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Grey prediction model-based GM (1, 1) 100m performance prediction applied research

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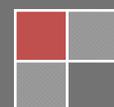
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ABSTRACT

Grey system theory is a kind of ideal method in processing small samples dynamic development problems, and 100m race speed problem has "poor data information", "small samples" and "dynamics", it lets grey system to have advantages over traditional probability statistics and fuzzy mathematics in its problem research, grey mathematics' application in competitive sports performances prediction and analyses are more and more extensive. The paper applies documents literature, makes statistics of year1992~2012 every session Olympic Games' 100m race best performances. Establish GM (1, 1) grey model on them. By using GM (1, 1) model, it predicts next session Olympic Games 100m best performances, and states GM (1, 1) model methods application in multiple events sports competition.

KEYWORDS

Sports performance; 100m performance; BP neural network; Prediction model.



INTRODUCTION

In recent years, Olympic Games has attracted more and more attentions from people, especially after year 2008 Beijing Olympic Games and year 2012 London Olympic Games, Chinese passions on Olympic Games are doubled. And Olympic Games 100m race almost becomes a synonym of Olympic Games, there is no other events as 100m race being long-standing, Olympic games 100m race has glorious history. Since American Donald created 10.6s in 1912 to federal Germany athlete Armin ran into entirely 10 seconds in 1960; from American Lewis created 9.86s in 1991, to American Green broke through with 9.79s performance in 1999 then to Jamaica famous sportsman Bolt fixed 100m record into 9.63s in 2012, in hundred years, 100m record only promotes 0.97seconds. Traditional research about uncertain factors mathematical methods, they were mainly regression analysis, variance analysis, principal component analysis and other mathematical statistics methods that carried out statistical analysis on system. Mathematical statistics method required a great deal of samples and data, data changes were required to have certain rules, factors relations to be static and so on, which had higher requirements on system, data samples had good distribution rules, the analysis might not can get effective statistical rules, event got statistical rules, in most cases, it was impossible to be used to analyze and predict system. Grey prediction modeling according to grey system behavior features, utilized accumulative sequence to overcome original sequence volatility and randomness, excavated information data's explicit information and hidden information, so that arrived at relative precise short-term prediction model. For sports competition performance prediction problem's "poor data information" and "dynamics" as well as other features, grey prediction model has already become one of main methods in competitive sports performances prediction. Regarding human 100m speed, there are lots of scholars have made researches.

In November, 2007, Xie Hui-Song, Zhou Tie-Min, Qin Xia and Zhou Cun-Sheng by researching on domestics and overseas excellent men100m three different factors described speed curves, found out low level athletes and high level athletes speed curve differences, and what kind of rules that their curve changes followed, provided diagnosis techniques and improved training thoughts scientific methods for coaches and athletes^[1].

In August, 2009, Chen Qi-Lian and others stated Bolt talent conditions and modern technology penetrating effects on sports so as to reveal the secret of Bolt breaking through world record, and explored human speed extreme problems^[2].

In September, 2012, Liu Xi-Jun by data analysis of 100m race world records from year 1930upto now, established human 100m speed extreme predicted mathematical model, and then utilized sequence extreme relevant knowledge to predict human completing 100m race used time extreme was 9.45s^[3].

What on earth human 100m race speed extreme is? So we select Olympic Games men 100mperformances as modeling materials. Predict human 100m speed extremes.

GREY PREDICTION MODEL

White system refers to system internal features are totally known; black system refers to system internal information is totally unknown; while grey system is a system between white system and black system, grey system its internal partial information is known, other partial information is unknown or uncertain.

Grey prediction refers to prediction on system behaviors features values development changes, prediction on system that has known information and uncertain information, which is prediction on grey process that changes in certain range and is related to time sequence. Though in grey process, indicated phenomenon is random, chaotic, after all, it is ordered, bounded, its obtained data set has potential rules. Grey prediction is utilizing the rules to establish grey model to predict grey system.

