$

s.t. (4),(5),(6),(12),(13)

The following is the iterative procedure of column generation:

Step1. Generate columns pool.

Gain an initial feasible solution of TTP and create a column pool.

Step2. Search the initial feasible solution.

Get an initial train scheme based on operation scheme. Find trains' departure order and stop strategies; convert them into $\alpha_{i,j}$. Initialize $\eta_j, x^\phi_{i,j}, w_{i,j}, \xi_j$.

Step3. Solvethe RMP.

Step4. Solve the price problem (PP).

Step5. Update $\eta_j, x^\phi_{i,j}, w_{i,j}$; update $\xi_{i,j}$.

Step6. Optimality judgment

Use the simplex multiplier π_s and RMP to solve the PP, with the help of simplex method. If obtain the optimal solution go to Step 7, otherwise to Step 6. Judge all the u_t. If they are all greater than 0. Go to Step 9, otherwise go to Step 3.

Step7. Update columns in column pool.

Step8. Update RMP.

Compute $\lambda_{11}, \lambda_{12}, \cdots \lambda_{p1}, \lambda_{p2} \cdots$ and $\pi_s{}'$, return to Step.3.

Step9. Output the result.

Convert the matrix $\alpha_{i,j}$ into a solution in the form of train diagram.

Due to limitations of the column generation, it cannot guarantee that values are integers. Branch and bound are used to eliminate bad values.

Branch and bound

In our paper, TTP is transferred into a large-scale integer linear programming, and this ILP may get non-integer solutions, which are not suitable for TTP. We use branch and bound to eliminate bad solutions. When complete the column generation, we search all the variables and find the variable $\alpha_{i',j'}$ that is most distant from the integer, in following equation:

$$\left|\alpha_{i',j'} - 0.5\right| = \min\{\left|\alpha_{i,j} - 0.5\right|\} \quad \textbf{(22)}$$

Then we need to set $\alpha_{i,j} = 1 \text{ or } 0$ and divide the original problem into two new sub problems. If all the variables $\alpha_{i,j} = 1 \text{ or } 0$, all variables are appropriate and the problem gain the optimal solution.

Solution procedure

The flow chart of the heuristic is shown inFigure 1.

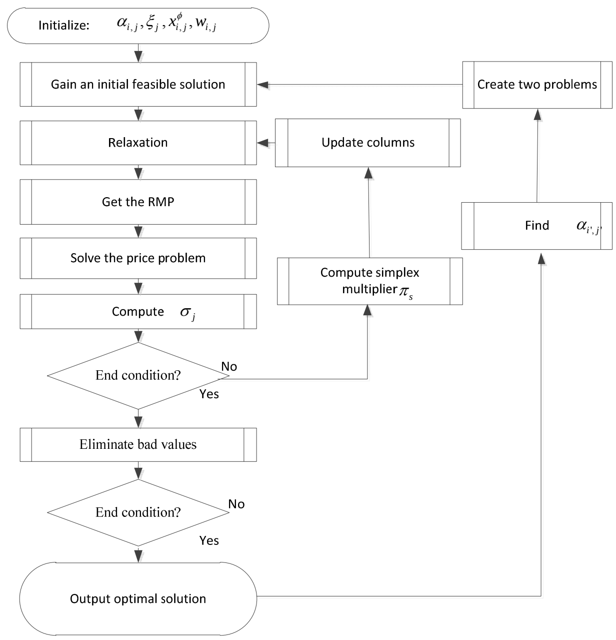

Figure 1 : Procedure of the solution methodology

CASE STUDY

Real world examples used in this paper come from theBeijing-Shanghai high-speed railway in China. Datum are from Wikipedia.

Beijing-shanghai high-speed railway

Beijing-Shanghai high-speed railway connects Beijing with Shanghai, which has a total length of 1318 km. It is designed for a maximum speed of 350km/h. There are two types of trains in this line: Grade G and Grade D. The highest speed of Grade G is 300km/h, and Grade D is 250km/h.

Results and analysis

We took some tests by using improved column generationalgorithm, and got an approximateoptimal operation scheme.

