ISSN : 0974 - 7435 Volume 8 Issue 9

BioTechnology
An Indian Journal

FULL PAPER

BTAIJ, 8(9), 2013 [1280-1289]

Global stability of impulsive hopfield neural networks with multiple delays

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ABSTRACT

In this paper, the global stability problem of discrete impulsive Hopfield neural networks with multiple delays is studied. By means of the Lyapunov stability theory and discrete Halanay inequality technique, we develop sufficient conditions for the global asymptotical stability and global exponential stability for discrete impulsive Hopfield neural networks with multiple delays. Finally, a numerical example is presented to illustrate the efficiency of our results.

KEYWORDS

Hopfield neural networks;
System with impulsive and multiple delays;
Global asymptotical stability;
Global exponential stability;
Halanay inequality.

INTRODUCTION

Since Hopfield neural networks was designed by John Hopfield in 1982[1,2], it has applied successfully in many areas such as combinatorial optimization, signal processing and pattern recognition, see e.g.[2-4]. Recently, it has been realized that the axonal signal transmission delays often occur in various neural networks, and may cause undesirable dynamic network behaviors such as oscillation and instability. Consequently, the stability analysis problems for delayed neural networks have gained considerable research attention. Up to now, a lot of results have been reported in the literature, see e.g.[5-16] and references therein. On the other hand, Impulsive phenomena can been encountered in real nervous systems These practical systems are characterized by the fact that abrupt jumps happen suddenly at some time points and the system state variables jump out of the original trajectory governed by the continuous or discrete systems at these time points. For instance, the climate changes have an impulsive impact on plant population and the supply and demand of productions will jump abruptly due to the sharp changes of financial environments. Those systems with impulsive effects are usually called impulsive systems and described by impulsive differential or difference equations see e.g.[17,18]. It is well known that impulses and time delays frequently cause instability and performance deteriorations. Thus, ignoring them always results in incorrect conclusions. This motivates us to study the global stability performance of Hopfield neural networks with impulsive and time delays. The problem of global stability analysis for neural networks with impulsive effects and multiple delays emerges as a research topic of primary importance, see e.g.[19-22].It should be pointed out that, in most existing literature, the global asymptotical stability issue and global exponential stability issue have been treated separately. To the best of the authors' knowledge, the global stability analysis problem for impulsive Hopfield neural networks with multiple delays has not been fully investigated, and remains important and challenging.

In this paper, we focus our attention on the global stability analysis of impulsive Hopfield neural networks with multiple delays. We consider the following impulsive Hopfield neural networks with multiple delays,

$$x_i(m+1)=a_i x_i(m)+\sum_{j=1}^{n} T_{ij} f_j\left(x_j\left(m-\tau_{ij}(m)\right)\right), m\neq N_k,$$
$$\Delta x_i(m)=x_i(m+1)-x_i(m)=g_i^{(k)}\left(m,x_1(m),\cdots,x_n(m)\right), m=N_k, \quad \textbf{(1)}$$
$$x_i(m)=\phi_i(m), m\in N(-\tau,0), i\in N(1,n), m\in N(1),$$

Where $x=(x_1,x_2,\cdots,x_n)\in R^n$ is the state vector, $a_i\in[0,1), i\in N(1,n)$ is a constant, $f_j:R\to R, g_i^{(k)}:N(1)\times R^n\to R$ and $\phi_i:R\to R$ are continuous functions and $f_j(0)=0, \phi_i(0)=x_0$, $g_i^{(k)}(m,0,0,\cdots,0)=0$. The impulsive behaviors can be described by $\Delta x_i(m)=x_i(m+1)-x_i(m)$ and the initial condition is given as $\phi_i(m),(m=-\tau,-\tau+1,\cdots,0)$. $\tau_{ij}(m)\geq 0$ ÿ T_{ij} are the connection weights and transmission delays of the i th neuron and the j th neuron, respectively, $\tau=\max_{i,j\in N(1,n)}\{\tau_{ij}(m)\}$, and $m-\tau_{ij}(m)\to\infty$ as $m\to\infty$. $\{N_k,g_i^{(k)}\}$ is impulsive law, $0<N_k<N_{k+1}, k\in N(0)$ and $N_k\to\infty$ as $k\to\infty$. $N(k,l)=\{k,k+1,k+2,\cdots,l\}$, $N(k)=\{k,k+1,k+2,\cdots\}$.

As Halanay inequality technique[23] and[24] provides a general frame to investigate stability performance of delayed dynamical systems, it is also a powerful tool to study impulsive Hopfield neural networks with multiple delays. In this paper, sufficient conditions for the global asymptotical stability and global exponential stability for discrete impulsive Hopfield neural networks with multiple delays are established. The rest of the paper is organized as follows. In Section II, some stability concepts of impulsive discrete systems with multiple delays and lemmas are introduced. In Section III, sufficient conditions for the global stability of the impulsive Hopfield neural networks with multiple delays are established via discrete Halanay inequality. In Section IV, a numerical example is presented to show the validity of our results. Finally, section V concludes the paper.

PRELIMINARIES

In this paper, we need the following three assumptions.

Assumption A. The sequence $\{N_k\}$ of the impulsive time points satisfies

$N_k+2<N_{k+1}$.

Assumption B. The impulsive function $\{g_i^{(k)}\}$ satisfies: if there exists $\omega_{ij}^{(k)}\geq 0$, such that for any $(x_1(t),x_2(t),\cdots,x_n(t))\in R^n$ □ $t\in R, k\in N(0)$, the following inequality holds,

$$\left|x_i(t)+g_i^{(k)}(t,x_1(t),\cdots,x_n(t))\right|\leq\sum_{j=1}^{n}\omega_{ij}^{(k)}\left|x_j(t)\right|, i\in N(1,n)$$

Assumption C. The function $\{f_j\}$ satisfies: if there exists $\delta_j>0$, such that for any $t_1,t_2\in R$, the following inequality holds,

$$\left|f_j(t_1)-f_j(t_2)\right|\leq\delta_j\left|t_1-t_2\right|, j\in N(1,n).$$

First, we need to introduce some stability concepts lemmas, which are needed throughout this paper, for the impulsive delayed discrete system (1).

