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Formation control of multi-robots using leader-follower approach at dynamic level

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ABSTRACT

In this paper, formation control of multiple nonholonomic mobile robots is studied using the leader-follower approach at dynamic level. The controllers are proposed firstly at kinematic level, then the results are extended to design dynamic controller using uncertain dynamic model of the system. Simulation results prove the effectiveness of the proposed algorithm.

KEYWORDS

Formation control;
Nonholonomic constraints;
Leader-follower;
Dynamic.

INTRODUCTION

In our daily life, there are many cases in which the single robot couldn't afford, for example, search, roundup, rescue, etc. Formation control of multi-mobile robots becomes an effective solution[1-3]. It could significantly improve the efficiency of completing a task of the robot system; it is also an important issue in the field of mobile robotics research which has been applied more and more widely. Formation control of multiple robots is referred to control multiple robots to form up and move in specified geometrical shapes, and at the same time adapt to environmental constraints. It allows robots having a more accurate and efficient access to the environment, improving work efficiency, strengthening their capability of resisting outside invasion, benefiting the cooperation among robots, and so on.

Various approaches have been presented in the literature for the formation control of multiple robots[4-8]. Behaviour-based formation control problem is studied in[4]. By behaviour-based approach, each robot is considered to have several behaviours, such as goal seeking, formation keeping and obstacle avoidance. The final action of each robot is generated by weighting certain importance of each behaviour. Though this method has an advantage for its parallelism distributivity, it's hard to guarantee a stable formation control.

Artificial potential field method is proposed in[5]. The robot is considered to move in a virtual force field, in which the target attracts it while the obstacle in the environment repels the robot, and as a result of the composite force, the robot moves in the direction of the minimum potential energy. This method is easy for real-time control, but there are local extremum and the potential field function is somewhat difficult to design.

In the leader-follower approach[6,7], one of the robots is designated as the leader, with the following being followers. The follower robots need to maintain a desired separation and bearing with respect to the leader. When the motion of the leader is known, the desired positions of the followers can be achieved by local control law on each follower. What's more, inspired by the trajectory tracking of a single nonholonomic robot[8], we could treat the formation control as an extension of trajectory tracking problem.

As can be seen from the literature, most of the controllers for the formation control problem of multi-robots were proposed at kinematic level. The problems are manageable with a constrained robot if the exact robot dynamic model is available for controller design. In real applications, however, perfect cancellation of the robot dynamics is almost impossible. In this paper, formational control problem is studied at dynamic level. Firstly, the controllers are designed using kinematic model of the system, then the results are extended to dynamic controllers using uncertain system's dynamic model.

SYSTEM MODELING

In this section, the kinematic model is discussed for the leader-follower based formation control of multiple mobile robots. For simplicity, a team composed of two robots is taken into consideration firstly.

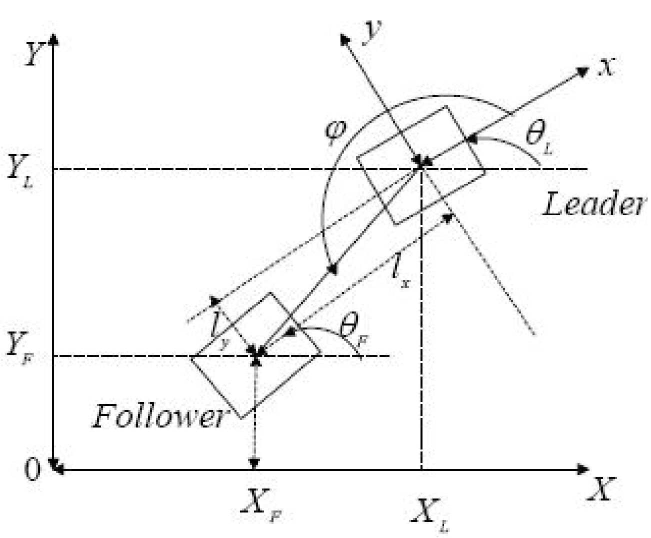

Figure 1 : A leader-follower formation of two robots

Let there be a leader L for follower F. Then for the follower, the desired position is defined as follows:

$$L_x = -(X_L - X_F)\cos(\theta_L) - (Y_L - Y_F)\sin(\theta_l)$$
$$L_y = (X_L - X_F)\sin(\theta_L) - (Y_L - Y_F)\cos(\theta_l) \quad (1)$$

in which L_x and L_y are the follower's relative positions along x and y directions respectively; (X_L, Y_L) and (X_F, Y_F) are the global positions of the leader and the follower separately; θ_L and θ_F are their orientation angles; l and φ are the follower's relative separation and bearing with respect to the leader. And our purpose is to control $l \to l_d, \varphi \to \varphi_d$, in which subscript 'd' denotes desired.

Since we have

$$L_{xd} = L_d \cos(\phi_d)$$
$$L_{yd} = L_d \sin(\phi_d) \quad (2)$$

Then we could get

$$\overset{\circ}{L}_{xd} = \dot{L}_d \cos(\varphi_d) - L_d \overset{\circ}{\phi}_d \sin(\phi_d)$$
$$\overset{\circ}{L}_{yd} = \dot{L}_d \sin(\varphi_d) + L_d \overset{\circ}{\phi}_d \cos(\phi_d) \quad (3)$$

Define

$$e_F = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} L_{xd} - L_x \\ L_{yd} - L_y \\ \theta_F - \theta_L \end{bmatrix} \quad (4)$$

then from (1), we can get

$$\overset{\circ}{L}_x = L_y w_L + v_F \cos(\theta_e) - v_L$$
$$\overset{\circ}{L}_y = -L_x w_L + v_F \sin(\theta_e) \quad (5)$$

If the desired distance L_d between the leader and follower keeps constant, then we can get:

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} w_L y_e - v_F \cos(\theta_e) + f_1 \\ -w_L x_e - v_F \sin(\theta_e) + f_2 \\ w_L - w_F \end{bmatrix} \quad (6)$$

In which,

$$f_1 = -L_d \dot{\varphi}_d \sin(\varphi_d) - w_L L_d \sin(\varphi_d) + v_L$$
$$f_2 = L_d \dot{\varphi}_d \cos(\varphi_d) + w_L L_d \cos(\varphi_d) \quad (7)$$

NONLINEAR KINEMATIC CONTROLLER

In the leader-follower formation control, if the leader's position and the follower's relative distance and angle with respect to the leader are known, we could calculate the follower's position through these information.

Control law for the leader robot

In this subsection, the control law for the leader robot is studied. As we know, since the motion of the leader robot is not affected by its followers, the kinematics of the leader robot can be analysed independently. Moreover, the leader's trajectory tracking state could have a great effect on the followers, so we should pay attention to the design of the leader's control law. In this paper, the control law[9] adopted for the leader robot is:

$$\begin{bmatrix} V_L \\ W_L \end{bmatrix} = \begin{bmatrix} v_R \cos(\theta_{Le}) + k_{i1} x_{Le} \\ w_R + k_{i2} v_R y_{Le} + k_{i3} v_R \sin(\theta_{Le}) \end{bmatrix} \quad (8)$$

In which, the position error of the leader is defined as follows:

$$e_L = \begin{bmatrix} x_{Le} \\ y_{Le} \\ \theta_{Le} \end{bmatrix} = \begin{bmatrix} \cos(\theta_L) & \sin(\theta_L) & 0 \\ -\sin(\theta_L) & \cos(\theta_L) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x_R - x_L \\ y_R - y_L \\ \theta_R - \theta_L \end{bmatrix} \quad (9)$$

and the derivatives of (9) is:

$$\dot{e}_L = \begin{bmatrix} \dot{x}_{Le} \\ \dot{y}_{Le} \\ \dot{\theta}_{Le} \end{bmatrix} = \begin{bmatrix} w_L y_{Le} - v_L + v_R \cos(\theta_{Le}) \\ -w_L x_{Le} + v_R \sin(\theta_{Le}) \\ w_R - w_L \end{bmatrix} \quad (10)$$

