Giant Magnetic Flares about Kerr Black Holes: A Review

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Abstract
Since the discovery of Quasars, giant magnetic flares about Kerr black holes (BHs) have drawn much attention to elucidate the mechanism of astrophysical high-energy phenomena and processes, such as quasi-periodic oscillations (QPOs), γ-ray bursts (GRBs), and outflow jets. Up to now, three kinds of flares are suggested: quasi-solar flares, cascade flares, and outflow flares. Both BH-dynamo theory in Gravitomagnetic (GM) field and force-free magnetosphere are developed in the curved 4D space-time by the manipulation of the 3+1 split of geometry (i.e., 3D in space+1D in time). We introduce first of all the disk-corona model of giant flares. Then, we describe the BH dynamo processes in the GM Field. Furthermore, we overview the magnetic topology of BH magnetosphere, helicity transfer, and the Penrose process. Finally, we provide a summary on the future work of the giant flare physics. It is predicted that the dynamics of the high-energy radiation in giant flares will be brought to light by the investigation of the general relativistic magnetohydrodynamic (GRMHD) dynamo action outside the central BHs residing in a tokamak-like cosmic magnetic field.

Keywords: Magnetic flare; Kerr black hole; Gravitomagnetic field; Force-free magnetosphere; Penrose process; General relativistic magnetohydrodynamic dynamo

Introduction
Since the discovery of Quasars in 1963, the nature of high-energy phenomena and processes, especially the energetic radiation and power generation of black holes (BHs) has been one of the most important subjects in modern astrophysics and astronomy [1,2]. In this field, the most striking enigma is the mechanism of the origin and evolution of the observed giant flares which manifest themselves via quasi-periodic oscillations (QPOs) [3,4], γ-ray bursts (GRBs) [5,6], outflow jets [7,8] etc., in the X/γ-ray spectra.

The concept of flare is originated from solar physics. It is defined as a sudden, rapid, and intense variation in brightness of the Sun with a period from several seconds to a few days. During the period, following processes can exist: catastrophic dissipation of stored magnetic energy, destructive alteration of original field topologies, enhanced particle acceleration and eruptive electromagnetic radiation emitted across virtually the entire electromagnetic spectrum from radio waves to X/γ-rays [9]. Flare occurs in regions of stressed magnetic fields. The existence of the fields and their dominance over the apparently numerous and complicated activities is a manifestation of a dynamo in the convection zone [10]. The exact site of the dynamo, found by SOHO in 1996, is in the thin transition layer (also called the tachocline layer) with a thickness of 38,000 miles at the bottom of the zone.

(about 135,000 miles deep below the Sun’s surface) where a high level of turbulence, shear flows and convective overshooting take place [11]. At this depth, the Sun undergoes a fundamental change in the nature of its rotation: inside this boundary the Sun rotates like a solid object; closer to the surface the rotation rates at different latitudes begin to diverge, with the equatorial regions rotating more rapidly than the middle latitudes and polar regions. This observation not only supports directly Parker’s mean-field $\alpha$-$\Omega$ interface dynamo model [12] (where $\alpha$ is the effect of convective turbulence; $\Omega$ is the angular velocity of differential rotation), which focuses primarily on the interpretation of global magnetic fields and/or large-scale ones in active regions, but also is indirectly in favor of Tong’s p-$\Omega$ dynamo model (where p reflects the divergence of the momentum density of plasma turbulence) and Cattaneo’s local dynamo model [13,14], which explain the origin of small-scale fields, such as the intra-network field, the network field, and magnetic elements at the granule and supergranule boundaries. In fact, during the last century, especially in last two decades when space observations (such as SMM, Yohkoh, TRACE, etc., in addition to SOHO) played extremely important roles, systematic measurements of the vector magnetic fields, data analyses, and theoretical approaches led to a much better understanding of the magnetic nature of solar flares and a much more comprehensive and thoroughgoing investigation of the coupling between solar flare eruption and solar dynamo action [15,16].

Based on the analogy and/or application of the concepts and methods in solar physics, the studies on flares related to either galactic sources or extragalactic ones are developed rapidly [17-19]. Similar to the Sun, these objects manifest themselves via electromagnetic radiation in the whole band. Observations show highly variable light curves on a wide range of timescales. For example, in the X-ray flux of galactic or AGN sources there are three components generally: a linear change continuously, a secularly sinusoidal fluctuation with a long period, and an episodic ingredient with a factor of several orders of magnitude on much shorter timescales but quasi-periodically occurrences with a longer interval [20,21]. Typically, such a sporadic event, considered as an astrophysical giant flare, has a luminosity of up to $10^{12-15}$ times of the solar one and last from milliseconds to hours for galactic objects and hours to days, even up to years, for extragalactic sources. With the same features as solar ones, these flares bring about a predominantly non-thermal radiation emitted from a compact region of up to $10^{3-7}$ solar radius, with the most possibility of harboring either a massive BH in galactic systems or a supermassive one in AGNs [22].

**Disk-Corona Model of Giant Flares**

With a premise of the presence of magnetic field due to some kind of dynamo action, various accretion disc-corona models were suggested to fit the observed broad-band and time-averaged X/$\gamma$-ray spectra of flares of BH candidates. In terms of Comptonization of soft photons in the structured optically-thin but hot corona, situated above a geometrically-thin and cold disc, these models focus on electron-positron plasmas in the pair and energy equilibria to explain properties of first two components of the light curve [23].

**Quasi-solar-flare model**

The first disk-corona model is a quasi-solar-flare model, proposed to clarify the connection between the primary energy source and the non-thermal phenomena of AGNs [24].

In the simple modeling, $\alpha$-$\Omega$ dynamo with magnetic buoyancy describes the behavior of the field generated in the accretion disk around a BH. The field is responsible not only for the angular momentum transfer but for the vertical energy transfer as well owing to the buoyancy [25]. The magnetic energy stored in the coronal region above the disk surface is released through magnetic reconnection. This model applies the solar flare process to the energy transport of AGNs and presents such a similar picture: At the center of a magnetized, thin accretion disk, there is a rotating BH. The differential rotation between the BH
and the disk generates a large-scale magnetic field in the disk by the $\alpha$-$\Omega$ dynamo of smaller scale turbulent motions in the velocity and vorticity field. A fluid element with stronger magnetic field but weaker gas pressure and turbulent pressure, thus smaller density, rise with Alven velocity because of the buoyant force, which may be the result of Rayleigh-Taylor instability. The magnetic flux escapes from the disk vertically with the Alven velocity. The field is then deformed above the disk to some configuration favorable to experience magnetic reconnections. Thus, the energy dissipates, transforms into the kinetic and thermal energy of plasma and particle acceleration.