Definition 1

Given $\varepsilon>0$, if there exists a $\delta(\varepsilon)>0$ such that

$\max_{-\tau\leq i\leq 0}\{\|x(i)\|\}\leq\delta(\varepsilon)$ implies $\|x(m)\|<\varepsilon, \forall m\in K$,

Then the impulsive delayed discrete system (1) is said to be stable.

Definition 2

If the impulsive delayed discrete system (1) is stable and $\lim_{m\to+\infty}\|x(m)\|=0$, then the impulsive delayed discrete system (1) is said to be asymptotically stable.

Definition 3

If there exist $K>0$ and $r\in(0,1)$ such that

$$\|x(m)\|\leq Kr^m, \forall m\in N^+, \quad \textbf{(2)}$$

holds, then the impulsive delayed discrete system (1) is said to be exponentially stable and r is called the exponential convergence rate of the impulsive delayed discrete system (1), the impulsive delayed discrete sys-

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tem (1) is said to be global exponentially stable if (2) holds for any $x(m) \in R^n, m \in N(-\tau,0)$.

Lemma 1

[25](Discrete-time Halanay-type Inequality) Suppose that the real numbers sequence $\{\alpha_n\}_{n\geq -h}$ satisfied

$$\Delta\alpha_n = -\varepsilon\alpha_n + g(n,\alpha_n,\alpha_{n-1},\cdots,\alpha_{n-h}), n\in N(1), \varepsilon\in(0,1],$$

if there exists a $\delta \in (0,\varepsilon)$ such that

$$\Delta\alpha_n \leq -\varepsilon\alpha_n + \delta \max_{i\in N(n-h,n)}\{\alpha_i\}, \forall n \in N(0).$$

Then, there exists a $\lambda \in (0,1)$ such that

$$\Delta\alpha_n \leq \lambda^n \max_{i\in N(-h,0)}\{\alpha_i\}, \forall n \in N(0), \quad \textbf{(3)}$$

where $g: N(0)\times R^{h+1} \to R$, $(\alpha_{-h},\alpha_{-h+1},\cdots,\alpha_0)$ is the initial condition, $h\in N(0)$ is a constant and λ is the smallest root in the interval $(0,1)$ of the following equation,

$$\lambda^{h+1} + (\varepsilon-1)\lambda^h - \delta = 0.$$

We have the following lemma from lemma 1.

Lemma 2

Suppose that the real numbers sequence $\{\alpha_n\}_{n\geq 0}$ satisfied

$$\Delta\alpha_n = -\varepsilon\alpha_n + g(n,\alpha_n), n\in N(1), \varepsilon\in(0,1],$$

if there exists a $\delta \in (0,\varepsilon)$ such that

$$\Delta\alpha_n \leq (-\varepsilon+\delta)\alpha_n, \forall n \in N(0).$$

Then, there exists a $\lambda = 1+\delta-\varepsilon \in (0,1)$ such that

$$\Delta\alpha_n \leq \lambda^n\alpha_0, \forall n \in N(0).$$

MAIN RESULTS

In this section, we consider the global stability of impulsive Hopfield neural networks with multiple delays (1). First, we will apply the discrete Halanay inequality to the following no-impulsive Hopfield neural networks with multiple delays,

$$\begin{aligned} &x_i(m+1) = a_i x_i(m) + \sum_{j=1}^{n} T_{ij} f_j\left(x_j\left(m-\tau_{ij}(m)\right)\right), \\ &x_i(m) = \phi_i(m), m\in N(-\tau,0), i\in N(1,n), m\in N(1),. \end{aligned} \quad \textbf{(4)}$$

For the global exponential stability of the no-impulsive Hopfield neural networks with multiple delays (4), we have the following result.

Theorem 1

If the following inequality holds,

$$a+\delta<1, \quad \textbf{(5)}$$

Where $a = \max_{i\in N(1,n)}\{a_i\}$, $\delta = \max_{i\in N(1,n)}\left\{\sum_{j=1}^{n}|T_{ij}|\delta_j\right\}$.

Then, the no-impulsive Hopfield neural networks with multiple delays (4) is global exponentially stable.

Proof

Let the solution of the discrete system (4) be $\{x(m)\}$, we have

$$x_i(m) = a_i^m x_i(0) + \sum_{s=0}^{m-1} a_i^{m-1-s}\sum_{j=1}^{n} T_{ij} f_j\left(x_j\left(s-\tau_{ij}(s)\right)\right), m\in N(1), i\in N(1,n).$$

Let

$$d_m = \begin{cases} \max_{i\in N(1,n)}\{|x_i(m)|\}, m\in N(-\tau,0), \\ a^m \max_{i\in N(1,n)}\{|x_i(0)|\} + \delta\sum_{s=0}^{m-1} a^{m-1-s} \max_{t\in N(s-\tau,s)}\max_{j\in N(1,n)}\{|x_j(t)|\}, m\in N(1), \end{cases}$$

it is clear that, $|x_i(m)| \leq d_m, i\in N(1,n)$ as $m\in N(-\tau,0)$.

