Gravitational wave generator apparatus

Abstract
An apparatus or structure is proposed for generating high-frequency gravitational waves (HFGWs) between pairs of force-producing elements by means of the simultaneous production of a third time derivative of mass motion of the pair of force-producing elements. The elements are configured as a cylindrical array in the proposed structure and are activated by a radiation wavefront moving along the axis of symmetry of the array. The force-producing elements can be micro-electromechanical systems or MEMS resonators such as film-bulk acoustic resonators or FBARs. A preferred cylindrical array is in the form of a double helix and the activating radiation can be electromagnetic as generated by microwave transmitters such as Magnetrons. As the activating radiation wavefront moves along the axis of the structure it simultaneously activates force elements on opposite sides of the structure and thereby generates a gravitational wave between the pair of force elements. It is also indicated that Earth is completely transparent to the HFGWs. Thus a sensitive HFGW detector, such as the Li-Baker under development by the group led by Fangyu Li at Chongqing University, China, can sense the generated HFGW at an Earth-diameter distance and could, in theory, be a means for implementing transglobal HFGW communications.

Keywords
Gravitational waves; High-frequency gravitational wave generator; HFGW; Communication; MEMS; FBARs.

INTRODUCTION
As will be discussed there exist several sources for high-frequency gravitational waves (HFGWs) or means for their generation. Historically the first generation means, which is the same for gravitational waves (GWs) of all frequencies, is based upon the quadrupole equation first derived by Einstein[8] 1918. A formulation of the quadrupole that is easily related to the orbital motion of binary stars or black holes, rotating rods, laboratory HFGW generation, etc. is based upon the jerk or shake of mass (time rate of change of acceleration), such as the change in centrifugal force vector with time; for example as masses move around each other on a circular orbit. Figure 1 describes that situation. Recognize, however, that change in force \( \Delta f \) need NOT be a gravitational force (see Einstein; Infeld quoted by Weber[8], Grishchuk and Sazhin[9]). Electromagnetic forces are more than \( 10^{15} \) times larger than gravitational forces and should be employed in laboratory GW generation. As Weber[8] points out: “The non-gravitational forces play a decisive role in methods for detection and generation of gravitational waves...” The quadrupole equation is also termed “quadrupole formalism” and holds in weak gravitational fields (but well over 100 g’s), for speeds of the generator “components” less than the speed of light and for the distance between two masses \( r \) less than the GW wavelength. Certainly there would be GW generated for \( r \) greater than the GW wavelength, but the quadrupole “formalism” or equation might not apply exactly. For very small time change \( \Delta t \) the GW wavelength, \( \lambda_{GW} = c \Delta t \) (where \( c \sim 3 \times 10^8 \text{ m s}^{-1} \), the speed of light) is very small and the GW frequency \( \nu_{GW} \) is high. The concept is to produce two equal and opposite jerks or \( \Delta f \)’s at two masses, such as are involved in micro-electromechanical systems (MEMS), for example film-bulk acoustic resonators (FBARs), a distance \( 2r \) apart. This arrangement is completely analogous to binary stars on orbit as shown in Figures 1 and 2.
ARRAY OF GRAVITATIONAL-WAVE SOURCES

Next we consider an array of GW sources. Consider a stack of binary star orbit planes, each one involving a pair of masses circling each other on opposite sides of a circular orbit as shown in Figure 3. Let the planes be stacked one light hour apart (that is, $60 \times 60 \times 3 \times 10^{12} = 1.08 \times 10^{12}$ meters apart) and each orbit exactly on top of another (coaxial circles). Let us also suppose that the periods of the orbits were 10 hours. The orbital “frequency” would then be $1/10 \times 60 \times 60 = 2.8 \times 10^{-5}$ Hz.

According Landau and Lifshitz\(^4\) on each plane a GW will be generated that radiates from the center of each circular orbit. The details of that generation process are that as the masses orbit a radiation pattern is generated. In simplified terms (from the equations shown in an exercise on page 356 of Landau and Lifshitz\(^4\)) an elliptically shaped polarized arc of radiation is formed on each side of the orbit plane (mirror images). As the two masses orbit each other 180° the arcs sweep out a figure of revolution and the resulting integrated GW radiation is circularly polarized. Together these figures of revolution become shaped like a peanut as shown in Figure 2. This situation occurs when the orbiting masses move half an orbital period 180° or 5 hours on their orbit. Thus the frequency of the GW generated is twice the orbital frequency or $5.6 \times 10^{-5}$ Hz.

