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HFTVSC for nonholonomic systems with AKC

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ABSTRACT

In this paper, a hybrid finite time variable structure controller(HFTVSC) guaranteeing the system global stability and finite time convergence for uncertain nonholonomic systems with affine kinematic constraints is proposed. Making use of the elementary transformation This paper proposes a global variable structure relay control scheme with finite time convergence for the nonholonomic control systems with affine kinematic constraints by the dynamical model of the nonholonomic control systems with affine kinematic constraints(ACK),The chattering can be eliminated since the proposed terminal sliding mode controller doesn't include switching item.

KEYWORDS

Hybrid intelligent system finite time convergence;
Variable structure control;
Affine kinematic constraints.

INTRODUCTION

There is a great number of research results which has been produced in studies on the dynamical models of nonholonomic control systems. I. Kolmanovsk et al. studied a class of nonholonomic control systems in extended power form. Wang et al. proposed a stable motion tracking control law for mechanical systems subject to both nonholonomic and holonomic constraints, developed a control law at the dynamic level and can deal with model uncertainties, and the proposed control law ensured the desired trajectory tracking of the configuration state of the closed-loop system. A large class of nonlinear systems, such as a space robot with an initial angular momentum, a coin or a ball on a rotating table, a pneumatic tire, underactuated manipulators and underwater vehicles and so on, which are affine in velocities. As we know, there have been much less research results on the nonholonomic control systems with affine kinematic constraints than those on the nonholonomic control systems with linear constraints has been produced in the control of nonholonomic control systems with linear constraints due to the demand for control of the referred systems. The controller problem of the nonholonomic control systems with affine kinematic constraints still has no paper to investigate it so far.

Motivated by Wu, in section 3, we propose a global variable structure relay control scheme with finite time convergence, and give a design approach for finite time tracking control by using a relay control method so that the boundedness of the control signal is guaranteed and the singularity phenomenon is avoided for the nonholonomic control systems with affine constraints.

SYSTEM MODEL

A mechanical system whose state is defined by gen-

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eralized coordinates $q=(q_1,q_2,\cdots,q_n)^T$ and velocities $\dot q = (\dot q_1,\dot q_2,\cdots,\dot q_n)^T$ which evolve in a smooth n-dimensional manifold with constraints restricting the motion of the system to a smooth m-dimensional manifold represented locally as a $J^T(q)\dot q = A(q)$, where is the number of coordinates, m is the number of constraint equations, $J(q)\in R^{n\times m}$, $A(q)\in R^m$.

By the property of full rank matrix, thus there exist matrices $P_1,P_2,\cdots,P_r$, such that $J^T(q)P_1P_2\cdots P_r = \begin{bmatrix} J_1(q) & J_2(q)\end{bmatrix}$, where $P_k\in R^{n\times n}$ is the matrix produced by exchanging row i and row j of the identity matrix, $J_1(q)\in R^{m\times m}$ is nonsingular.

Defining, $\bar N = P_1P_2\cdots P_r\in R^{n\times n}$, $S(q)=\bar N\begin{bmatrix} J_1^{-1}(q)J_2(q)\\ -I_1\end{bmatrix}$

where $I_1\in R^{(n-m)\times(n-m)}$ is an identity matrix. It is easy to see that $S(q)$ is of full rank also, then we can easy deduce the following relation: $J^T(q)S(q)=0$. and there exists a full rank matrix $S_1(q)\in R^{(n-m)\times n}$ by the property of full rank matrix, which satisfies $S_1(q)S(q)=0$.

Now, we define

$$E=\begin{bmatrix} J_1^{-1}J_2 & J_1^{-1}A\\ -I_1 & 0_1\\ 0_2 & -1\end{bmatrix}\in R^{(n+1)\times(n-m+1)},\quad N=\begin{bmatrix}\bar N & 0_2^T\\ 0_2 & 1\end{bmatrix}\in R^{(n+1)\times(n+1)}.$$