As in the case of solar flares, major part of the energy in this picture was assumed to go into the kinetic ejection of plasma and generate the relativistic shock wave in the corona behind which electrons are heated up to relativistic energy. Subsequently, they produce X/$\gamma$-rays by the Inverse Compton Scattering of low energy photons and emit photons from radio up to soft X bands by the synchrotron radiation. Although the picture was found to be compatible with observational facts such as the superluminal expansion, the paucity of non-relativistic electrons on the compact radio sources, there are fatal obscurities in this model, such as, mechanisms of components in the light curves, the initiation of the magnetic reconnection, and, the effect of the actual geometry of the accretion flow.

**Cascade-flare model**

It was not until the study of both the stochastic pulse-avalanche flares [26] and the thundercloud ones [27] that a cascade mechanism for giant flares in an accretion disc-corona system was put forward to elucidate the tempest events of high energy radiation. The sketch is as follows:

- The accreting gas releases the total power $L$; the disc generates the coronal power $L_c$ as a fraction $f$ of the total ($L_c=fL$);
  the coronal power can be either dissipated locally to heat the corona, and ultimately radiated away with a luminosity $L_\alpha=(1-\eta)L_c$, or used to launch an outflow with a power $L_o=\eta L_c$.
- In the hot corona the energy is stored in a strong, highly intermittent magnetic field, amplified in the turbulent disc and buoyantly expelled in the vertical direction.
- Magnetic dissipation occurs at the smallest end of the turbulent energy cascade; such small flares heat the corona (with the power $L_{\alpha}$) and trigger an avalanche in their immediate neighborhood via magnetic reconnection. In avalanches, the heating of the corona proceeds in correlated trains of events. The amplitude of the avalanches obeys a power-law distribution and determines the size of the active regions where the spectrum is observed.
- The avalanche creates a larger active region, called the thundercloud area, where larger magnetic reconnection causes likely the compact heating with a typical size comparable to the accretion disc thickness. The spatial and temporal distribution of the thundercloud areas are determined by the avalanche. Due to the feedback effect of the X-ray radiation reprocessed in the disc, larger active regions produce softer spectra.

**Avalanche model**

The avalanche model aims at explaining the shortest time-scale flare, which has long been considered as the source of the rapid variability of the spectra [19]. In the model, some dynamo processes associated with the differential motion amplify magnetic fields in the disc and lines rise up into the corona where they reconnect with each other to liberate energy.

Although how this process takes place is not clear, there are indications that the emission expands and detaches from the disc and the overall evolution time-scale of the magnetic field configuration is of the order of Keplerian time-scale [28], lying
between ~1 ms and a few tenths of a second for a BH of 10 M_☉. The reconnection of the lines in one location can force the neighboring lines into unstable states to cause subsequent longer reconnections successively as the avalanche-phase of the flare. The evolution of an individual avalanche may roughly parallel to that of a solar active region [29].

When magnetic fields are moving away from the underlying accretion disc, the spectrum of the flare varies with time. In the beginning of the flare on the disk surface, soft photons are just the average background of luminosity, and the spectrum is very soft. With the increase of time, hard X-rays are produced in the flare emission region (ER) and then heat up the cold disc in the vicinity of the flare, creating extra soft photons which then dominate the cooling and harden the spectrum [30].

However, not all hard photons return to the ER. When ER moves sufficiently away from the disc, the spectrum becomes soft again. The hardening of the spectrum during the phase of maximal energy dissipation agrees well with the observation of short time-scale spectral evolution of Cyg X-1 and other galactic BHs [26,31]. In such a model, detected signals correspond to coronal hard X-ray magnetic fares, respectively, which possess a range of durations and are capable to trigger larger avalanches.

**Thundercloud model**

For longer time-scale flares, a thundercloud model [27] was suggested to explain the process. The basic idea is that the magnetic reconnection in the disc corona does not occur randomly in time and space, but rather relates to avalanche events. Because the spectrum produced in regions where a large number of heating events take place is different from the spectrum generated by an isolated event, large avalanches of magnetic flares may obscure the underlying disc as seen from the observer, and complicate the temporal behavior of the secondary spectral features, e.g., reflection hump and fluorescent lines. This concept both interprets the properties of the light curve and explains the observed spectral variability accordingly.

In the model, the background power-law flux is automatically satisfied due to previous demonstrations with a simple analytic model and with Monte Carlo simulation [32]: If a dynamo-generated magnetic field with a coherence scale of the order of the disc thickness emerges from the accretion disc, the inverse cascade process of stochastic magnetic reconnection of flux tubes may lead to much larger coherent fields and active regions, with a power-law distribution in size. This means that, in principle, the magnetic field reconnection of flux tubes in a tenuous corona is able to produce a power-law distribution of active region sizes. Such a behavior may be a universal one in compact accreting systems.

The basics geometric properties of the model required to explain the observed spectral and temporal variability of AGN are as the following: the corona is not uniform but structured and heated intermittently; the fundamental heating event, a flare likely caused by magnetic reconnection, is compact with typical size comparable to the accretion disc thickness; the height of the reconnection site is an order of magnitude larger than its size; the spatial and temporal distribution of the flares are not random but proceed in correlated trains of events in an avalanche fashion; the size of the avalanches determines the size of the active regions and their luminosity, and are distributed as a power-law.

It deserves to mention that this model assumes a static disk-corona [33] which extends down to the innermost stable orbit, and the X-ray spectrum is produced by the thermal Comptonization in spherical active regions which are lifted above the disc to a height with permeated magnetic field. There are two kinds of photons for Comptonization: synchrotron ones produced locally by the interaction between the hot electrons and the magnetic field, and black-body ones coming from the underlying
cold disc above which part of the flux in an active region is reflected and part is absorbed and reprocessed. If more accretion power is dissipated in the corona and the reprocessed radiation is the dominant source of soft photons for Comptonization, the spectrum is then softer.

In general, the dominant source of soft photons can be determined by comparing the local energy densities of the synchrotron radiation and of the thermal emission emerging from the disc, both from the intrinsic dissipation and reprocessed radiation. In the radiation, the synchrotron decreases with an increasing central source mass and is a strong function of the temperature in only relevant, magnetically dominated, compact active region [34], which can be viewed as magnetic thunder-clouds in the highly inhomogeneous, stochastic accretion disc-corona system. These thunderclouds are charged by the differential rotation of the underlying disc and/or the turbulent motions in the accretion flow. Their sizes are distributed as a power-law. The energy release, triggered by magnetic reconnection on the smallest scales, heats progressively larger active regions. Each active region (or the thundercloud) produces the observed flares or strokes by inverse Compton scattering soft photons, coming mainly from the underlying optically thick accretion disc. If the coronal optical depth is thick enough, active regions may obscure the X-ray spectral features produced in the cold disc such as, in particular, the fluorescent lines.