The general concept of the present HFGW generator is to utilize an array of force-producing elements arranged in pairs in a cylindrical formation. They could be piezorods connecting the two masses or individual resonators. In any event they would be analogous to the binary arrays of Figure 3 in which an imaginary cylinder could be formed or constructed from the collection of circular orbits. As a wavefront of energizing radiation proceeds along the cylindrical axis of symmetry of such a cylindrical array, the force-producing element pairs (such as pairs of FBARs) are energized simultaneously and jerk, that is they exhibit a third time derivative of mass motion, in concert. The jerking generates gravitational waves focused midway between the jerking pairs exactly analogous to centrifugal force jerks of the orbiting binaries.

Double helix

A convenient cylindrical array is a double helix exhibited in Figure 4. In this case the MEMS or FBARs are placed along the opposing ribbons of the helices. As activating radiation (e.g., magnetron-generated microwaves) moves along the axis of symmetry of the helices, the opposing FBARs are energized and jerk thereby producing a HFGW. It is important that the activating radiation be phase-coherent. In order to understand this concept better let us return to the orbit-plane stack of Figure 3. A GW generated by the first binary (at the base of the stack) should reach the second member of the stack just as the GW arc is formed with the correct polarization and phase. We imagined the polarization plane as the plane of an elliptical arc. Since the orbit planes are one light hour apart the orbiting binaries must be synchronized one hour of mo-

Figure 1 : Change in centrifugal force of orbiting masses, $\Delta f_{cf}$ and radiation pattern.

Figure 2 : Radiation pattern calculated by Landau and Lifshitz\(^4\) (1975) Section 110 Page 356.

Figure 3 : GW flux growth analogous to stack of N orbital planes.
Superradiance

The $N^2$ build up, termed “Superradiance,” is attributed to two effects: one $N$ from there being $N$ HFGW power sources or generation elements and the other $N$ from the narrowing of the beam so that the HFGW is more concentrated and the flux ($W m^{-2}$) thereby increased. Utilizing General Relativity, Dehnen and Romero-Borja[7], computed a superradiance build up of “…needle-like radiation …” HFGW beam from a closely packed but very long linear array of crystal oscillators. Their oscillators were essentially two vibrating masses that were a distance $b$ apart whereas a pair of vibrating FBAR masses is a distance $2r$ apart as shown in Figure 5. However, the FBAR operates in an analogous fashion as piezoelectric crystals.

Superradiance also occurs when emitting sources such as atoms “…are close together compared to the wavelength of the radiation …” (Scully and Svidzinsky[6] p.1510). Note that it is not necessary to have the MEMS or FBAR elements perfectly aligned (that is, the FBARs exactly across from each other) since it is only necessary that the energizing wave front (from Magnetrons in the case of the MEMS or FBARs as in Baker, Woods and Li[8]) reaches a couple of nearly opposite FBARs at the same time so that a coherent radiation source or focus is produced between the two FBARs. The energizing transmitters, such as Magnetrons, can be placed along the helixes’ array axes between separate segments of the array or, more efficiently, at the base of the double helix so that a superradiance microwave beam is projected up the axis of the helices. The force change, $\Delta f$, produced by energizing one off-the-shelf FBAR is 2 N according to Woods and Baker[9]

Analogy and fabrication technique

In order to clarify the double-helix concept and its fabrication, let us consider a totally different yet analogous situation. It is a storage facility for mattresses. Each mattress is, say, 7 feet by 6 feet and one foot thick (analogous to a gigantic MEMS or FBAR). The storage-facility is composed of many coaxial cylindrical structures that are analogous to the cylindrical array of MEMS. The cylindrical structures consist of 7-foot wide compartments between the cylinders’ inside and outside walls and each of these compartments is 6-feet high. Thus one can store one mattress on its side in each compartment. In order to reach a given compartment, imagine that two escalators are installed on the inside wall of each cylindrical structure. They are in the form of spiral escalators “stairsway” and are constructed on exactly opposite sides of each cylindrical storage structure (essentially the ribbons of a double helix of MEMS). As an example, let us consider one of the cylindrical structures that happen to have a diameter of 100 feet. The circumference of the inside wall of the cylinder is about 314 feet so that the foot of the opposite escalator is about 157 feet distant from its opposite. We take the tread of each escalator step as one foot wide (enough room to slide a mattress in or out of its compartment when the escalator is periodically halted).