$$y=N^{-1}\begin{bmatrix} q\\ -t\end{bmatrix}=(\xi^T,z^T,-t)^T,$$

$$\xi=(\xi_1,\xi_2,\cdots,\xi_m)^T\in R^m,$$

$z=(z_1,z_2,\cdots,z_{n-m})^T\in R^{n-m}$, then the affine kinematic constraints $J^T(q)\dot q = A(q)$ can be expressed as $\dot\xi = -J_1^{-1}J_2\dot z + J_1^{-1}A$ and one can obtain $z=B_1\bar N^{-1}q$, $\dot q = S(q)\dot z+\eta$, $\dot y = E\begin{bmatrix}\dot z\\ 1\end{bmatrix}+\begin{bmatrix}\eta\\ 1\end{bmatrix}$, where $B_1=\begin{bmatrix}0 & I_1\end{bmatrix}\in R^{(n-m)\times n}$. Differentiating the constraints with respect to t, it can be readily obtained that $\ddot q = S\ddot z+\dot S\dot z+\dot\eta$, $\ddot y = B_4(S\ddot z+\dot S\dot z+\dot\eta)$, where $_4=\begin{bmatrix} I\\ 0_2\end{bmatrix}\in R^{(n+1)\times n}$.

Using DAlembert-Lagrange principle, one can get

$$\left(-\frac{d}{dt}\frac{\partial L}{\partial\dot q}+\frac{\partial L}{\partial q}+\bar B(q)u'(t)+J(q)\lambda\right)^T\delta q=0, \quad \textbf{(1)}$$

Under (1), the dynamics of the mechanical system can be described by the following differential equations ([1]):

$$\frac{d}{dt}\frac{\partial L}{\partial\dot q}-\frac{\partial L}{\partial q}=\bar B(q)u'(t)+J(q)\lambda. \quad \textbf{(2)}$$

Working out the details for the case of Lagrangian yields

$$M(q)\ddot q+F(q,\dot q)+G(q)=J(q)\lambda+\bar B(q)u'(t), \quad \textbf{(3)}$$

where

$$F(q,\dot q)=\frac{dM(q)}{t}\dot q-\frac{1}{2}\frac{\partial}{\partial q}[\dot q^TM(q)\dot q]$$

$$G(t,q)=\frac{\partial U(t,q)}{\partial q}$$

Now, (3) can be rewritted as

$$\begin{bmatrix} M(q) & 0_2^T\\ 0_2 & 1\end{bmatrix}\ddot y+\begin{bmatrix} F(q,\dot q)+G(q)\\ 0\end{bmatrix}=\begin{bmatrix} J(q)\\ A^T(q)\end{bmatrix}\lambda+\begin{bmatrix}\bar B(q)u'(t)\\ -A^T(q)\lambda\end{bmatrix} \quad \textbf{(4)}$$

Left multiplying $B_4^TE^T$ on both sides of (4) and eliminating $\ddot y$ by (2), it yields

$$\ddot q = W_1(q)\dot q+W_2(q,\dot q)+S(S^TMS)^{-1}S^T\bar Bu'(t), \quad \textbf{(5)}$$

where

$$W_1(q)=-S(S^TMS)^{-1}S^TM\dot SS_1+\dot SS_1$$

$$W_2(q,\dot q)=-S(S^TMS)^{-1}(-S^TM\dot SS_1\eta+S^TM\dot\eta$$
$$+S^TF+S^TG)-\dot SS_1\eta-\dot\eta$$

Define $x=\begin{bmatrix} q\\ \dot q\end{bmatrix}$, Then system (18) can be expressed by the following dynamics

$$\dot{x}=\begin{bmatrix}\dot{q}\\ W_1(q)\dot{q}+W_2(q,\dot{q})\end{bmatrix}+\begin{bmatrix}0\\ I\end{bmatrix}S(S^TMS)^{-1}\tau(t) \quad .(6)$$

Suppose $u(t), q, \dot{q}$ are measurable. To make $q, \dot{q}$ track the ideal reference model, the reference model is chosen as [20]

$$\dot{x}_b=\begin{bmatrix}\dot{q}_b\\ \ddot{q}_b\end{bmatrix}=\begin{bmatrix}0 & I\\ R & Q\end{bmatrix}x_b+\begin{bmatrix}0\\ B_1\end{bmatrix}r(t)=A_bx_b+\bar{B}r(t) \quad (7)$$

where $R, Q, \bar{B}$ are constant matrices such that system (7) is stable, $r(t), q_b, \dot{q}_b$ are measurable signals, $x_b=\begin{bmatrix}q_b\\ \dot{q}_b\end{bmatrix}$. Define the tracking error vector $e(t)$ between x and the desired torque x_b as

$\varepsilon(t)=q(t)-q_b(t),\ e(t)=x(t)-x_b(t)$,

$\dot{e}(t)=[\dot{\varepsilon}^T(t),\ddot{\varepsilon}^T(t)]^T$ .And defining $B=\begin{bmatrix}0\\ I\end{bmatrix}$. We have $\dot{e}(t)=A_be(t)+B[M^{-1}(q)Eu(t)+g(r,q,\dot{q})]$,

where $g(r,q,\dot{q})=M^{-1}(q)[-F(q,\dot{q})\dot{q}-G(q)-\Lambda]\cdot$

$|-Rq-Q\dot{q}-B_1r(t)$.