At $m\in N(1)$, we obtain

$$\begin{aligned} |x_i(m)| &\leq a_i^m|x_i(0)| + \sum_{s=0}^{m-1} a_i^{m-1-s}\sum_{j=1}^{n}|T_{ij}|\left|f_j\left(x_j\left(s-\tau_{ij}(s)\right)\right)\right| \\ &\leq a_i^m \max_{i\in N(1,n)}\{|x_i(0)|\} + \sum_{s=0}^{m-1} a_i^{m-1-s}\sum_{j=1}^{n}|T_{ij}|\delta_j\left|x_j\left(s-\tau_{ij}(s)\right)\right| \\ &\leq a_i^m \max_{i\in N(1,n)}\{|x_i(0)|\} + \sum_{s=0}^{m-1} a_i^{m-1-s}\sum_{j=1}^{n}|T_{ij}|\delta_j \max_{t\in N(s-\tau,s)}\{|x_j(t)|\} \\ &\leq a_i^m \max_{i\in N(1,n)}\{|x_i(0)|\} + \sum_{s=0}^{m-1} a_i^{m-1-s}\sum_{j=1}^{n}|T_{ij}|\delta_j \max_{j\in N(1,n)}\max_{t\in N(s-\tau,s)}\{|x_j(t)|\} \\ &\leq a^m \max_{i\in N(1,n)}\{|x_i(0)|\} \\ &\quad + \sum_{s=0}^{m-1} a^{m-1-s}\max_{i\in N(1,n)}\left\{\sum_{j=1}^{n}|T_{ij}|\delta_j\right\}\max_{t\in N(s-\tau,s)}\max_{j\in N(1,n)}\{|x_j(t)|\} \\ &\leq a^m \max_{i\in N(1,n)}\{|x_i(0)|\} + \delta\sum_{s=0}^{m-1} a^{m-1-s}\max_{t\in N(s-\tau,s)}\max_{j\in N(1,n)}\{|x_j(t)|\} \\ &= d_m. \end{aligned}$$

Thus, for any $m\in N(-\tau)$, the following inequality

$$\left|x_i(m)\right| \le d_m, i \in N(1,n), \tag{6}$$

holds.

Since

$$\Delta d_m = d_{m+1} - d_m$$

$$= -(1-a)d_m + \delta \max_{t \in N(m-\tau,m)} \max_{j \in N(1,n)} \left\{\left|x_j(t)\right|\right\}$$

$$\le -(1-a)d_m + \delta \max_{t \in N(m-\tau,m)} \{d_t\}, \forall m \in N(1),$$

It follows from (5) and Lemma1 that there exists a $\lambda \in (0,1)$ such that

$$\Delta d_m \le \lambda^m \max_{t \in N(-\tau,0)} \{d_t\}, \forall m \in N(0).$$

By (6), we have

$$\|x(m)\|_\infty = \max_{i \in N(1,n)} \left\{\left|x_i(m)\right|\right\} \le d_m, \forall m \in N(-\tau).$$

Let $K = \max_{t \in N(-\tau,0)} \left\{ \max_{i \in N(1,n)} \left\{\left|x_i(t)\right|\right\} \right\}$, the following inequality holds,

$$\|x(m)\|_\infty \le K\lambda^m, \forall m \in N(0).$$

According to Definition3, it can be seen that the no-impulsive Hopfield neural networks with multiple delays (4) is global exponentially stable, attenuation rate λ is the smallest root in the interval $(0,1)$ of the following equation,

$$\lambda^{\tau+1} - a\lambda^\tau - \delta = 0,$$

And this completes the proof of the theorem.

We consider the following no-impulsive Hopfield neural networks without,

$$\begin{aligned} &x_i(m+1) = a_i x_i(m) + \sum_{j=1}^n T_{ij} f_j\left(x_j(m)\right), m \in N(1), \\ &x_i(0) = x_{i0}, i \in N(1,n). \end{aligned} \tag{7}$$

We have the following corollary for it's the global exponentially stability.

Corollary 1

If the following inequality holds,

$$a + \delta < 1,$$

Where $a = \max_{i \in N(1,n)} \{a_i\}$, $\delta = \max_{i \in N(1,n)} \left\{ \sum_{j=1}^n \left|T_{ij}\right| \delta_j \right\}$.

Then, the Hopfield neural networks without impulse and delays (7) is global exponentially stable.

Proof

Let the solution of the discrete system (7) be $\{x(m)\}$, we have

$$x_i(m) = a_i^m x_i(0) + \sum_{s=0}^{m-1} a_i^{m-1-s} \sum_{j=1}^n T_{ij} f_j\left(x_j(s)\right), m \in N(1), i \in N(1,n).$$

Let

$$d_m = \begin{cases} \max_{i \in N(1,n)} \left\{\left|x_{i0}\right|\right\}, m = 0, \\ d_m = a^m \max_{i \in N(1,n)} \left\{\left|x_{i0}\right|\right\} + \delta \sum_{s=0}^{m-1} a^{m-1-s} \max_{j \in N(1,n)} \left\{\left|x_j(s)\right|\right\}, m \in N(1), \end{cases}$$

At $m \in N(1)$, we obtain

$$\left|x_i(m)\right| \le a_i^m \left|x_{i0}\right| + \sum_{s=0}^{m-1} a_i^{m-1-s} \sum_{j=1}^n \left|T_{ij}\right| \left|f_j\left(x_j(s)\right)\right|$$

$$\le a_i^m \max_{i \in N(1,n)} \left\{\left|x_{i0}\right|\right\} + \sum_{s=0}^{m-1} a_i^{m-1-s} \sum_{j=1}^n \left|T_{ij}\right| \delta_j \left|x_j(s)\right|$$

$$\le a_i^m \max_{i \in N(1,n)} \left\{\left|x_{i0}\right|\right\} + \sum_{s=0}^{m-1} a_i^{m-1-s} \sum_{j=1}^n \left|T_{ij}\right| \delta_j \max_{j \in N(1,n)} \left\{\left|x_j(s)\right|\right\}$$

$$\le a^m \max_{i \in N(1,n)} \left\{\left|x_{i0}\right|\right\} + \sum_{s=0}^{m-1} a^{m-1-s} \max_{i \in N(1,n)} \left\{ \sum_{j=1}^n \left|T_{ij}\right| \delta_j \right\} \max_{j \in N(1,n)} \left\{\left|x_j(s)\right|\right\}$$

$$\le a^m \max_{i \in N(1,n)} \left\{\left|x_{i0}\right|\right\} + \delta \sum_{s=0}^{m-1} a^{m-1-s} \max_{j \in N(1,n)} \left\{\left|x_j(s)\right|\right\} = d_m.$$

Thus, for any $m \in N(0)$, the following inequality

$$\left|x_i(m)\right| \le d_m, i \in N(1,n),$$

holds.