We want to be able to reach each mattress so the escalators must rise 6 feet in 157 feet in the first 6-foot-high floor of the storage structure. Thus the height of each escalator step when it is moving is 6/157 of a foot or about 1/32 of an inch. Two people start up on each escalator simultaneously, which is analogous to a wavefront from a Magnetron moving up a double helix of FBARs. They proceed up from compartment to compartment. At each of the 157 “levels” (N) they reach opposite pairs of mattresses. In the analogous manner the wave front reaches opposite FBARs and excites them and produces...
a jerk and, therefore, HFGW radiation pattern focused between the FBARs. But what about the other coaxial cylindrical mattress storage cylinder structures? In order to transport the mattresses the tread width needs to be kept constant that is, more levels on cylinder structures having inside diameters of more than 100 feet and fewer levels on cylinder structures having diameters less than 100 feet. Thus each level is distinct and every mattress pair is on a uniquely different level (there are \( N \) such different levels and, hence, mattress pairs). Also the escalators for each cylinder could be located at different starting points on the circumference of a given cylinder structure. For example, if there were ten structures, then one could place them on different azimuths such as 0, 18, 36, 54, 72, 90, 108, 126, 144 and 162 degrees or at random. Such options may be considered in the fabrication or building process of the imaginary mattress—storage cylinders’ construction or, analogously, the FBAR array fabrication. In order to develop the double helix winding, a column could be fabricated with the mattress joined that is, glue the mattresses back to back one mattress to the next in a long line. This would create a 6-foot by 7-foot cross-section tube or, for the analogous FBARs, a 110 µm by 110 µm thread (or whatever the dimensions of the trimmed FBAR MEMS are). Then place one tube on top of the other after 157 feet. Thus the composite tube exhibits a 7-foot by 2×6 = 12-foot rectangular cross-section. The analogous FBAR—storage cylinders’ construction or, analogously, the FBAR array fabrication. In order to develop the double helix winding, a column could be fabricated with the mattress joined that is, glue the mattresses back to back one mattress to the next in a long line. This would create a 6-foot by 7-foot cross-section tube or, for the analogous FBARs, a 110 µm by 110 µm thread (or whatever the dimensions of the trimmed FBAR MEMS are). Then place one tube on top of the other after 157 feet. Thus the composite tube exhibits a 7-foot by 2×6 = 12-foot rectangular cross-section. The analogous FBAR construction would be a 110 µm by 220 µm rectangular cross-section thread. The FBAR fabrication would continue by tightly-winding the composite threads around a microwave-transparent cylinder or spool, layer after layer. Thus the resulting double-helix structure could be inserted in the microwave guide. Returning to the mattress analogy, it is recognized that each escalator passenger may take off at slightly different time, analogous to slightly irregular wave front. They all, however, will ascend at the same speed: the speed of light in the structure. Such wavefront irregularities would however be mitigated or eliminated by a properly designed waveguide.

**NUMERICAL EXAMPLE**

As a numerical example of a double-helix FBAR array, we will choose the median radius of the overall array as \( r = 20 \) cm (convenient laboratory size though usually somewhat greater than \( \lambda_{GW} \)), \( \Delta f = 2 \) N for an off-the-shelf FBAR and \( \Delta t = 4\times10^{-10} \) s (equivalent to about a \( v_{EM} = 2.5 \) GHz frequency or pulse of the jerk or energizing radiation frequency) so that \( \lambda_{EM} = 12 \) cm and \( \lambda_{GW} = 6 \) cm (the frequency of the GW is twice that of the frequency of the energizing EM wave) and the power, \( P \) from the basic GW equation (its derivation can be found in, for example, Baker\[10\], found by hyperlink at http://www.gravwave.com/docs/Astronomische%20Nachrichten%202006.pdf))

\[
P = 1.76\times10^{-42} \left(2r\Delta f/\Delta t\right)^2 W
\]