Grey prediction model prediction general process:

(1) First order accumulated generating operation (1-AGO)

Set there is a original non-negative data sequence that its variable as $X^{(0)}$:

$$X^{(0)} = [x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)] \tag{1}$$

Then $X^{(0)}$ first order accumulated generating operation sequence:

$$X^{(1)} = [x^{(1)}(1), x^{(1)}(2) \dots x^{(1)}(n)] \tag{2}$$

In formula,
$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), k = 1, 2, \dots, n$$

(2) Make quasi-smooth test and quasi-exponent rules test on $X^{(0)}$
Set:

$$\rho(k) = \frac{x^{(0)}(k)}{x^{(1)}(k-1)} \quad k=2, 3, \dots, n \quad (3)$$

If it meets $\rho(k) < 1$, $\rho(k) \in [0, \varepsilon]$ [$\varepsilon < 0.5$], $\rho(k)$ is in declining trend, then call $X^{(0)}$ as quasi-smooth sequence, and $X^{(1)}$ has quasi-exponent rules.

Otherwise, carry out first order weaken processing:

$$x^{(0)}(k) = \frac{1}{n-k+1} (x(k) + x(k+1) + \dots + x(n)) \quad k=1, 2, \dots, n \quad (4)$$

And let $x^{(0)}(k) = x^{(0)}(k)$, that is letting $X^{(0)}$ to be replaced by $X^{(0)}$.

(3) By step 2, it is clear $X^{(1)}$ has approximate exponent growth rules, so it can be thought that sequence $X^{(1)}$ meets following first order linear differential equation :

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u \quad (5)$$

It solves:

$$\begin{bmatrix} \hat{a} \\ \hat{u} \end{bmatrix} = (B^T B)^{-1} B^T Y_n \quad (6)$$

Among them:

$$Y_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad B = \begin{bmatrix} -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -\frac{1}{2}[x^{(1)}(2) + x^{(1)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(1)}(n-1) + x^{(1)}(n)] & 1 \end{bmatrix}$$

Input solved \hat{a} , \hat{u} into differential equation (5), it has:

$$\frac{dx^{(1)}}{dt} + \hat{a}x^{(1)} = \hat{u}$$

MODEL ESTABLISHMENTS

Generally, grey model is n order h piece of variables, it records as $GM(n, h)$, for men 100 running performance corresponding accumulated generating operation value, it is a individual variable, therefore take $h = 1$, n generally is below 3 orders, n gets bigger, then connotation becomes more plentiful. However, calculation is so complicated, systems that orders are too high, their features methods and solution are difficult, not only calculated quantity is big, calculated time is long, but also precise may not be high, its result is also analytic. Therefore, we can only get rid of establishing $n > 3$ GM model. And meanwhile, we consider accumulated generating operation values monotone increasing, sports performances trend basic smoothness and practical calculated quantity, so take $h = 1$, one $GM(1,1)$ as sports performance development rules grey model type.

Establish $GM(1,1)$ model

TABLE 1 is previous Olympic Games men 100m race performances.

TABLE 1 : Previous olympic games 100m performances table

Year	1992	1996	2000	2004	2008	2012
n	1	2	3	4	5	6
$x^{(0)}(n)/s$	9.96	9.84	9.87	9.85	9.69	9.63

Accumulated generating operation (AGO), set:

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$$

As original sequence, its successive accumulated generating operation sequence is :

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n))$$

Among them:

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), k = 1, 2, \dots, n.$$

$$x^{(1)}(1) = 9.96$$

$$x^{(1)}(2) = 9.96 + 9.84 = 19.8 \quad x^{(1)}(3) = 19.8 + 9.87 = 29.67$$

$$x^{(1)}(4) = 29.67 + 9.85 = 39.52 \quad x^{(1)}(5) = 39.52 + 9.69 = 49.21$$

$$x^{(1)}(6) = 49.21 + 9.63 = 58.83$$

By accumulated generating operation, it processes with sports performances sequence, the purpose is to provide medium information for modeling, and weaken original sports performances that sequence's randomness.