Since

$$\Delta d_m = d_{m+1} - d_m$$

$$= -(1-a)d_m + \delta \max_{j \in N(1,n)} \left\{\left|x_j(m)\right|\right\}$$

$$\le -(1-a)d_m + \delta d_m, \forall m \in N(0),$$

From Lemma2, we have

$$\Delta d_m \le (a+\delta)^m d_0, \forall m \in N(0).$$

And

$$\|x(m)\|_\infty = \max_{i \in N(1,n)} \left\{\left|x_i(m)\right|\right\} \le d_m, \forall m \in N(0).$$

Let $K = \max_{i \in N(1,n)} \left\{\left|x_{i0}\right|\right\}$, the following inequality holds,

$$\|x(m)\|_\infty \le K(a+\delta)^m, \forall m \in N(0).$$

According to Definition3, it can be seen that the no-impulsive Hopfield neural networks without delays

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(7) is global exponentially stable, attenuation rate $\lambda = \varepsilon + \delta$, and this completes the proof of the theorem.

Now, we will establish global exponentially stable criterion for the impulsive Hopfield neural networks with multiple delays (1).

Theorem 2

If the following two inequalities hold,

$$a + \delta < 1, \tag{8}$$

And

$$\sum_{j=0}^{k} \ln l_j - (k+1)\ln a \le 0 \text{, for any } k \in N(0), \tag{9}$$

Where $a = \max\limits_{i\in N(1,n)}\{a_i\}$, $\delta = \max\limits_{i\in N(1,n)}\left\{\sum\limits_{j=1}^{n}|T_{ij}|\delta_j\right\}$,

$l_k = \max\limits_{i\in N(1,n)}\left\{\sum\limits_{j=1}^{n}\omega_{ij}^{(k)}\right\}$.

Then, the impulsive Hopfield neural networks with multiple delays (1) is global exponentially stable.

Proof

For any $m \in N(1)$, without loss of generality, let $m \in (N_k, N_{k+1}]$ and then we obtain that

$$|x_i(m)| \le a_i^{m-N_k-1}|x_i(N_k+1)| + \sum_{s=N_k+1}^{m-1} a_i^{m-1-s}\sum_{j=1}^{n}|T_{ij}|\left|f_j\left(x_j\left(s-\tau_{ij}(s)\right)\right)\right|$$

$$\le a_i^{m-N_k-1}\sum_{j=1}^{n}\omega_{ij}^{(k)}|x_j(N_k)| + \sum_{s=N_k+1}^{m-1} a_i^{m-1-s}\sum_{j=1}^{n}|T_{ij}|\delta_j\left|x_j\left(s-\tau_{ij}(s)\right)\right|$$

$$\le a_i^{m-N_k-1}\sum_{j=1}^{n}\omega_{ij}^{(k)}\max_{j\in N(1,n)}\{|x_j(N_k)|\} + \sum_{s=N_k+1}^{m-1} a_i^{m-1-s}\sum_{j=1}^{n}|T_{ij}|\delta_j\max_{j\in N(1,n)}\{|x_j(s-\tau_{ij}(s))|\}$$

$$\le a^{m-N_k-1}l_k\max_{j\in N(1,n)}\{|x_j(N_k)|\} + \delta\sum_{s=N_k+1}^{m-1} a^{m-1-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}. \tag{10}$$

Thus,

$$|x_i(N_{k+1})| \le a^{N_{k+1}-N_k-1}l_k\max_{j\in N(1,n)}\{|x_j(N_k)|\} + \delta\sum_{s=N_k+1}^{N_{k+1}-1} a^{N_{k+1}-1-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}.$$

Using above iteratively, we have

$$|x_i(N_k)| \le a^{N_k-N_0-k}\prod_{j=0}^{k-1}l_j\max_{j\in N(1,n)}\{|x_j(N_0)|\}$$
$$+\delta\prod_{j=1}^{k-1}l_j\sum_{s=N_0+1}^{N_1-1}a^{N_1-k-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}$$
$$+\delta\prod_{j=2}^{k-1}l_j\sum_{s=N_1+1}^{N_2-1}a^{N_k-k-s+1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}+\cdots$$
$$+\delta l_{k-1}\sum_{s=N_{k-2}+1}^{N_{k-1}-1}a^{N_k-s-2}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}$$
$$+\delta\sum_{s=N_{k-1}+1}^{N_k-1}a^{N_k-s-1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}.$$

Since,

$$|x_i(N_0)| \le a^{N_0}\max_{j\in N(1,n)}\{|x_j(0)|\} + \delta\sum_{s=0}^{N_0-1}a^{N_0-1-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\},$$