Let us construct an augmented linear system as $\dot{z}=A_bz+Bv$, where $z(t)=\begin{bmatrix}\xi(t)\\ \dot{\xi}(t)\end{bmatrix}$. Define the vectors $\zeta(t)=\varepsilon(t)-\xi(t),\eta(t)=e(t)-z(t)=\begin{bmatrix}\zeta(t)\\ \dot{\zeta}(t)\end{bmatrix}$-$\begin{bmatrix}\eta_1(t)\\ \eta_2(t)\end{bmatrix}$.

We get

$$\dot{\eta}(t)=A_b\eta(t)+B[M^{-1}(q)\tau(t)+g(r,q,\dot{q})-v(t)] \quad (1)$$

the controllability Grammian matrix and the control function of linear system with the form

$$G_c(0,t_f)=\int_0^{t_f}Exp(-A_bt)BB^TExp(-A_b^Tt)dt,$$

$$v(t)=B^TExp(-A_b^Tt)G_c^{-T}(0,t_f)[Exp(-A_bt_f)z(t_f)-z(0)],$$

where $z(0)$ and $z(t_f)$ are the initial state and final state, respectively. A switching plane is defined as

$$s_i=\dot{\zeta}_i+c_i\zeta_i,\ \ \zeta=(\zeta_1,\zeta_2,\cdots,\zeta_n)^T, \quad (2)$$

where c_i' are positive constants. Define

$S_1=(s_1,s_2,\cdots,s_n)^T$, $C=diag(c_1,c_2,\cdots,c_n)$.

CONTROLLER DESIGN

Theorem 1

Consider system (1) with the pre-TSM controller

$$u(t)=-\frac{\beta\rho(t)}{\|S_1\|}S_1, \quad (3)$$

$$\rho(t)=\delta_1\|\eta(t)\|+\delta_2(D(q,\dot{q})+\|Rq+Q\dot{q}+B_1r(t)+v(t)\|+1)$$

where β,δ_1,δ_2 are positive constants to be determined. Then the system solution $\eta(t)$ converges to zero exponentially.

Proof

According to the equation (2), take the time derivative of S_1 along (1) together with (3), then

$$\dot{S}_1=[C,I_{nn}]\dot{\eta}+C\eta_1+\eta_2=[C,I_{nn}][A_b\eta(t)+$$
$$B(M^{-1}(q)Eu(t)+g(r,q,\dot{q})-v(t))]. \quad (4)$$

A candidate Lyapunov function $V_1(t)$ is defined as

$$V_1(t)=\frac{1}{2}S_1^TS_1. \quad (5)$$

Differentiating it along (1) together with (3) and (5) satisfies the following inequality

$$\dot{V}_1=S_1^T\dot{S}_1=S_1^T[C,I_{nn}]\dot{\eta}$$
$$=S_1^T[C,I_{nn}][A_b\eta(t)+B(M^{-1}(q)\tau(t)+g(r,q,\dot{q})-v(t))]$$
$$=S_1^T[(R,C+Q)\eta(t)+M^{-1}(q)Eu(t)+g(r,q,\dot{q})-v(t)]$$
$$\le\delta_1\|S_1^T\|\|\eta(t)\|-S_1^TM^{-1}(q)E\frac{\beta\rho(t)}{\|S_1\|}S_1+$$
$$\cdot\|S_1^T\|\|M^{-1}(q)\|D(q,\dot{q})+\|S_1^T\|\|Rq+Q\dot{q}+B_1r(t)+v(t)\|$$
$$<\beta\alpha_1\|S_1\|. \quad (6)$$

where $\delta_1\ge\|(R,C+Q)\|$, $\beta>\frac{1}{\alpha_1}$ are chosen positive constants, $\delta_2>\max\{\alpha_2,1\}$. Since is positive definite,

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the inequality (6) implies that reaches zero in finite time and keeps zero forever. This guarantees that tends to zero in finite time, i.e, the trajectories of the systems (41), (44) and (46) reaches the fast nonlinear switching surface .

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