Let $z^{(1)} = (x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(n))$ to be sequence $x^{(1)}$ adjoining neighborhood generating sequence, that:

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k - 1) \quad (k = 2, 3 \dots n)$$

It has $Z^{(1)}$ to be $x^{(1)}$ adjoining neighborhood mean value generating sequence:

$$Z^{(1)} = (Z^{(1)}(2), Z^{(1)}(3), \dots, Z^{(1)}(n))$$

$$Z^{(1)}(2) = 0.5 * (19.8 + 9.96) = 14.8$$

$$Z^{(1)}(3) = 0.5 * (29.67 + 19.8) = 24.735$$

$$Z^{(1)}(4) = 0.5 * (39.52 + 29.67) = 24.735$$

$$Z^{(1)}(5) = 0.5 * (49.21 + 39.52) = 44.365$$

$$Z^{(1)}(6) = 0.5 * (58.83 + 49.21) = 54.02$$

Define $x^{(1)}$ grey derivative is :

$$d(k) = x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1)$$

Then define GM(1,1) grey differential equation model as:

$$d(k) + az^{(1)}(k) = b$$

That:

$$x^{(0)}(k) + az^{(1)}(k) = b \quad (7)$$

In formula (7), $x^{(0)}(k)$ is called grey derivative, $z^{(1)}(k)$ is called whitening background value, a is called development coefficient, b is called grey action. (a, b to be solved)

Input moment $k = 2, 3, 4, 5$ into formula (7), it has:

$$\begin{cases} x^{(0)}(2) + az^{(1)}(2) = b \\ x^{(0)}(3) + az^{(1)}(3) = b \\ x^{(0)}(4) + az^{(1)}(4) = b \\ x^{(0)}(5) + az^{(1)}(5) = b \end{cases} \quad (8)$$

Introduce matrix vector mark:

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ x^{(0)}(4) \\ x^{(0)}(5) \end{bmatrix}, u = \begin{bmatrix} a \\ b \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ -z^{(1)}(4) & 1 \\ -z^{(1)}(5) & 1 \end{bmatrix}$$

Data vector Parameter vector Data matrix

Then GM(1,1) model can be expressed as $Y = Bu$. Now problem is concluded into solving value of a, b . Use unary linear regression, that least square method to solve their estimation values as :

$$u = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y$$

It solves: $a = 0.0047$ $b = 9.9525$

Note: The equation is solving by Matlab.

For GM(1,1) grey differential equation (1), if regard grey derivative $x^{(0)}(k)$ moment $k = 2, 3, \dots, n$ as continuous variable t , then $x^{(1)}$ is regarded to be time t function $x^{(1)}(t)$, then $x^{(0)}(k)$ corresponds to derivative $\frac{dx^{(1)}(t)}{dt}$, let background value $z^{(1)}(k)$ to correspond to derivative $x^{(1)}(t)$. Then GM(1, 1) grey differential equation corresponding white differential equation is :

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \quad (9)$$

It is called as GM(1,1) whitening type.

By ordinary differential knowledge, it is clear that formula (2) solution with initial value $x^{(1)}(t = 1) = x^{(0)}(1)$ is:

$$\hat{x}^{(1)}(t) = (x^{(0)}(1) - \frac{b}{a})e^{-a(t-1)} + \frac{b}{a} \text{ (Among them, } t = 2,3,4,5,6)$$

t=2, $\hat{x}^{(1)}(2) = 19.8424$

t=3, $\hat{x}^{(1)}(3) = 29.6786$

t=4, $\hat{x}^{(1)}(4) = 39.4685$

t=5, $\hat{x}^{(1)}(5) = 49.2126$

t=6, $\hat{x}^{(1)}(6) = 58.9110$

TABLE 2 : Calculated value and original data comparison

Original data	Calculated value
$x^{(1)}(2) = 19.8$	$\hat{x}^{(1)}(2) = 19.8424$
$x^{(1)}(3) = 29.67$	$\hat{x}^{(1)}(3) = 29.6786$
$x^{(1)}(4) = 39.52$	$\hat{x}^{(1)}(4) = 39.4685$
$x^{(1)}(5) = 49.21$	$\hat{x}^{(1)}(5) = 49.2126$

TABLE 3 : Original performance and predicted performance comparison

Original performance	Predicted performance	Difference	Error analysis	Prediction precise P
$x^{(0)}(2) = 9.84$	$\hat{x}^{(0)}(2) = 9.8362$	0.0038	0.0003862	99.9%
$x^{(0)}(3) = 9.87$	$\hat{x}^{(0)}(3) = 9.7899$	0.0801	0.0081155	99.2%
$x^{(0)}(4) = 9.85$	$\hat{x}^{(0)}(4) = 9.7441$	0.1059	0.0107513	98.9%
$x^{(0)}(5) = 9.69$	$\hat{x}^{(0)}(5) = 9.6984$	-0.0084	0.0008669	99.9%