We have

$$|x_i(N_k)| \le a^{N_k-k}\prod_{j=0}^{k-1}l_j\max_{j\in N(1,n)}\{|x_j(0)|\}$$
$$+\delta\prod_{j=0}^{k-1}l_j\sum_{s=0}^{N_0-1}a^{N_k-k-s-1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}$$
$$+\delta\prod_{j=1}^{k-1}l_j\sum_{s=N_0+1}^{N_1-1}a^{N_1-k-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}$$
$$+\delta\prod_{j=2}^{k-1}l_j\sum_{s=N_1+1}^{N_2-1}a^{N_k-k-s+1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}+\cdots$$
$$+\delta l_{k-1}\sum_{s=N_{k-2}+1}^{N_{k-1}-1}a^{N_k-s-2}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}$$
$$+\delta\sum_{s=N_{k-1}+1}^{N_k-1}a^{N_k-s-1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}.$$

From (10), the following inequality holds,

$$|x_i(m)| \le a^{m-k-1}\prod_{j=0}^{k}l_j\max_{j\in N(1,n)}\{|x_j(0)|\}$$
$$+\delta\prod_{j=0}^{k}l_j\sum_{s=0}^{N_0-1}a^{m-k-s-2}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}$$
$$+\delta\prod_{j=1}^{k}l_j\sum_{s=N_0+1}^{N_1-1}a^{m-k-s-1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}+\cdots$$
$$+\delta l_k\sum_{s=N_{k-1}+1}^{N_k-1}a^{m-s-2}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}$$
$$+\delta\sum_{s=N_k+1}^{m-1}a^{m-1-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}.$$

By (9), we obtain

$$\left|x_i(m)\right| \le a^m \max_{j\in N(1,n)}\left\{\left|x_j(0)\right|\right\}$$
$$+\delta\sum_{s=0}^{N_0-1} a^{m-s-1} \max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}$$
$$+\delta\sum_{s=N_0+1}^{N_1-1} a^{m-s-1} \max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}+\cdots$$
$$+\delta\sum_{s=N_{k-1}+1}^{N_k-1} a^{m-s-1} \max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}$$
$$+\delta\sum_{s=N_k+1}^{m-1} a^{m-1-s} \max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}$$
$$=c_m, \forall i\in N(1,n), m\in N(1).$$

Let

$$d_m=\begin{cases}\max_{i\in N(1,n)}\left\{\left|x_i(m)\right|\right\}, m\in N(-\tau,0),\\ c_m, m\in N(1),\end{cases}$$

Then, the rest of the proof follows readily from similar arguments as those given for the proof of Theorem 1.

We consider the following impulsive Hopfield neural networks without delays,

$$\begin{aligned}&x_i(m+1)=a_ix_i(m)+\sum_{j=1}^{n}T_{ij}f_j\left(x_j(m)\right), m\ne N_k,\\ &\Delta x_i(m)=x_i(m+1)-x_i(m)=g_i^{(k)}\left(m,x_1(m),\cdots,x_n(m)\right), \qquad \textbf{(11)}\\ &x_i(0)=x_{i0}, i\in N(1,n), m\in N(1), m=N_k,\end{aligned}$$

We have the following corollary for it's the global exponentially stability.

Corollary 2

If the following two inequalities hold,

$a+\delta<1$,

And

$\sum_{j=0}^{k}\ln l_j-(k+1)\ln a\le 0$, for any $k\in N(0)$,

Where $a=\max_{i\in N(1,n)}\{a_i\}$, $\delta=\max_{i\in N(1,n)}\left\{\sum_{j=1}^{n}\left|T_{ij}\right|\delta_j\right\}$,

$$l_k=\max_{i\in N(1,n)}\left\{\sum_{j=1}^{n}\omega_{ij}^{(k)}\right\}.$$

Then, the impulsive Hopfield neural networks without delays (11) is global exponentially stable.

Proof

For any $m\in N(1)$, without loss of generality, let $m\in(N_k,N_{k+1}]$ and then we obtain that

$$\left|x_i(m)\right|\le a^{m-N_k-1}l_k\max_{j\in N(1,n)}\left\{\left|x_j(N_k)\right|\right\}$$
$$+\delta\sum_{s=N_k+1}^{m-1}a^{m-1-s}\max_{j\in N(1,n)}\left\{\left|x_j(s)\right|\right\}.$$

Thus,

$$\left|x_i(N_{k+1})\right|\le a^{N_{k+1}-N_k-1}l_k\max_{j\in N(1,n)}\left\{\left|x_j(N_k)\right|\right\}$$
$$+\delta\sum_{s=N_k+1}^{N_{k+1}-1}a^{N_{k+1}-1-s}\max_{j\in N(1,n)}\left\{\left|x_j(s)\right|\right\}.$$

Using above iteratively, we have

$$\left|x_i(N_k)\right|\le a^{N_k-N_0-k}\prod_{j=0}^{k-1}l_j\max_{j\in N(1,n)}\left\{\left|x_j(N_0)\right|\right\}$$
$$+\delta\prod_{j=1}^{k-1}l_j\sum_{s=N_0+1}^{N_1-1}a^{N_1-k-s}\max_{j\in N(1,n)}\left\{\left|x_j(s)\right|\right\}$$
$$+\delta\prod_{j=2}^{k-1}l_j\sum_{s=N_1+1}^{N_2-1}a^{N_k-k-s+1}\max_{j\in N(1,n)}\left\{\left|x_j(s)\right|\right\}+\cdots$$
$$+\delta l_{k-1}\sum_{s=N_{k-2}+1}^{N_{k-1}-1}a^{N_k-s-2}\max_{j\in N(1,n)}\left\{\left|x_j(s)\right|\right\}$$
$$+\delta\sum_{s=N_{k-1}+1}^{N_k-1}a^{N_k-s-1}\max_{j\in N(1,n)}\left\{\left|x_j(s)\right|\right\}.$$

Since,

$$\left|x_i(N_0)\right|\le a^{N_0}\max_{j\in N(1,n)}\left\{\left|x_{j0}\right|\right\}+\delta\sum_{s=0}^{N_0-1}a^{N_0-1-s}\max_{j\in N(1,n)}\left\{\left|x_j(s)\right|\right\}.$$

We obtain

$$\left|x_i(m)\right|\le a^m\max_{j\in N(1,n)}\left\{\left|x_{j0}\right|\right\}+\delta\sum_{\substack{s=0\\ s\ne N_i\\ i\in N(0,k)}}^{m-1}a^{m-s-1}\max_{j\in N(1,n)}\left\{\left|x_j(s)\right|\right\}.$$

Let

$$d_m=\begin{cases}\max_{i\in N(1,n)}\left\{\left|x_{i0}\right|\right\}, m=0,\\ c_m, m\in N(1),\end{cases}$$

Then, the rest of the proof follows readily from similar arguments as those given for the proof of Theorem 1.