For this equation the calculation of the combined \( \Delta f \) of all the pulsating MEMS or FBARs requires more calculation. We will set the length of a double-helix array cylinder as 20 m, but recognize that it can be separated into segments along the same axis with energizing transmitters, e.g., Magnetrons installed on the cylinder axis between the segments. As mentioned the transmitters could also be phase coherent and arranged in a line along the double-helix axis at its base. If, for example, there were 1000 synchronized one-kilowatt Magnetrons (similar to those installed in a conventional microwave oven and feeding in on one hundred 12-cm, \( \lambda_{EM} \), wide levels) and each of their beams covered a 10-cm radius circle, then the energizing radiation flux would be \( 3.2\times10^4 \) W m\(^{-2}\). According to superradiance there would result a needle-like microwave radiation directed along the axis of the double helixes amounting to 32 gigawatts per square meter. In order to create a perfectly planner wave front, with no irregularities, the cylindrically symmetric MEMS array would be contained in a waveguide or possibly a very wide coaxial “cable,” surrounded by a robust one-megawatt heat sink. To increase instantaneous power to the array, bursts of gigawatt power, for example, every millisecond could be employed that would maintain a megawatt average power input. The walls of the cylindrical array are taken to be 30 cm thick. Thus the volume of the array is \( \pi (r_1^2 - r_2^2) \times 20 \) m, where \( r_1 \) is the outside radius = 0.35 m and \( r_2 \) is the inside radius = 0.05 m. Thus the volume is 7.5 m\(^3\). A FBAR (Figure 6) is a mechanical (acoustic) resonator consisting of a vibrating membrane (typically about 100×100µm\(^2\) in plan form, and about 1µm thickness), fabricated using well-established integrated circuit (IC) micro fabrication technology. A typical off-the-shelf FBAR as shown schematically in Figure 6, usually has overall dimensions 500 µm by 500 µm by approximately 100 µm thick. For our purposes, in which a high number density is important, we will trim the FBARs to a minimum size. In order to account for fabrication margins we will take the dimensions as 110 µm by 110 µm by 20 µm for an FBAR volume of \( 2.42\times10^{-13} \) m\(^3\). However, it could be smaller as shown in Figure 1 of Chan, et al.\[11\] (the MEMS resonator shown there is about 50 µm square by 2 µm thick for a volume of about \( 10^{-14} \) m\(^3\)). Thus the total number of FBARs in the double-helix cylindrical array is \( 3.1\times10^{13} \) and the number of pairs is half of that. Thus there will be \( N = 1.55\times10^{13} \) FBAR pairs in the double-helix cylindrical array. Since each FBAR exhibits a jerking force of 2 N the combined \( \Delta f \) of all the jerking FBAR pairs is \( 3.1\times10^{13} \) N if the jerking pairs (or “orbits”) were collapsed and moved in concert analogous to the orbit plane with the synchronized mass motion. A more conservative approach would be that there are \( N \) individual GW power sources each with a \( \Delta f = 2 \)
N. Thus from Eq. (1), with $2r_{\text{rms}} = 2[(r_1^2 + r_2^2)/2] = 0.5$ m, the total power produced by the double-helix array is $P = 1.55 \times 10^{13} \times 1.76 \times 10^{-52} (0.5\times2/4 \times 10^{-10})^2 = 1.69 \times 10^{-20}$ W. But due to the $N$ levels, each one of which represents an individual GW focus, there exists a “Superradiance” condition in which the HFGW beam becomes very narrow as shown schematically in Figure B of Scully and Svidzinsky[6]. Thus the HFGW flux, in W m$^{-2}$, becomes much larger at the cap of the peanut shaped radiation pattern. According to the analyses of Baker and Black[12] the area of the half-power cap is given by:

$$A_{\text{cap}} = A_{1/2(N=1)}/N \text{ m}^2$$

(2)

where $A_{1/2(N=1)} = 0.1358$ m$^2$ for a single level ($N=1$) at a distance of 0.282 m (radius of a one square meter area sphere) or $(1m/0.282m)^2(0.1358) = 1.71$ m$^2$ at a distance of one meter. Thus Eq. (2) becomes $A_{\text{cap}} = 1.71/N \text{ m}^2$ (actually one fourth of the HFGW power reaches the cap since half goes to the other side of the peanut shaped radiation pattern in the $-z$ direction in Figures 2 and 3). Thus the HFGW flux at a one-meter distance from the end of the double-helix cylindrical array is:

$$S(1) = (P/4)/(1.71/N)$$

$$= (1.69 \times 10^{-52}/4)/(1.71/1.55 \times 10^{13}) = 3.8 \times 10^{-8} \text{ W m}^2$$