We mention here that it is also feasible to seek for an analogy between high-energy astrophysical processes and terrestrial lightning [27]. In terrestrial lightning’s, the durations, peak currents, intervals between the strokes in the flashes, and the flash charges are log normally distributed, which have been found to be similar to that of GRB-light curves [35].

Outflow flare model
On longer timescales, variations of the fraction of power released into the corona and of the fraction of coronal power used to launch an outflow may be associated, for example, to changes of the accretion rate and/or of the geometry of the inner disc. The thundercloud model can also show that, under reasonable assumptions on the nature of the disc viscosity, the strength of the coronal fraction increases as the accretion rate decreases, and that the energetically dominant corona is the site where powerful outflow-flares are produced [36].

In this case, the evolution of the corona is governed by the global properties of the accretion flow, the evolution of the accretion rate and/or of the inner disc geometry. At a low accretion rate, the strength of a magnetic corona produced by buoyant magnetic flux tube amplified in an underlying standard accretion disc increases. If the energy in the corona is stored in the magnetic field, and the height of a reconnection site is much larger than its size, which is of the order of the disc thickness, powerful MHD outflows can be launched from the inner corona. Observational evidences show that there are common features for two classes of systems, Galactic Black Hole Candidates (GBHC) in their low/hard state and the Low Luminosity AGN (LLAGN). They contain not only the weak quasi-thermal spectral component in the soft X-ray/EUV spectral range, and the hard X-ray power-law spectra rolling over ~100 keV, the latter of which behaves as the evidence of thermal Comptonization in an optically thin medium, but also the compact (unresolved) flat or inverted radio core accompanied by jet-like features, and a clear temporal correlation between the radio and hard X-ray fluxes.

If angular momentum transport happens in a standard, geometrically thin and optically thick accretion disc due to magnetic turbulent stresses, the magnetic energy density and the effective viscous stresses inside the disc are proportional to the geometric mean of the total (gas plus radiation) pressures, and the fraction of the gravitational power released in a magnetic
corona increases as the accretion rate decreases, because the disc is more and more gas pressure dominated, even in its innermost parts. When the disc is completely gas pressure dominated (at accretion rates smaller than the critical value), the strength of the corona depends on the poorly understood mechanisms of vertical flux tube transport in the disc.

Models and simulations of jet production [37] show that it is the poloidal component of the magnetic field which mainly drives powerful jets with stronger output power and outflow power if the coronal scale height is larger with respect to the distance from the central source. This would help to increase the relative strength of the poloidal component of the magnetic field, which is ultimately responsible for the powering of the jet and carry away a substantial fraction of the coronal power. The X-ray mission Chandra has observed a dramatic X-ray flare (a brightening by a factor of 50 for three hours) from Sgr A*, the Galactic Center supermassive BH [38]. Because Sgr A* has never shown variability of this amplitude in the radio band, it was therefore argued that a jump in the accretion rate does not seem to have any effects. Based on the model for jet-dominated emission in the quiescent state of Sgr A*, it was suggested that the flare is a consequence of extra electron heating near the BH. This can either lead to a direct heating of thermal electrons to $T_e \approx 6 \times 10^{11}$ K and significantly increased synchrotron-self Compton emission, or result in a non-thermal particle acceleration with increased synchrotron radiation and electron Lorentz factors up to $\gamma_e > 10^5$. While the former scenario is currently favored by the data, simultaneous VLBI, submm, mid-infrared and X-ray observations should ultimately be able to distinguish between the two possibilities.

**Deficiencies of disk-corona flare models**

Disk-corona models seems to be successful in elucidating the mechanism of giant flares. However, there emerge following concerns:

- Essentially different from stars with primary fusion energy, compact objects are at the last stage of the stellar evolution when most of the nuclear fuel has been consumed. It is the gravity coupling with the electromagnetic field that supports all kinds of activities [39]. In high-energy processes of compact objects, the gravitational effect plays such a key role that it must be taken into account [40].

- The high-energy radiation of pulsars and BHs, which are observationally showing the evidence of axisymmetrically rotating [41], is directly related to the rotational energy of the central bodies [42-45]. The energy is transmitted to particles by electromagnetic engine of the core. The dynamo action of the disk looks less favorable to account for the high-energy production [46,47]. Consequently, the conventional 3D Minkowski geometry is invalid but the 4D Kerr metric must be considered.

- Superimposing upon the continuum, atom Fe line emission is also detected in high-energy radiations. This line cannot be originated from the disk-corona but from near the innermost part of the disk, where the configuration may be a torus rather than a disk [48]. This structure was supported by early theoretical work of bound geodesics of particles near a BH in the Kerr metric [49].

**BH Dynamo in Gravitomagnetic Field**

Fortunately, the gravitational electrodynamics of both pulsars and BHs was developed to explain the energy transfer electromagnetically via the Blandford Znajek (BZ) mechanism in 1970s [47,50,51]. The general relativistic magneto hydrodynamics (GRMHD) was also established in the 1980s to describe the coupling of the electromagnetic field with the fluid one in the 4-D curved space time [52-54]. To be more important, on the basis of GRMHD, a theory of the mean-field $\alpha$-$\alpha_g$-$\omega$-$\Omega$ dynamo was developed for a BH-disk system (where $\alpha_g$ is the factor of gravitational redshift and $\omega$ is the angular
velocity of zero-angular-momentum-observers, or, ZAMOs) in the 1990s [55-57]. The theory demonstrated an extraordinary ability to describe the topology of the gravitational magnetic field and the evolution of the magnetosphere of compact objects, and, consequently, reveal bright prospects in modeling energetic giant-flares of compact objects [24,58].

**Gravitomagnetic field and FD effect**

In the conventional electromagnetism, a moving charge (an electric current) causes an ordinary magnetic field by Ampere’s law, and, a changing magnetic field induces electrical currents by Faraday’s law. Similarly, a moving body (a matter current) produces a new quasi-magnetic field, named as gravitomagnetic (GM) field, and a changing GM field induces matter currents [59]. This description is a variant of the conventional gravitational field described by the simpler math of Sir Isaac Newton. This GM field can be so strong that it not only is possible to rival the old one in strength but can radiate discernible gravitational waves as well. This striking analogy is so apt that the equations describing the new field components can essentially be adapted from Maxwell’s equations in electromagnetism by replacing the charge density with the mass density and the charge current with the mass current.