Proof: Consider the following Lyapunov function as

$$V = \frac{1}{2}(x_{Le}^2 + y_{Le}^2) + \frac{1 - \cos(\theta_{Le})}{k_{i2}} \quad (11)$$

It's clear that $V > 0$ and $V = 0$ only when $e_L = 0$.
Differentiating (11), we can get

$$\begin{aligned} \dot{V} &= x_{Le}\,\dot{x}_{Le} + y_{Le}\,\dot{y}_{Le} + \frac{1}{k_{i2}} \sin(\theta_{Le})\,\dot{\theta}_{Le} \\ &= x_{Le}(w_L y_{Le} - v_L + v_R \cos(\theta_{Le})) + y_{Le}(-w_L x_{Le} + v_R \sin(\theta_{Le})) \\ &\quad + \frac{1}{k_{i2}} \sin(\theta_{Le})(w_R - w_L) \\ &= -x_{Le}(v_R \cos(\theta_{Le}) + k_{i1} x_{Le}) + v_R x_{Le} \cos(\theta_{Le}) + v_R y_{Le} \sin(\theta_{Le}) \\ &\quad + \frac{1}{k_{i2}} \sin(\theta_{Le}) w_R - \frac{1}{k_{i2}} \sin(\theta_{Le})(w_R + k_{i2} v_R y_{Le} + k_{i3} v_R \sin(\theta_{Le})) \\ &= -k_{i1} x_e^2 - \frac{k_{i3}}{k_{i2}} v_R \sin^2(\theta_{Le}) \le 0 \end{aligned}$$

Clearly, the error system is asymptotically stable, and $e_L \to 0$ as $t \to \infty$.

Control law for the follower robots

According to the paper[10], the design of the control law for the follower can be separated for two situations:

1) when $w_L \neq 0$, the control for the follower can be designed as:

$$\begin{bmatrix} v_F \\ w_F \end{bmatrix} = \begin{bmatrix} k_2 x_e + y_e [k_1^2 w_L^{-1} - k_1 (w_L^{-1})' - k_1 k_2 w_L^{-1}] \\ + f_1 - (w_L^{-1})' f_2 - k_2 w_L^{-1} f_2 - w_L^{-1} \dot{f}_2 \\ w_L - k_3 \theta_e \end{bmatrix} \quad (12)$$

This controller can guarantee y_e and θ_e asymptotically converge to desired states, and x_e is proved to be bounded.

2) when $W_L = 0$, just as explained in paper[10], the use of sign-function,

namely $sign(x) = \begin{cases} -1 & x \ge 0 \\ 1 & x = 0 \end{cases}$ when designing the control law, may cause some chattering, In order to solve this problem, without using sign function, a new controller is proposed to guarantee the stability of the system[11].

Consider the following Lyapunov function

$$V = \ln(\cosh(x_e)) + \ln(\cosh(y_e)) + \frac{1}{2}\theta_e^2 \quad (13)$$

Differentiating (13), we have

$$\begin{aligned} \dot{V} &= \tanh(x_e)\,\dot{x}_e + \tanh(y_e)\,\dot{y}_e + \vartheta_e\,\dot{\vartheta}_e \\ &= -v_F[\tanh(x_e)\cos(\theta_e) + \tanh(y_e)\sin(\theta_e)] + w_F\theta_e + f_{11}\tanh(x_e) + f_{22}\tanh(y_e) \end{aligned}$$

Using back-stepping method, if

$$\begin{bmatrix} v_F \\ w_F \end{bmatrix} = \begin{bmatrix} c_1[\tanh(x_e)\cos(\theta_e) - \tanh(y_e)\sin(\theta_e)] + \zeta \\ -c_2\theta_e - c_3.\theta_e.\tanh^2(y_e) \end{bmatrix} \quad (14)$$

in which

$$\zeta = \frac{f_1 \tanh(x_e) + f_2 \tanh(y_e)}{\delta + \tanh(x_e)\cos(\theta_e) + \tanh(y_e)\sin(\theta_e)} \quad (15)$$