(3)

From Baker, et al[3], Eq. (6A) of the Appendix, the amplitude of the dimensionless strain in the fabric of spacetime is:

$$A = 1.28 \times 10^{-48} \overline{S}/c_{\text{GW}} \text{ m/m}$$

(4)

So that at a one-meter distance $A = 5 \times 10^{-32} \text{ m/m}$. If the FBARs in all of the helix levels are not activated as individual pairs, then the situation changes. For example, let all of the FBARs in a 6-cm wide level ($\frac{1}{2} \lambda_{\text{EM}}$) be energized in concert. The number of levels would be reduced to $N = 20 \text{ m}/0.06 \text{ m} = 333$. But, because the FBAR-pairs in each level act together, “$f = (2N)(1.55 \times 10^{13}) / 333$.” Then the changes in Eq. (1) cancel out and there is no change in HFGW flux. From Woods, et al[14] the current estimated sensitivity of the Chinese Li-Baker HFGW Detector is $A = 1.0 \times 10^{-30} \text{ m/m}$ to $1.0 \times 10^{-32} \text{ m/m}$ with a signal to noise ratio of over 1500 (Woods, et al[14], p. 511) or if we were at a 1.3x10$^7$ m (diameter of Earth) distance, then $S = 3.8 \times 10^{-30} \text{ W m}^2$ and the amplitude $A$ of the HFGW is given by $A = 3.8 \times 10^{-30} \text{ m/m}$. Although these theoretical sensitivities are higher than the Li-Baker HFGW detector’s on the order of $10^{-32}$ m/m, its sensitivity can be increased dramatically (Li and Baker[18]) by introducing superconductor resonance chambers into the interaction volume (which also improves the Standard Quantum Limit; Stephenson[16]) and two others between the interaction volume and the two microwave receivers. Together they provide an increase in sensitivity of five orders of magnitude and result in a theoretical sensitivity of the Li-Baker detector to HFGWs having amplitudes of $10^{-37}$ m/m. There also could be a HFGW superconductor lens, as described by Woods[17] that could concentrate very high frequency gravitational waves at the detector or receiver. Thus with Chinese Li-Baker HFGW detector successful and the Wood’s lens practical, the Li-Baker detector will exhibit sufficient sensitivity to receive the generated HFGW signal globally.

The HFGW beam is very narrow. From Eq. (4b) of Baker and Black (2009)[12], for $N = 1.55 \times 10^{13}$ it would be $\sin^{-1}(0.737)/(1.55 \times 10^{13}) = 1.87 \times 10^{-7}$ radians. For $N = 333$ the angle is 0.0022 radians. This is still narrow, but the double helix configuration certainly reduces the width of the HFGW beam. Additionally, multiple HFGW carrier frequencies can be used, so the signal is very difficult to intercept, and is therefore useful as a low-probability-of-intercept (LPI) signal, even with widespread adoption of the HFGW technology.

**RESULTS**

The overall concept is shown in Figure 7 in very simplified form. In theory the preferred double-helix array of force-producing FBARs can generate significant superradiant HFGW radiation. A numerical example of a 20-meter long array is presented. Activation-energy radiators or transmitters (for example off-the-shelf synchronized continuous-wave Magnetrons) can be utilized to energize MEMS such as off-the-shelf FBARs found in cell phones. Thus point-to-point communication, even at a distance of the diameter of the Earth, might be realized using very sensitive HFGW Chinese detectors or receivers and HFGW lenses to concentrate the HFGW signal at the receivers.

A HFGW amplitude of the time-varying strain of the

**HFGW Generator**

Using Magnetron-FBAR (Piezoelectric Crystals)

Similar to Romero and Dehnen (1981)

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**Figure 7**: Simplified concept of the HFGW generator.
REFERENCES

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