For a rotating body (say, a BH), Einstein’s GR theory predicted that it drags the 4D space time of the curved inertial frames around itself as it rotates. This is called the frame-dragging (FD) effect in the GM field. It is so important and fascinating for the test of relativity that, soon after the publication of GR theory, Lense and Thirring (LT) predicted in 1918 the advance of the pericenter and line of nodes of a particle orbiting a rotating mass in the weak-field limit. For this reason, the FD effect is also called the LT effect. However, an exact expression of the effect was not presented systematically under Boyer-Lindquist coordinates in Kerr metric until a half century later [49]. Results show that the dragged bound geodesics of a node is not restricted only in the equatorial plane of the central body but can cross it repeatedly; besides, the node is able to trace out a helix-like path lying on a sphere enclosing the central object with the extreme Kerr case. More strikingly, different from the case of a test particle with a finite mass in equatorial orbits [60], which is the simplest accretion-disk model commonly used so far, general geodesics of a test particle for arbitrary rotating state of a BH show that its three constants of motion (energy, angular momentum parallel to the symmetry axis, and Carter’s polar-angle constant) have upper limits and lower ones. The particle can take every set of the three constants between the two limits, both of which correspond to that of the prograde disk (either a tilted one or an equatorial one) and the retrograde one, respectively [61-63].

Nevertheless, like many other aspects of GR theory, the FD effect is so subtle to be detected in our flat Minkowski space that no conventional method could measure it. The precision is equivalent to about one part in a few trillion. This means that it is necessary to either look at an object very massive, or build an instrument that is incredibly sensitive. So far, there are two kinds of tentative methods to measure the weak effect: (1) indirect ones which make use of observations of either quasi-periodic oscillations (QPOs) [64] or broad Fe K lines [65] of compact objects; (2) direct ones which either measure the first two of the three main FD effects (precession of a gyroscope, orbital planes, and the pericenter) around the Earth with satellites [66,67] or, detect the gravitational waves in the solar system with spacecraft [68,69].

**Stationary, axisymmetric, force-free magnetosphere of a Kerr BH**

As an essential ingredient in energy releases (e.g., electromagnetic radiation, energetic burst, particle acceleration, etc.) of an AGN, the magnetic field is observationally verified to exist at least on scales of several kpc [70], and, undoubtedly, the field is continuous till the BH in the core, which is rotating axisymmetrically, a justified geometry for most observed compact objects in nature [41,71]. Thus, the magnetospheric physics of a Kerr BH has been a hot topic since a few decades ago.
Blandford and Znajek revealed the well-known Blandford-Znajek (BZ) process in 1977 [51], which was proposed to illustrate the energy transfer in a BH’s magnetosphere from both the central BH and the surrounding disk into an intense electromagnetic flux, the energizer of radiation in AGNs. In the process, the rotating BH, threaded by magnetic fields of the surrounding disk, is imaged to drag the field lines around, and, cause them to eject plasmas (extracting rotational energy) in two opposite jets along the rotation axis of the BH.

Besides, a membrane paradigm of the BH in the gravitomagnetic magnetosphere was also suggested in the 1980s [72]. In the picture, the presence of a thin, frozen, but highly complex boundary layer (also considered as the stretched horizon) is presumed to be just above the true event horizon and covers up all physical properties, such as, electrical resistivity, viscosity, etc. The new viewpoint not only simplifies the relic of the BH’s past, but provides a feasible model to study particles and fields very near the BH’s horizon.

Unfortunately, the GR language and mathematical formalism of electrodynamics theory in the 4D curved spacetime was very abstruse and alien in real applications for observational astronomers. For example, 4D tensors are used to represent parameters in the coupling fields of gravitomagnetic electromagnetism. Only after the 3+1 (3D absolute space+1D global time) split of the Kerr metric was realized and applied to the BZ theory, can the coupling mechanism among the three fields--the GM field, the electromagnetic field, and the fluid field--be able to be dealt with practical cases with parameters in flat space, which are familiar for most observers [73]. With the simplified metric, the analytical expressions of two parameters (the magnetic flux and current) have been derived in a stationary, axisymmetric, force-free magnetosphere of a Kerr BH [74].

A global structure of the magnetosphere was described in early 1990s [75]. The magnetosphere spreads out over all the space from the BH’s stretched-horizon (instead of the traditional BH’s horizon), the outer boundary of the BH within which ZAMOs cannot communicate with the exterior, to the infinity horizon (instead of the traditional BH’s infinity), the load region in which high-energy astrophysical processes can be treated as in flat space.

In addition, similar to that of an unmagnetized BH’s surroundings, there exist the static-limit surface, the effective ergosphere, and the inner/outer light surfaces. The static-limit surface is also called the infinitely-redshift surface within which ZAMOs cannot remain at rest but must rotate in the same direction as that of the hole. The effective ergosphere (instead of the traditional BH’s ergosphere) is the region between the stretched-horizon and the static-limit surface, in which there exist orbits of negative-energy-at-infinity. At inner/outer light surfaces (for a nonrotating BH, the inner one is coincident with the horizon and the other at infinity), the angular velocity of a particle equals the light-speed to avoid superluminous motion. Within the former surface (close to the stretched horizon), the particle must slide inward along magnetic lines, while beyond the latter one (corresponding to the familiar pulsar light cylinder) outward along magnetic lines. More important, there are two surfaces which characterize the BH’s magnetosphere. One is the null surface between the two light surfaces; at which ω is equal to that of field lines (the surface where ω is half of the BH’s rotation is called the middle surface). Relative to ZAMOs, the toroidal velocity of the line and the poloidal component of the electric field on this surface is zero. Hence, the surface divides the force-free magnetosphere toroidally into such two regions, the inner one and the outer one, that each of the both possesses its own light surface through which the plasma current flows inward or outward. The velocity of lines relative to ZAMOs begins to increase from zero at the null surface and equals light velocity at the outer light surface, tending to infinity toward the region far from the outer light
surface, while it changes sign at the inner light surface and equals negative light velocity and decreases to negative infinity toward the stretched horizon. Another surface is the critical field-line surface which divides the force-free magnetosphere poloidally into such two regions, the polar one and the equatorial one, that each of the both possesses its own charge distribution through which the convective electric currents flows inward or outward. The current flows in from infinity to the stretched-horizon surface in the polar region, and out of the surface to infinity in the equatorial region. The outgoing wind blows from near the middle, null surfaces, consisting of electrons in the polar region and of positrons in the equatorial region, respectively. The ingoing wind blowing towards the stretched-horizon surface consists of positrons in the polar region and of electrons in the equatorial region, respectively. Nevertheless, the topology of the magnetosphere needs further clarification. In neutron-star/BH binaries, it was suggested that a torus-type magnetosphere may be formed around the spinning BH [76]. The torus is presumably magnetized by the remnant magnetic field of the neutron star. Outside the innermost stable circular orbit of the BH, the breakup process of electrically neutral matter takes place. In seyfert AGNs, the observation of a comparable width of the Fe line with the optical ones supports the assumption of a hot-torus outside the central BH [48]. Data-fitting of the torus model to the observed Fe K$\alpha$ lines detected by Advanced Satellite for Cosmology and Astrophysics (ASCA) also verifies a thin-torus structure, extending to both sides of the BH’s equatorial plane within a polar angle of tens of degree in a radial position no more than 40 gravitational radii [77,78]. As a result, the commonly used model of the standard thin disk may not still be valid in this special region [79,80].