$$\delta = \begin{cases} 0 & if \quad \tanh(x_e)\cos(\theta_e) + \tanh(y_e)\sin(\theta_e) \neq 0 \\ \delta_0 & if \quad \tanh(x_e)\cos(\theta_e) + \tanh(y_e)\sin(\theta_e) = 0 \end{cases} \quad (16)$$

$$f_{11} = -L_d \dot{\varphi}_d \sin(\varphi_d) + v_L \quad (17)$$

$$f_{22} = L_d \dot{\varphi}_d \cos(\varphi_d) \quad (18)$$

Then it can be obtained :

$$\begin{aligned} \dot{V} &= -c_1 \tanh^2(x_e)\cos^2(\theta_e) - c_2\theta_e^2 - \tanh^2(y_e)[c_3\theta_e^2 - c_1 \sin^2(\theta_e)] \\ &\quad - \zeta[\tanh(x_e)\cos(\theta_e) + \tanh(y_e)\sin(\theta_e)] + [f_{11}\tanh(x_e) + f_{22}\tanh(y_e)] \end{aligned} \quad (19)$$

Substitute the above control law (14) into (19), we could get $\dot{V} \le 0$.

In assigning the control laws in next section, these variables V_L, W_L, V_F, W_F are the desired velocities to make the kinematic model stable, which will be presented as v_d and w_d respectively.

CONTROLLER DESIGN UNDER DYNAMIC MODEL

The dynamic equations of the mobile robot can be described as follows:

$$M(q)\ddot{q}+C(q)\dot{q}+G(q)=B(q)\tau+J^T(q)\lambda \quad (20)$$

In which, $\tau = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$.

Assuming $\tau_l = \frac{1}{R}(\tau_1+\tau_2)$, $\tau_a = \frac{L}{R}(\tau_1-\tau_2)$, where R is the radius of wheels and L is the distance of rear wheels.In this section, the following control laws[12] are used to prepare tracking of the desired velocities proposed in the third part:

$$\begin{bmatrix}\tau_l\\ \tau_a\end{bmatrix} = \begin{bmatrix} m\dot{v}_d + k_l(v_d - v)\\ I\dot{w}_d + k_a(w_d - w)\end{bmatrix} \quad (21)$$

Where m and I are the mass and inertia of robot respectively.

Here, m and I are assumed to be known. In fact, not only the measurements of these parameters have uncertainties, but also they change in a large area in most applications. So, the control laws should be written in this form:

$$\begin{bmatrix}\tau_l\\ \tau_a\end{bmatrix} = \begin{bmatrix} \hat{m}\dot{v}_d + k_l(v_d - v)\\ \hat{I}\dot{w}_d + k_a(w_d - w)\end{bmatrix} \quad (22)$$

Define $\frac{\hat{m}}{m}=\theta_1$, $\frac{k_l}{m}=\theta_2$, we could have

$$\dot{v}=\theta_1\dot{v}_d+\theta_2(v_d-v)$$

The reference model for velocity error is

$$\dot{v}_e+Tv_e=0$$

Assuming $v_e=v_d-v_m$, $e=v-v_m$, in which v_m is the velocity of the reference model.

Define $V=e^2+\frac{1}{\gamma_1}(\theta_1-1)^2+\frac{1}{\gamma_2}(\theta_2-1)^2$

Then, it's derivate can be obtained as

$$\dot{V}=-Te^2+(\theta_1-1)[e\dot{v}_d+\frac{1}{\gamma_1}\frac{d\theta_1}{dt}]+$$
$$(\theta_2-T)[ev_d-ev+\frac{1}{\gamma_2}\frac{d\theta_2}{dt}]$$

Then, to make $\dot{V}$ negative definite, we can define

$$\frac{d\theta_1}{dt}=-\gamma_1 e\dot{v}_d$$
$$\frac{d\theta_2}{dt}=-\gamma_2 e(v_d-v)` \quad (23)$$

Similarly, we can get

$$\frac{d\theta_3}{dt}=-\gamma_3 e\dot{w}_d$$
$$\frac{d\theta_4}{dt}=-\gamma_4 e(w_d-w) \quad (24)$$