Now, we give two other sufficient conditions for the global exponentially stable of impulsive Hopfield

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neural networks with multiple delays (1).

Theorem 3

If the following inequality holds,

$\eta+\delta<1$,

Where $a=\max_{i\in N(1,n)}\{a_i\}$, $l_k=\max_{i\in N(1,n)}\left\{\sum_{j=1}^{n}\omega_{ij}^{(k)}\right\}$,

$\eta=\sup_{k\in N(0)}\{a,l_k\}$, $\delta=\max_{i\in N(1,n)}\left\{\sum_{j=1}^{n}\left|T_{ij}\right|\delta_j\right\}$.

Then, the impulsive Hopfield neural networks with multiple delays (1) is global exponentially stable.

Proof

For any $m\in N(1)$, without loss of generality, let $m\in(N_k,N_{k+1}]$ and then we obtain that

$$\left|x_i(m)\right|\le a_i^{m-N_k-1}\sum_{j=1}^{n}\omega_{ij}^{(k)}\max_{j\in N(1,n)}\left\{\left|x_j(N_k)\right|\right\}$$
$$+\sum_{s=N_k+1}^{m-1}a_i^{m-1-s}\sum_{j=1}^{n}\left|T_{ij}\right|\delta_j\max_{j\in N(1,n)}\left\{\left|x_j\left(s-\tau_{ij}(s)\right)\right|\right\}$$
$$\le\eta^{m-N_k}\max_{j\in N(1,n)}\left\{\left|x_j(N_k)\right|\right\}$$
$$+\delta\sum_{s=N_k+1}^{m-1}\eta^{m-1-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}. \quad \textbf{(12)}$$

Thus,

$$\left|x_i(N_{k+1})\right|\le\eta^{N_{k+1}-N_k}\max_{j\in N(1,n)}\left\{\left|x_j(N_k)\right|\right\}$$
$$+\delta\sum_{s=N_k+1}^{N_{k+1}-1}\eta^{N_{k+1}-1-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}.$$

Using above iteratively, we have

$$\left|x_i(N_k)\right|\le\eta^{N_k-N_0}\max_{j\in N(1,n)}\left\{\left|x_j(N_0)\right|\right\}$$
$$+\delta\sum_{s=N_0+1}^{N_1-1}\eta^{N_1-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}$$
$$+\delta\sum_{\substack{s=N_1+1\\ s\ne N_i,\\ i\in N(2,k-1)}}^{N_k-1}\eta^{N_k-s-1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}.$$

Since,

$$\left|x_i(N_0)\right|\le\eta^{N_0}\max_{j\in N(1,n)}\left\{\left|x_j(0)\right|\right\}$$
$$+\delta\sum_{s=0}^{N_0-1}\eta^{N_0-1-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}.$$

We have

$$\left|x_i(N_k)\right|\le\eta^{N_k}\max_{j\in N(1,n)}\left\{\left|x_j(0)\right|\right\}$$
$$+\delta\sum_{\substack{s=0\\ s\ne N_i,\\ i\in N(0,k-1)}}^{N_k-1}\eta^{N_k-s-1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}.$$

From (12), the following inequality holds,

$$\left|x_i(m)\right|\le\eta^{m}\max_{j\in N(1,n)}\left\{\left|x_j(0)\right|\right\}$$
$$+\delta\sum_{\substack{s=0\\ s\ne N_i\\ i\in N(0,k)}}^{m-1}\eta^{m-s-1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\left\{\left|x_j(t)\right|\right\}\}$$
$$=c_m,\forall i\in N(1,n),m\in N(1).$$

Let

$$d_m=\begin{cases}\max_{i\in N(1,n)}\left\{\left|x_i(m)\right|\right\}, m\in N(-\tau,0),\\ c_m, m\in N(1),\end{cases}$$

Then, the rest of the proof follows readily from similar arguments as those given for the proof of Theorem 1.

Theorem 4

If the following inequality holds,

$\beta<\frac{1}{2}$,

Where $a=\max_{i\in N(1,n)}\{a_i\}$, $l_k=\max_{i\in N(1,n)}\left\{\sum_{j=1}^{n}\omega_{ij}^{(k)}\right\}$,

$\delta=\max_{i\in N(1,n)}\left\{\sum_{j=1}^{n}\left|T_{ij}\right|\delta_j\right\}$, $\beta=\sup_{k\in N(0)}\{a,\delta,l_k\}$.

Then, the impulsive Hopfield neural networks with multiple delays (1) is global exponentially stable.