**$\alpha G\omega$-Mechanism and Mean-Field $\alpha$-$\alpha G\omega$-$\Omega$ Dynamo in 3+1 Kerr space time**

In the BH’s magnetosphere, the laws are couched in terms of the fields measured by ZAMOs in the locally nonrotating frames (LNRF) [60,81]. In LNRF, FD effect is condensed into only two scalar congruent functions: the gravitational redshift $\alpha_g$ of ZAMOs’ global time and the toroidal angular velocity $\omega$ of ZAMOs relative to the absolute space. Thus, FD effect is also referred to the $\alpha G\omega$ mechanism in the 3+1 GM field where the electric field, magnetic field, electric charge density, and current density, etc., are the same variables as those in flat-space electrodynamics [74]. On the horizon of a BH, $\alpha_g$ is zero and $\omega$ is the uniform angular velocity of the body, while in a region far from the body $\alpha_g$ becomes 1 and $\omega$ would be negligible.

Although the $\alpha G\omega$ mechanism reflects the particularity of the GM field, the magnetic flux threading through the BH is known to be provided by the dynamo process in the surrounding accretion disk [55,82,83]. It was found that any seed field advected into the differentially rotating accretion disk with the angular velocity, $\Omega$, will be sheared into a helical field, which is then transported by the turbulent diffusion and the radial advection in the disk to the central body. The distribution and the strength of the field strongly depend on the large scale conductivity of the accreting plasma, and, similar to that in the solar convection layer, the poloidal component of the field can be amplified by the $\alpha$-effect of the plasma [84,85]. Without no doubt, the $\alpha-\Omega$ dynamo action exists in the disk [86,87].

Nevertheless, different from the one in the flat space, the dynamo action is certainly modified, more or less, by the FD effect in the GM field. The commonly used MHD equations should be replaced by the GRMHD ones to exhibit the amplification of the initial magnetic field, the configuration of the BH’s magnetosphere, the electromagnetic process of a BH-disk-jet system, and, eventually, the energy mechanism (energy origin, energy storage, and, energy dissipation in the GM field) of AGNs. With the advance of the study on relativistic effects of the MHD wind in a stationary, axisymmetric system [52-54] and on the GRMHD flow in Kerr geometry [88-90], Khanna and Camenzind developed the kinematic theory of the axisymmetric
mean-field $\alpha$-$\alpha_g$-$\omega$-$\Omega$ dynamo with the formulation of 3+1 split [39,56,57,91]. In several aspects, the theory brings to light characteristics of a BH dynamo in a GM field.

Firstly, the Cowling’s theorem is not valid in the GM field which falls off rapidly with the increase of the distance from a central BH, and, causes the FD effect to be localized only in the innermost part (or the transition layer) of the accretion disk, covering a region between the BH’s horizon and a few Schwarzschild radii outside the BH. In the region, besides the conventional $\alpha$-$\Omega$ dynamo action, there also exists an extra current term in the induction equation, producing an additional closed poloidal magnetic structure around the BH. Thus, relative to the case in Minkowski space, the poloidal magnetic field is augmented by the $\alpha_g$-$\omega$-effect, and, a self-excited axisymmetric dynamo action can be present without any need of turbulent plasma motions even in an axisymmetric system. In a zero angular momentum (i.e., $\omega=\Omega$) flow, there is still a FD shear and the dynamo can still be operated. The unified $\alpha$-$\alpha_g$-$\omega$-$\Omega$ dynamo action builds up a stellar-like magnetosphere which blends into the disk field topology in the outer boundary of the transition layer.

Secondly, there is an additional battery term, the electromotive force originated by the GM field, in the generalized Ohm’s law, which is likely to saturate at higher field strength than the classical Biermann battery. In an initially unmagnetized plasma, the battery can induce toroidal magnetic fields in an axisymmetric system, which also generates a poloidal one due to the dynamo action. The current system driven by the battery is closed in the corona of the BH and extracts the rotational energy of the BH. Thus, the efficiency of the BZ mechanism can be promoted.

Finally, the FD effect is responsible for the growth of linkages between the toroidal and poloidal magnetic components, and, the particle acceleration with energy extraction. In the GM field, the transition layer behaviors like the boundary layer of the accretion plasma, where three dynamo modes (the dipolar, the dominant quadrupolar, and, the mixed modes) are prone to be in favor of resistive MHD instabilities (such as, the rippling mode and/or the tearing mode), as well as the anomalous or turbulent magnetic diffusivity, which provide a potential mechanism of particle acceleration in outflows observed as jets in AGNs and/or YSOs, conveying the energy extracted from the BH and depositing it in the disk corona and/or the load region (such as the radio lobes) in infinity. In fact, as seen by local stationary observers, the plasma in the transition layer appears to behave as particles since the electron collision time becomes longer than dynamical timescales.

New concern lies in the connection of the dynamo action with the origin of flares [92]. According to the relativistic Ohm’s law in stationary axisymmetric geometry, the axisymmetric magnetic fields developed against ohmic dissipation by a dynamo action are usually originated from two essential elements: a background magnetic field and an azimuthal convection current. The latter is so effective that, if the plasma rotation becomes highly relativistic with a sharp gradient of the angular velocity, it is the flare-like instability permissible near the light cylinder, but not the generation of a stronger field, can be triggered because of the magnetic reconnection caused by the plasma shear in the GM field. In this case, at least part of the energy transported by outflows is released in the form of flare eruption.

However, it is worthwhile mentioning three significant gaps between the mean-field dynamo action and its applications. One problem is that the disk model does not apply in the region where the mean-field $\alpha$-$\alpha_g$-$\omega$-$\Omega$ dynamo is formulated. In the innermost part of the accretion disk where the FD effect is valid, the accretion matter adopts a torus structure but not a disk
one [49,60-63], and, at the null surface (also called the dyadosphere) where the electromagnetic field exceeds the Heisenberg-Euler critical value, e⁻e* pairs are produced and can be detected in infinity [93]. This means the disk model must be replaced by a torus one which can be verified by observations.

Besides, it is not clear about the evolution of magnetic helicity in the GM field. As a result, the estimation of the energy storage and release by dynamo actions will be hampered. Magnetic helicity is the volume integral of the product of magnetic field and its vector potential [94]. It measures the knottedness and twistedness of field lines. In flat space, either for the domain of integration is periodic, infinite [95], or, at a boundary where the normal component of the field vanishes [96], or even the boundary is open to the magnetic field [97,98], the helicity will be gauge invariant and hence meaningful. Once helicity is known, we can estimate the variation of line topology, which determines the difference of the initial and final energy during one period of field change, and thus calculate the radiation emission used for data-fit modeling.