$$\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t + 1) - \hat{x}^{(1)}(t)$$

$$\hat{x}^{(0)}(2) = \hat{x}^{(1)}(3) - \hat{x}^{(1)}(2) = 29.6786 - 19.8424 = 9.8362$$

$$\hat{x}^{(0)}(3) = \hat{x}^{(1)}(4) - \hat{x}^{(1)}(3) = 39.4685 - 29.6786 = 9.7899$$

$$\hat{x}^{(0)}(4) = \hat{x}^{(1)}(5) - \hat{x}^{(1)}(4) = 49.2126 - 39.4685 = 9.7441$$

$$\hat{x}^{(0)}(5) = \hat{x}^{(1)}(6) - \hat{x}^{(1)}(5) = 58.9110 - 49.2126 = 9.6984$$

Model test

Apply backward error-detection method, test on established grey prediction model, define model prediction reliability and precise, calculate backward error value C , C gets small, it represents S_1 is small S_2 is big, S_1 is small that represents prediction error discreteness is small, S_2 is big that represents original data discreteness is big, calculated small

error probability P gets bigger that shows the one that error is smaller, its probability is bigger, which directly shows error precise is high.

TABLE 4 : Every moment residual table

n	1	2	3	4	5
$x^{(0)}(n)$	9.96	9.84	9.87	9.85	9.69
$\hat{x}^{(0)}(n)$		9.8362	9.7899	9.7441	9.6984
$e_{(n)}^{(0)}$		0.0038	0.0801	0.1059	-0.0084

Backward error-detection

Calculate original sequence average value:

$$\bar{x}^{(0)} = \frac{1}{n} \sum_{i=1}^n x^{(0)}(i) = 9.842$$

Calculate residual average value:

$$\bar{e}^{(0)} = \frac{1}{4} \sum_{i=1}^4 e^{(0)}(i) = -0.000525$$

Calculate original sequence $X^{(0)}$ mean variance:

$$S_1 = \left(\frac{\sum_{i=1}^5 [x^{(0)}(i) - \bar{x}^{(0)}]^2}{5-1} \right)^{1/2} = 0.308$$

Calculate residual mean variance:

$$S_2 = \left(\frac{\sum_{i=1}^4 [e^{(0)}(k) - \bar{e}]^2}{4-1} \right)^{1/2} = 0.0337$$

Calculate variance ratio C:

$$C = \frac{S_2}{S_1} = 0.1221$$

Calculate small residual probability:

$$P' = p \left\{ e^{(0)}(i) - \bar{e} < 0.6745 S_1 \right\}$$

Let $S_0 = 0.6745 S_1$, $e_i = |e^{(0)}(i) - \bar{e}|$, that $P' = p\{e_i < S_0\}$. It is easily getting probability $P' = 1$.

If given $C_0 > 0$ when $C < C_0$, call the model as mean variance ratio qualified model; if given $P_0 > 0$, when $P > P_0$, call the model as small residual probability qualified model.

TABLE 5 : Precise detection grade table

Prediction precise grade	P precise	C
GOOD	>0.95	<0.35
QUALIFIED	>0.8	<0.5
JUST MARK	>0.7	<0.45
UNQUALIFIED	≤ 0.7	≥ 0.65

By $C=0.1221$ and all $P>0.95$, it is clear that precise grade is GOOD, so prediction scheme is feasible.

To sum up, it can apply formula $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t+1) - \hat{x}^{(1)}(t)$

It solves year 2016men 100m race performance is (ignore body function extreme):

$$t = 7, \hat{x}^{(1)}(7) = 68.5439$$

$$\hat{x}^{(0)}(6) = \hat{x}^{(1)}(7) - \hat{x}^{(1)}(6) = 68.5439 - 58.9110 = 9.6229$$

CONCLUSION

The paper analyzes Olympic Games 100m performances prediction, discusses grey prediction model feasibility in 100m race performance prediction modeling, it provides guidance for applied mathematics performance prediction and decision in competitive sports, pushes applied mathematics application in competitive sports decision, let competitive sports decision to be more scientific; by statistical data modeling analysis, and applying grey model group model to establish model on 100m performances prediction, by grey model group to screen out proper data samples dimension and model on them, it more flexible applies grey prediction model, and promotes grey prediction precise.

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