Proof

For any $m\in N(1)$, without loss of generality, let $m\in(N_k,N_{k+1}]$ and then we obtain that

$$\left|x_i(m)\right|\le a_i^{m-N_k-1}\sum_{j=1}^{n}\omega_{ij}^{(k)}\max_{j\in N(1,n)}\left\{\left|x_j(N_k)\right|\right\}$$
$$+\sum_{s=N_k+1}^{m-1}a_i^{m-1-s}\sum_{j=1}^{n}\left|T_{ij}\right|\delta_j\max_{j\in N(1,n)}\left\{\left|x_j\left(s-\tau_{ij}(s)\right)\right|\right\}$$
$$\le\beta^{m-N_k}\max_{j\in N(1,n)}\left\{\left|x_j(N_k)\right|\right\}$$

$$+\sum_{s=N_k+1}^{m-1}\beta^{m-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}. \quad \textbf{(13)}$$

Thus,

$$|x_i(N_{k+1})|\le \beta^{N_{k+1}-N_k}\max_{j\in N(1,n)}\{|x_j(N_k)|\}$$

$$+\sum_{s=N_k+1}^{N_{k+1}-1}\beta^{N_{k+1}-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}.$$

Using above iteratively, we have

$$|x_i(N_k)|\le \beta^{N_k-N_0}\max_{j\in N(1,n)}\{|x_j(N_0)|\}$$

$$+\sum_{s=N_0+1}^{N_1-1}\beta^{N_1-s+1}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}$$

$$+\sum_{\substack{s=N_1+1\\ s\ne N_i\\ i\in N(2,k-1)}}^{N_k-1}\beta^{N_k-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}.$$

Since,

$$|x_i(N_0)|\le \beta^{N_0}\max_{j\in N(1,n)}\{|x_j(0)|\}$$

$$+\sum_{s=0}^{N_0-1}\beta^{N_0-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}.$$

We obtain

$$|x_i(N_k)|\le \beta^{N_k}\max_{j\in N(1,n)}\{|x_j(0)|\}$$

$$+\sum_{\substack{s=0\\ s\ne N_i\\ i\in N(0,k-1)}}^{N_k-1}\beta^{N_k-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}.$$

From (13), the following inequality holds,

$$|x_i(m)|\le \beta^{m}\max_{j\in N(1,n)}\{|x_j(0)|\}+\sum_{\substack{s=0\\ s\ne N_i\\ i\in N(0,k)}}^{m-1}\beta^{m-s}\max_{t\in N(s-\tau,s)}\{\max_{j\in N(1,n)}\{|x_j(t)|\}\}$$

$$=c_m,\ \forall i\in N(1,n), m\in N(1).$$

Let

$$d_m=\begin{cases}\max_{i\in N(1,n)}\{|x_i(m)|\}, m\in N(-\tau,0),\\ c_m, m\in N(1),\end{cases}$$

Then, the rest of the proof follows readily from similar arguments as those given for the proof of Theorem 1.

NUMERICAL EXAMPLE

In this section, a numerical example is presented to verify and illustrate the usefulness of our main results. Consider no-impulsive Hopfield neural networks with multiple delays with the following specifications:

$$x_1(m+1)=\frac{1}{2}x_1(m)+\theta f_1\left(x_1\left(m-\frac{1}{2}\right)\right)-\frac{1}{4}f_2(x_2(m-1)),$$

$$x_2(m+1)=\frac{1}{3}x_2(m)+\frac{1}{3}f_1\left(x_1\left(m-\frac{1}{4}\right)\right)+\theta f_2\left(x_2\left(m-\frac{1}{3}\right)\right),$$

$$\begin{aligned}x_1(m)&=\phi_1(m),\\ x_2(m)&=\phi_2(m),\end{aligned}\quad m\in N(-1,0), m\in N(1), \quad \textbf{(14)}$$

Where

$f_1(t)=\sin t$, $f_2(t)=t$, $\phi_1(t)=t^2-8$, $\phi_2(t)=-t+6$.

Since,

$$|f_1(s)-f_1(t)|\le|s-t|, \forall s,t\in R,$$

$$|f_2(s)-f_2(t)|\le|s-t|, \forall s,t\in R,$$

and

$$\max_{i\in N(1,2)}\{a_i\}+\max_{i\in N(1,2)}\left\{\sum_{j=1}^{2}|T_{ij}|\delta_j\right\}$$

$$=\max\left\{\frac{1}{2},\frac{1}{3}\right\}+\max\left\{|\theta|+\frac{1}{4},|\theta|+\frac{1}{3}\right\}=\frac{5}{6}+|\theta|.$$

Therefore, from corollary 1, the Hopfield neural networks without impulse and delays (14) is the global exponential stability and attenuation rate $\lambda=\frac{5}{6}+|\theta|$ as $|\theta|<\frac{1}{6}$. It's state sequence chart with $\theta=0.125$ is shown in Figures 1.

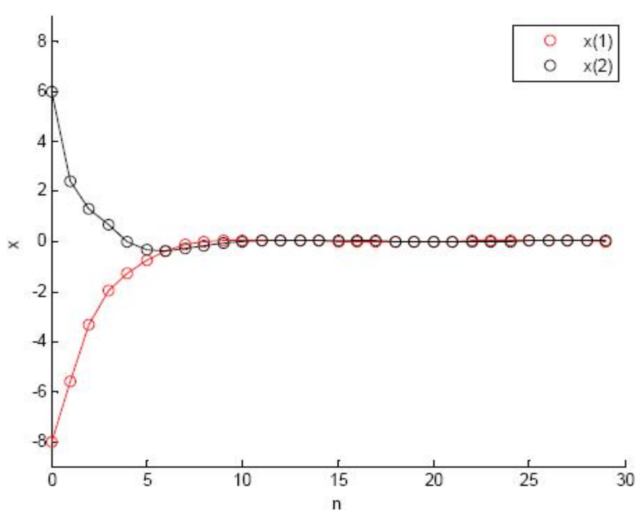

Figure 1 : The state sequence chart of (14) without impulse and delays as $\theta=0.125$**.**

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From Theorem 1, no-impulsive Hopfield neural networks with multiple delays (14) is the global asymptotical stability and global exponential stability and attenuation rate $\lambda = 0.25(1+\sqrt{1+16\left(|\theta|+\frac{1}{3}\right)})$ as $|\theta| < \frac{1}{6}$. The state sequence chart of (14) with $\theta = 0.125$ is shown in Figures 2.