Another major impediment is that there are few observations so far to support the presence of magnetic fields about a BH, generated by dynamo actions. According to Bisnovatyi-Kogan and Ruzmaikin’s results [99,100], the magnetic field near the marginally stable orbit could be $10^{10-11}$ Gs. However, it is completely impossible up to now to detect directly the magnetic field of compact objects within an order of several mpc with current ground/spaceborne instruments, without mentioning the discrimination of the transverse components from longitudinal ones as happened in solar observations. Luckily, evidences of iron line emission from the region close to a BH can provide indirect messages of magnetic fields, which helps to diagnose the dynamo action. For example, for Seyfert AGN MCG-6-30-15, the iron line reveals that the above mentioned magnetic strength is the minimum [101].

**Magnetic Topology of BH Magnetosphere, Helicity Transfer, and Penrose Process in GM Field**

In most regions outside the hole, the field is both force-free and degeneracy [51]. This means the plasma becomes sufficiently rarefied that it is no longer exerts significant forces on the magnetic field but it is still sufficiently dense to provide the charged particles with a high electrical conductivity to freeze the magnetic field into the plasma. However, the plasma in the inner region, especially the innermost one, of the disk is only degenerate but not force-free [74].

Here is the synthesis of magnetic structures described by previous authors [74,75,102,103], the BH magnetosphere is characterized by a quasi-tokamak cavity in the helical topologies of both toroidal and poloidal magnetic fields, both of which are not only axisymmetrical about the hole’s rotation axis, but reflection-symmetrical about the hole’s equatorial plane as well. The latter field is divided into two types, open and closed, depending on whether the lines are penetrating the stretched horizon of BH in the polar region at one end and extending to infinity at the other end, or, connecting both low-latitude regions of two hemispheres at two ends, respectively, with the help of a surrounding torus and/or disk. Particles on the open lines flow outward beyond the outer light surface to the infinity and flow inward within the inner light surface to the stretched horizon. The last open line is trapped to form a closed loop through the stretched horizon with opposite polarities in two hemispheres. All last open lines form a surface, within which a field-free, quasi-tokamak cavity is formed to convey a ring current around the BH in the low-latitude region. The ring current is induced by the curl of the closed field and flows in the cavity toroidally outside the stretched horizon. The quasi-tokamak cavity behaviors like a Tokamak waveguide where two plasma waves, the fast magnetosonic and Alfvén waves propagate [104].
Nevertheless, either the linkages driven by BH dynamo actions between the toroidal and poloidal magnetic components, or the helicity transport from the innermost region outside a central BH to infinity, are still unclear. This hampers the effort to elucidate the change of the magnetic topology and the process of huge energy dissipation during giant flares. Although the magnetic Reynolds number is large (the resistivity is small) and the helicity is almost independent of time under many astrophysical conditions, similar to the case of ideal plasma with zero resistivity, where the helicity is strictly conserved and the magnetic energy is dissipated at finite rates, an exact topology of BH magnetosphere and a clear picture of energy release are still under discussions. In the flat space, the helicity conservation equation can be split into two evolution equations of the sub-helicities which are associated with the large-scale mean field and the small-scale turbulent one, respectively. The conservation of the total helicity naturally leads to the result that two sub-helicities are equal but opposite, and, the ratio of the mean-field helicity to the turbulent-field one is in proportion to the ratio of the turbulent scale over the mean-field one. In the presence of closed and/or open boundaries, the mean-field helicity, the expression of the potential capability to release magnetic energy, can be either transported across space or separated from small-scale eddies, depending on the relevant dynamo process [105-107]. When the electrostatic/electron-diabatic turbulence is dominant, the mean-field helicity is taken across space without dissipation, and, when the electromagnetic turbulence is effective, the turbulent helicity is converted to the mean-field one. Besides, the mean-field will evolve naturally to a saturation value by the gradients of the large-scale fields relative to the small-scale one.

As said in the above, a new ingredient, FD effect, occurs during the dynamo process in the GM field. The new term in the dynamo equation is the relativistic EMF called the gravitomagnetic dynamo term. Even without any other initial source, this term causes similar amplification and maintenance of the magnetic field as in flat space [57]. Naturally, the effect can be classified into the contribution of a pseudo-turbulent α-effect in the ergosphere. In this case, the mechanism of helicity transfer will be similar to that in flat space. However, due to the everlasting existence of the FD effect in the ergosphere, it can be predicted that the saturation value of magnetic field will be endlessly increasing. To avoid this infinite accumulation for any compact-object system with a limited capacity, the central object has to find a way to get rid of the helicity excess to the infinity through the force-free region. Either fluid flows or electron-positron pairs near the null surface in the ergosphere can be conjectured to propagate the helicity excess in the GM field released by giant flares and/or CME-like jet-ejections, along with intermittent but constant collapses of the entire BH magnetosphere.

We propose a solution as follows. The initial total helicity (mean-field+turbulent one) of a BH magnetosphere is zero under the condition of an initially non-magnetic spacetime. This gives rise to two cases during the helicity transfer: (1) In each hemisphere, the helicity equals to that of another one but with opposite polarity. That is, any helicity outcome in one hemisphere should be balanced by the opposite-signed change in another one. (2) The external environment (e.g., the accretion disk or torus) provides the required helicity. The former corresponds to the case of two asymmetric jets/lobes in AGNs relative to the core, while the latter is related to symmetric ones. In either case, the amount of helicity transfer can be evaluated from the helicity change of jets/lobes by observations in an evolution period. Luckily, VLA radio, HST optical, and Chandra X-ray observations exhibit obvious successive occurrences of hotspots (or knots) in jets/lobes of AGNs such as, e.g., Cen A, M87, 3C273, etc. [108-110]. The helical appearances of these asymmetric or symmetric jets/lobes both directly present evidences of helicity transfer from the core, and indirectly verify the dynamo process and energy output from supermassive BH candidates. The most encouragingly, the formation of relativistic jets is exactly manifested by GRMHD simulations in Kerr spacetime, where the helical jets are driven by the FD dynamo action in the ergosphere [111-114].
It is interesting to note that the background of the asymmetric or symmetric jets/lobes may be contributed by a non-magnetized mechanism in the GM field, Pen-rose process. It takes advantage of the fact that time-like vectors outside become space-like ones inside the ergosphere, but yet matter can still emerge from within. When a particle A enters the ergosphere from infinity, it splits into two particles B and C inside the ergosphere. Particle B will plunge into the event horizon, carrying negative energy into the BH and the BH loses rotational energy and slows down. Particle C emerges from the ergosphere and goes back to infinity. It is possible for particle C to carry more energy than particle A. Considering particles of an accretion disk which fall into the ergosphere and scatter off particles in bound equatorially and non-equatorially confined orbits, simulations of Compton scattering and e⁻e⁺ pair production processes in the ergosphere of a supermassive BH suggest that the Penrose mechanism allows the rotational energy of a Kerr BH to be extracted by scattered particles escaping from the ergosphere to infinity, and, can produce the observed high-energy radiation (up to GeV) emitted by quasars and other AGNs, without the necessity of the external electromagnetic field of the accretion disk [113]. Calculations show that, inside the ergosphere, the LT effect results in a GM force, and, the GM vector field lines are frame-dragged into the positive azimuthal direction of the BH rotation. The force is exerted on the scattered escaping particles to produce symmetrical and asymmetrical (or one-sided) particle emissions in the polar direction; consistent with the astrophysical jets observed in radio strong AGNs.