First, generate an initial feasible solution based on operation scheme. The value of ξ_j in this solution may be higher, but the number of trains is lower and the value of $\sum_j C_j$ is lower, either. This solution is unable to make full use ofthe transport capacity and cannot meet the passenger travel demand.

Based on previous solution, we use improved column generationalgorithm to optimize train diagram. Through adding column (increase trains), improve transport capacity. In the meantime, increasing of trains makes more conflicts and ξ_j may be decrease. We replace columns (change trains' stop strategies), to increase ξ_j. In the iteration, $\sum_j C_j$ is increasing and the transport capacity is enhanced.

Figure 2 shows the iterative process. The horizontal axis represents the iteration number and the vertical axis represents $\sum_j C_j$. The dotted line is the values of the current train diagram and solid lines arethe valuesat each iteration. The value of $\sum_j C_j$ in the initial solution is less than current diagram's. Through the iteration of the column generation algorithm, the profit of trains is increasing. After about 2500 iterations (the average calculation time is about 2 minutes), the objective function value reaches the maximum.

Figure 3 shows numbers of trains in each section under different conditions and in Figure 3, in each section except Beijing-Tianjin or Nanjing-Shanghai, numbers of trains in optimal solution are greater current. In view of these two sections has other high-speed railways, it can meet passengers' demand.

TABLE 1 shows the comparisons of between ξ_j and $\sum_j \xi_j \sum_i \alpha_{i,j}$ between different diagrams. In TABLE 1, the number and profit of trains in the optimal diagram are greater than current. The number of trains increase by 19% and the profit increased by 24%. Based on analysis, the algorithmincrease transport capacity and improve train service.

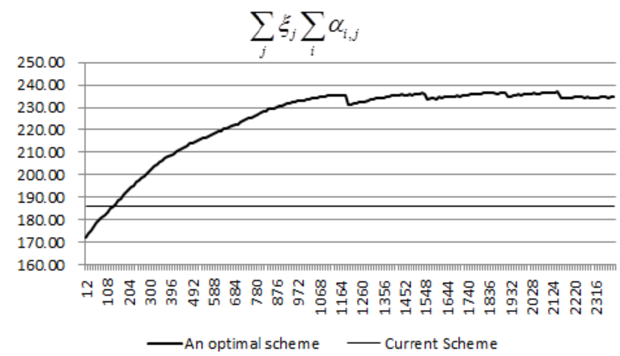

Figure 2 : Iterative process

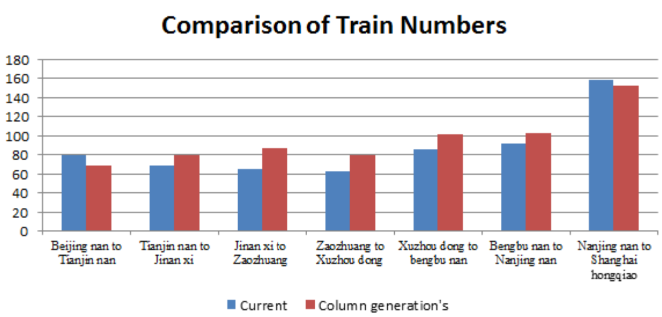

Figure 3 : Comparison of train numbers

TABLE 1 : Parameters under different conditions

	Current scheme	Initial feasible scheme	Optimal scheme
Number of trains	230	190	275
ξ_j	230	0.91	0.83
$\sum_j \xi_j \sum_i \alpha_{i,j}$	186.3	172.9	231

CONCLUSION

This paper was intended to solve the problem of scheduling train timetable on high-speed railway. The foremost problem was formulated to effectively schedule the use of line capacity in the train timetable.

We introduced an improved column generationalgorithm to solve TTP. With the case study of Beijing-Shanghai high-speed railway line, we get a better solution; the number of trains is 19% more than current operation scheme and it can provide a better service. In some case, our innovative and integrate heuristic can get a better solution rapidly and take full advantage of the potential capacity for applying in the real world.

Our future research will focus on two major directions. On one hand, we can establish an accurate mathematical model, which will include the usage of Electric Multiple Units (EMU), in order to get better and more practical results. On the other hand, this article only takes single Beijing-Shanghai high-speed railway line as study case, if we want to improve the method for real world high-speed railway network, we need modified the model and more experiments on computation algorithm.

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