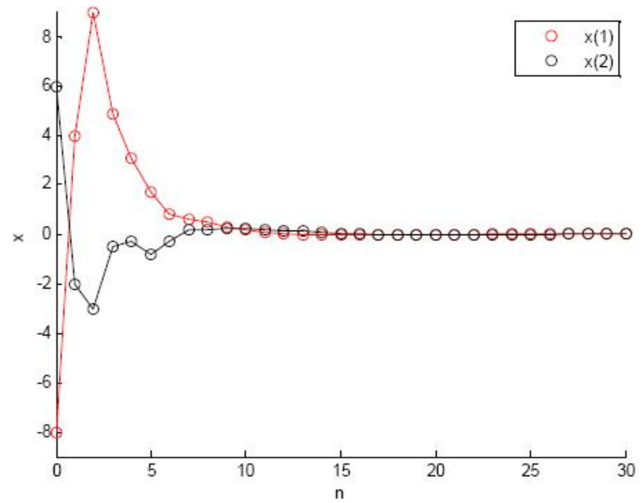

Figure 2 : The state sequence chart of (14) without impulse as $\theta=0.125$.

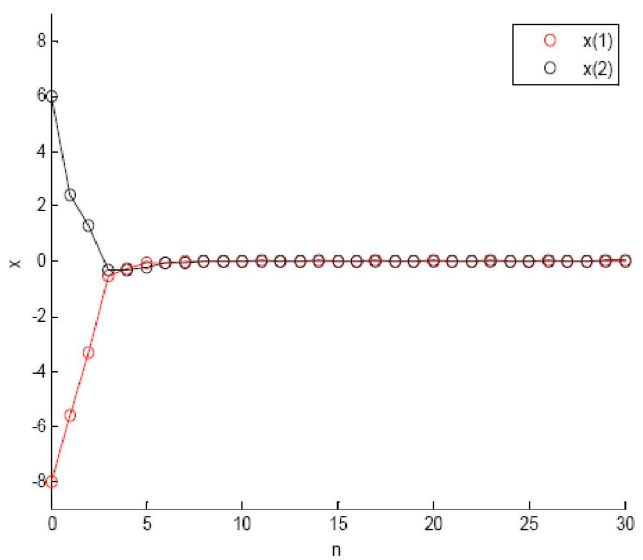

Figure 3 : The state sequence chart of impulsive system (14) without delays as $\theta=0.125$.

Now, we consider (14) with the following impulsive law,

$$\Delta x_1(m) = -\frac{3}{4}x_1(m) + \frac{1}{5}x_2(m),$$

$$\Delta x_2(m) = \frac{1}{5}x_1(m) - \frac{3}{4}x_2(m), m = N_k, m \in N(1),$$

Where $N_{k+1} = N_k + 3, N_0 = 4$.

We can compute that

$$l_k = \max_{i\in N(1,n)}\left\{\sum_{j=1}^{n}\omega_{ij}^{(k)}\right\} = \frac{9}{20} \text{ and } a = \max_{i\in N(1,n)}\{a_i\} = \frac{1}{2},$$

Therefore, from Theorem 2 and corollary 2, it is clear that the impulsive Hopfield neural networks with multiple delays (14) and the impulsive Hopfield neural networks without delays (14) are the global asymptotical stability and global exponential stability as $|\theta| < \frac{1}{6}$. Their state sequence chart under $\theta = 0.125$ are shown in Figures 3 and Figures 4.

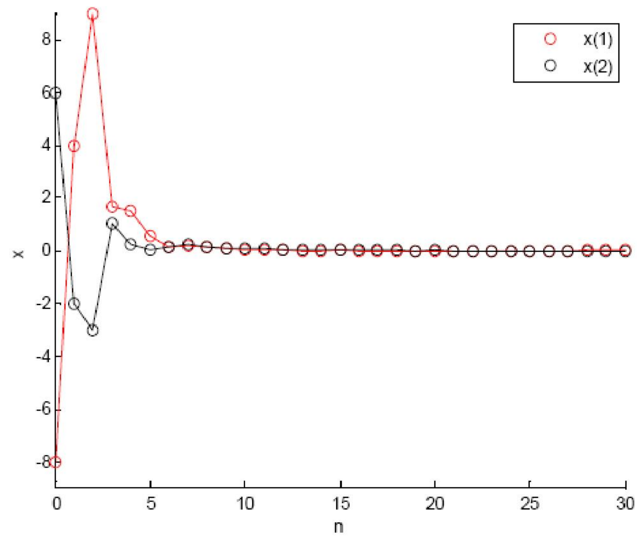

Figure 4 : The state sequence chart of impulsive system (14) as $\theta=0.125$.

CONCLUSION

We studied the global stability problems of impulsive Hopfield neural networks with multiple delays. Several sufficient conditions for global exponential stability and global asymptotical stability of impulsive Hopfield neural networks with multiple delays are derived based on the Lyapunov stability theory and discrete-time Halanay-type inequality technique respectively. Finally, a numerical example was given to show the effectiveness of our results.

ACKNOWLEDGMENT

This work was partially supported by the National

Natural Science Foundation of China under Grant 11171079, and the Natural Science Foundation of Guizhou Province under Grant LKM[2011]03, and the Construction Projects of Key Laboratory about Pattern Recognition & Intelligent Systems of Guizhou Provinc under Grant [2009]4002, and the Graduate Education Innovation Bases about Information Processing & Pattern Recognition of Guizhou Provinc.

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