However, since it requires a very large amount of internal energy to explode particle A, the Penrose process seems not astrophysically realistic, although it is a possible energy source theoretically. If the energy output of AGNs has a connection with the process, the observed flare-type jets/lobes should consist of two components: the exhibition of the electromagnetic energy via helicity dissipation in the GM field related to magnetic storm activities, and, the direct rotational energy of BH transported via Penrose process irrelevant of the magnetic field. In this case, not only the portion of particles to convey the energy of each part in the total energy release, but also the coupling of them with each other in the outword transport to infinity, are vital in dealing with the actual state of giant flares, as well as relevant astronomical phenomena. A qualitative arguments demonstrate that, for the simplest thin-disk structure, (1) the energy density of magnetic fields in matter accreting onto a BH inside the marginally stable orbit is found to be automatically comparable to the rest-mass energy density of the accretion flow, (2) magnetic effects must be dynamically significant but cannot be so strong as to dominate, and, (3) the outward energy transport in Alfven waves may alter the effective efficiency of energy liberation, and, vertical magnetic stresses may contribute to ”coronal” activity, e.g., flares [114].

By contrast, even if there is no initial magnetic field in the GM field, hot, dense e⁻e⁺ pairs can be produced in the region of null surface because of the GM force. They may be saturated with gamma-rays when recombination radiation occurs in the degenerate region, and, form a high-energy emission in directions the GM force directs. If a very large number of e⁻e⁺ pairs is created and reach thermodynamic equilibrium with a photon gas in the ensuing pair-electromagnetic pulse, which may trigger the seed magnetic field and then the field is amplified by FD dynamo actions, it will interact with some of the baryonic matter of the un-collapsed material in a very short time to produce giant flares, accompanying particle accelerations in the opposite directions controlled by the developing BH magnetosphere [115-117]. Thus, a large fraction of the extractable energy of the central compact object may be carried away by radiation from the short-life dyadosphere (no more than a year), a region which may be a mix of both the ergosphere and the quasi-tokamak cavity. Although more detailed calculations should be carried out, this qualitative picture provides a more reliable mechanism than other ones to exhibit the existence of magnetic field and the development of giant flares by dynamo process.
**Future Work on Giant Flare Physics**

The fact that up to 50% of the mass energy of a BH can be stored in its electromagnetic field stresses that BHs should be the most efficient engine to provide energy. The mean-field GRMHD dynamo action, influenced and even controlled by FD effect of a BH in the presence of an accretion disk in 4D curved spacetime, supports the extraction of rotating energy from the BH by magnetic fields and the energy transport and release to infinity. From a fundamental point of view, the process of magnetic flares due to magnetic reconnections of shear fields occurring in the dyadosphere, which may be closely coupled with the quasi-tokamak cavity, should represent the physics of efficient energy conversion, along with nuclear fusion happening in the tokamak cavity with a helical magnetic structure. If it is true, origins of atomic emission lines of Fe, Ni, etc., and the existence of neutrino-rich torus can be reasonably and quantitatively modelled to fit to observations [118,119].

Thus, the evolution of magnetic field driven by the GRMHD dynamo action in the BH magnetosphere, containing the quasi-tokamak cavity, the thin spherical torus, the active blobs in the corona, etc., has intrinsic connections with the macroscopic eruptive flare events, e.g., predicted fireballs, detected successive knots in jets/lobes, in the observational band of electromagnetic radiations. Nevertheless, there are a few crucial issues to be investigated in more details before realistic progress is achieved to be applicable to the elucidation of the nature of the observed giant flares.

One topic is the estimation of the field intensity, an important parameter to understand the central engine of AGNs, to predict the contribution of the electromagnetic field to the energy storage in the GM field, to evaluate the efficiency of the dynamo action in the transfer of magnetic helicity and the power outputs from the BH magnetosphere [118-125]. Early models conclude that in the vicinity of the marginally stable orbit of a disk surrounding a BH, the field intensity can be as high as $10^{11} \text{ Gs}$ [126,127]. However, for a BH of mass $M \approx 10^8$ solar mass rotating at 1 rad/ks, an output of roughly $10^{44} \text{ erg/s}$, comparable to the observation of radiation from radio sources, needs only an average magnetic field of $10^4 \text{ Gauss}$ in a force-free magnetosphere [51]. An interesting calculation compromises that, in the Poynting flow conveying an energy of $1.6 \times 10^{53} \left( M/\text{solar mass} \right) \text{ erg}$, equivalent to the energy of gamma-ray bursts, GRB971214, the magnetic field can change from $10^{15} \text{ Gauss}$ at $r=10^5 \text{ cm}$ down to $10^9 \text{ Gauss}$ at $r=10^{16} \text{ cm}$ for a BH rotating rapidly [47].

Recent observations of wide X-ray emission lines of heavy ionized elements in the spectra of AGNs by, e.g., ASCA, RXTE, Chandra and XMM-Newton, etc., also offer another method to predict the magnetic field intensity. Suppose (1) the iron $K\alpha$ line emission arises in the inner parts of the accretion discs where the FD effect cannot be neglected but dominate, and, (2) the circular motion of a hotspot near a BH is a clear model of the emission variability of AGNs, the magnetic fields in the photon-emission region is found below $10^{10-11} \text{ Gs}$ for Seyfert AGN MCG-6-30-15 by the fits of predicted profiles with Zeeman-effects to observed data [128-130]. If using the perspective facilities of measurement devices, e.g., Constellation-X mission, with higher resolution of instruments, it is expected that data-fits can reveal definite intensities of magnetic fields.

However, the fatal deficiency in the modelling or the data-fitting to estimate the magnetic field is the consideration of the equatorial disk structure of the accreting matter close to the central BH. Using a tilt-disk model, a rough estimation of the field is at least an order less than that of the equatorial disk model [131]. We hope this topic attracts more concerns in future, especially with the advance of torus magnetosphere in GM field, as discussed below.