SIMULATION

In this section, to verify the proposed plan, we will carry out two experiments. First, we simulate a team of 3 mobile robots travelling in a line, and then we will simulate a team of 3 mobile robots moving in a triangular formation. Under the control algorithm based on $l-\varphi$, the motion trajectories are as follows:

Figure 2 shows the case when the leader goes along a straight line at a const linear velocity 5px/s and an angular velocity 0px/s, the initial position of the leader is (400, 300); the other two robots keep a const relative distance and angle from the leader respectively, as

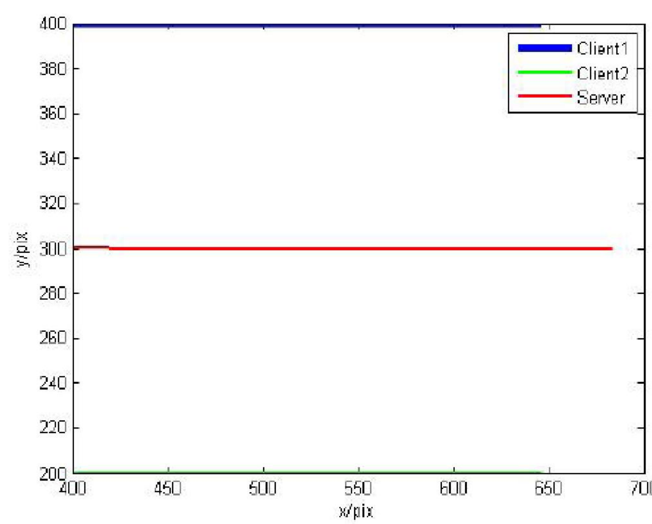

Figure 2 : 3 robots moving in a line

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shown in (25):

$$l_1 = 100 \quad , \quad \varphi_{01} = \pi / 2 ;$$
$$l_2 = 100 \quad , \quad \varphi_{02} = 3\pi / 2 ; \qquad \textbf{(25)}$$

Figure 3 shows the case when the leader goes along a straight line at a const linear velocity 5px/s and an angular velocity 0px/s, the initial position of the leader is (400, 300); the other two robots keep a const relative distance and angle from the leader respectively, as shown in (26):

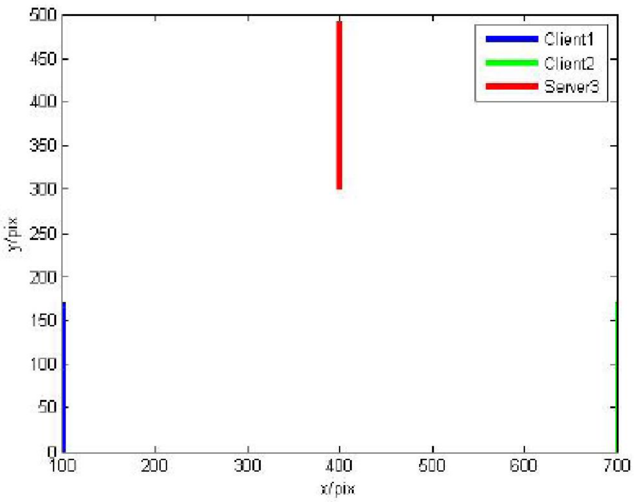

Figure 3 : Triangular formation of 3 robots

$$l_1 = 300\sqrt{2}, \quad \varphi_{01} = 3\pi / 4 ;$$
$$l_2 = 300\sqrt{2}, \quad \varphi_{02} = -3\pi / 4 ; \qquad \textbf{(26)}$$

Initially, the two followers are at (100, 0) and (700, 0). In our experiment, the linear and angular velocity are refreshed 0.5ms.

As we can see from the above simulation results, the followers could keep at a desired separation and bearing from the leader; the proposed plan could achieve the desired formation, and the whole system is stable.

CONCLUSION

From the above, we can see that the leader-follower based formation control of multiple mobile robots at dynamic model is feasible. All system states are proven to be stable. The simulation results show the effectiveness of the proposed control strategy.

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