The second topic is the description of BH’s magnetosphere and the formulation of torus dynamo action. Based on the early model of BH’s magnetosphere, in which BH’s magnetic fields are embedded in the hole’s horizon and a surrounding
accretion disk and transmit rotational and orbital energy along the hole’s axis to distant in the disk corona [44,74], many modern torus models are developed to describe observed high-energy events. For example, the coalescence of a neutral star and a Kerr BH in a binary system is believed to form a torus magnetosphere around the BH in the inner part of the surrounding disk, in which magnetic field lines penetrating the BH are either closed or open, depending on whether they connect to the torus and/or the disk, or to infinity, respectively [102]. The torus is a transition region with its inner part coupling to the BH and the outer one to the outside. The torus magnetosphere rotates with the Keplerian angular velocity of the torus. Corotation extends from the inner to the outer light surfaces. The inner light surface touches the last closed line to the torus, which encloses a trapped region. Between the inner light surface and the horizon, the magnetic field vanishes in the equatorial annulus of current sheet. By symmetry to the equatorial plane, closed field lines, the torus, the trapped region, the sheet, and the BH’s horizon form two closed loops of poloidal current induced by BZ process. These two loops trace out two cavities, respectively, from the horizon to the torus that is essentially free of the poloidal current. The behaviors of the cavity like a waveguide for plasma waves, in which the fast magnetosonic and the Alfvén waves propagate. If the BH rotates faster than the torus, a Poynting flux emerges from the horizon and extracts energy from the BH. The magnetic field near the horizon is similar to a Wald field, but with a toroidal field component resulted from poloidal currents.

To characterize the relativistic jet-forming activities with radio/infrared flares associated with a hard X-ray dip which terminates in a soft X-ray spike in the microquasar GRS 1915+105, a complete BH-torus model is developed from the suspended accretion model for long gamma-ray bursts and the van Putten-Levinson stability criterion for magnetized disks [132-135]. The model also posits a rotating BH and a thin accretion disk extending to somewhere near the last stable orbit. A seed magnetic field in the disk develops a large-scale field by a dynamo process. This leads to the creation of a net poloidal magnetic flux, associated with an approximately uniform magnetization of matter in the inner disk. A torus magnetosphere around the BH with net poloidal flux is thus produced and causes a connection between the inner disk and the BH. Incidence of most of the BH power output on the inner face of the torus creates a magnetic wall around the BH, blocking the passage of accreting material through the inner disk. This is named as the suspended accretion state. Matter then builds up in the torus at a rate determined by the rate of accretion in the extended disk. When the ratio of magnetic energy to kinetic energy reaches the van Putten-Levinson instability criterion, the field disconnects and the stored magnetic energy is rapidly dissipated as a magnetic bomb. The inner disk then refills on a viscous timescale, and the system return to the initial state, allowing the cycle to begin anew.

The fact of the successful match between the torus magnetosphere and observations leads to a reconsideration of the disk-related BH dynamo. Not mentioning that the accretion rate of matter is no longer constant, and, the fluid fields no longer keeps an invariable topology in the dynamo process in the new scenario, the new magnetic topology of the torus magnetosphere in the suspended accretion state implies that the trapped region, enclosed by current-free field lines of the cavity, possesses the magnetic structure of the mentioned fusion Tokamak. More important, the torus also provides a site in the equatorial plane of the BH where the X-type neutral point is formed. As the case in the flat space, the resultant X-type neutral line around the BH can tend to be so locally unstable that any original perturbation can be increased by magnetic force, and then, the ohmic heating, particle acceleration, electromagnetic radiation, etc., may be well dissipated by the X-line collapse in magnetic reconnections. With these new constraints, dynamo action to produce an initial magnetic field via FD effect, or, amplify seed fields brought by a disk should present another clear picture which is closer to reality.

Last but not least, it is crucial to estimate the energy release for giant flares by expressing the helicity evolution in the GM field in terms of the magnetic field amplified by the BH dynamo action from a weak but nonzero seed field. Up to now, it is
well known that in flat space the operation of the mean-field dynamo automatically leads to the growth of linkages between the poloidal and toroidal mean fields. If the domain of integration is closed, that is, periodic, infinite or with a boundary where the normal component of the field vanishes, the gauge-invariant magnetic helicity will represent the linkages, and, its evolution rate will present the lower limit of the energy loss in a magnetic structure available for the dissipation. Otherwise, the relative helicity, a new topologically well-defined and gauge-invariant parameter, should be used [94,95].

In the axisymmetric GM field, however, the operation of the dynamo leads to two different growths of linkages: one is between the poloidal and toroidal mean fields as that in flat space, and the other is between the poloidal magnetic field and the sheared poloidal field, the additional source of the curl of the electric field and an extra term introduced by the FD effect specifically. In this case, helicity evolution is contributed jointly by these linkages associated with the mean field. Different from the case in a closed region in flat space where current helicity tends to decrease the initial magnetic helicity and produce the energy loss, the new linkage increases the initial magnetic helicity in the FD region, and thus it is possible, relative to infinity, that a BH, the core of AGNs, can provide stronger initial energy to be released in infinity. In this topic, the key issue seems to be the split of the helicity conservation equation into evolution equations of the sub-helicities associated with the mean field and the fluctuating field, and then to estimate the energy release during the period of total helicity evolution [134].

In fact, there exist three caveats in the present topic. Firstly, the region concerned is absolutely not closed. This means that at least part of the whole surface of the region is open where magnetic field has a normal component and a flux of helicity flows through the surface. It is the evolution of relative helicity, but not the total helicity that should be considered [136]. Secondly, the additional FD helicity in the open region and its connection with the energy loss may characterize giant flares in GM field. It is possible that the negligence of this term reduces the prediction of total energy dissipation. Thirdly, the widely-accepted omission of the cross-helicity contributed by the product of the plasma velocity and the vector potential of the open field in flat-space study should be avoided for the prediction of the saturation value of the mean-field field with the GRMHD dynamo equation. It is shown that, in flat space, besides that of the potential field, the cross-helicity of the open field contributes to the evolution of the relative helicity in an open volume with two components: one is transverse representing the interaction of the transverse component of the open magnetic vector potential with that of the fluid field in the plane of open surface, and the other is longitudinal representing the interaction of the longitudinal component of the open magnetic vector potential with that of the fluid field in the direction vertical to the open surface [98].

More and more evidences exhibit that giant flares connect with magnetic fields in AGNs, even in Galactic sources. We are looking forward to future observations of space-borne high-energy missions. Hopefully, they should lay to rest the disk/non-disk magnetosphere debate and strong magnetic field controversy in AGNs in the presence of wider spectral band and better resolution than ever to probe into the core of galactic/extragalactic systems. If successful, the mechanism of giant flare related to the GM dynamo will eventually be elucidated via the fit of both analytical and numerical calculations to effective observations [137].
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