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FILTERS IN TERNARY SEMIGROUPS

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ABSTRACT

In this article the ternaryfilters in a ternary semigroup are considered. The characterization of a ternary filter of T interm of the primeideals and some relations between the semilattice congruence N and the set of primeideals of the ternarysemigroup T are given.

Key words: Filter, Principal Filter.

INTRODUCTION

Lee and Lee[1] introduced the notion of a left (right) filters in a po-semigroup and gave a characterization of the left (right) filters of T interms of the right (left) prime ideals. Kwon[2] and Kostaq[3] characterized filters in ordered semigroups. Rao et al.[4] defined some relations between the filters of partially ordered Γ-semigroups S. In this paper, the characterization of a ternary filter of T interm of the primeideals and some relations between the semilattice congruence N and the set of primeideals of the ternary semigroup T are given.

Definition 2.1: A ternary semigroup F of a ternary semigroup T is known as a filter of T if $a,b,c \in T;\ abc \in F \Rightarrow a,b,c \in F$.

Example 2.2: Let $T = \{x, y, z, w\}$ with the multiplication defined by

$$abc = \begin{cases} x, & \text{if} \quad a = b = c = \quad x \\ y, & \text{if} \quad a = b = c = \quad y \\ z, & \text{if} \quad a = b = c = \quad z \\ w & \text{if} \quad \text{otherwise} \end{cases}$$

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Then T is a ternary semigroup and $\{x, y, z, w\}$, $\{y\}, \{z\}, \{w\}$ are all filters of T.

Theorem 2.3: Let F_1, F_2 be the two filters of a ternary semigroup T. Then the intersection $F_1 \cap F_2$, if it is nonempty is a filter of T.

Proof: Let F_1, F_2 be the two filters of T. Let $a, b, c \in T$; $abc \in F_1 \cap F_2$.

$abc \in F_1 \cap F_2 \Rightarrow abc \in F_1$ and $a, b, c \in F_2$. $a, b, c \in T$; $abc \in F_1$; F_1 is a filter of T $\Rightarrow$ $a, b, c \in F_1$. $a, b, c \in T$; $a, b, c \in F_2$; F_2 is a filter of T $\Rightarrow a, b, c \in F_2$. $a, b, c \in F_1$; $a, b, c \in F_2 \Rightarrow a, b, c \in F_1 \cap F_2$. $a, b, c \in T$; $abc \in F_1 \cap F_2 \Rightarrow a, b, c \in F_1 \cap F_2$. Therefore $F_1 \cap F_2$ is a filter of T.

Theorem 2.4: The nonempty intersection of a family of ternary filters of a ternary semigroup T is also a ternary filter.

Proof: Let $F = \bigcap_{\alpha \in \Delta} F_\alpha$. Let $a, b, c \in T$; $abc \in F$. Now $abc \in F \Rightarrow abc \in \bigcap_{\alpha \in \Delta} F_\alpha \Rightarrow$ $abc \in F_\alpha$ for each $\alpha \in \Delta$. $abc \in F_\alpha$; F_α is a ternary filter of T $\Rightarrow a, b, c \in F_\alpha$ for each $\alpha \in \Delta$ $\Rightarrow a, b, c \in \bigcap_{\alpha \in \Delta} F_\alpha \Rightarrow a, b, c \in F$. Therefore F is a ternary filter of T.

Note 2.5: In general, the union of two ternary filters is not a ternary filter.

Example 2.6: As in the example 2.2, T is a ternary semigroups and {y}, {z}, {w} are ternary filters, but $\{y\} \cup \{z\} \cup \{w\}$ is not a ternary filter of T, because $yzw = x$ is not in $\{y\} \cup \{z\} \cup \{w\}$.

In this paper, the characterization of a ternary filter of T interm of the primeideals is given.

Theorem 2.7: Let T be a ternary semigroup and F be a nonempty subset of T. The succeeding are equivalent:

(i) F is a filter of *T*.

(ii) $T \setminus F = \phi$ or $T \setminus F$ is a primeideal.

Proof: (i) $\Rightarrow$ (ii) Let $T \setminus F = \phi$. Then $T \setminus F$ is a primeideal of T. Infact: Since $T \setminus F \neq \phi$, we take $a, b \in T$; $c \in T \setminus F$. If $abc \in F$; F is a filter of T, we have $a, b, c \in F$. It is impossible. Thus we have $aaa \in T \setminus F$, $bbb \in T \setminus F$. i.e. $TT(T \setminus F) \subseteq T \setminus F$; $(T \setminus F)TT \subseteq T \setminus F$ and $T(T \setminus F)T \subseteq T \setminus F$. Let $a, b, c \in T$ and $abc \in T \setminus F$. If $a \in F$; $b \in F$ and $c \in F$ then since F is a sub ternary semigroup of T, $abc \in F$. It is impossible. Hence we have $a \in T \setminus F$ or $b \in T \setminus F$ or $c \in T \setminus F$.

(ii) $\Rightarrow$ (i) Let $T \setminus F = \phi$. Since $T = F$; F is a filter of T. Suppose that $T \setminus F$ is a primeideal of T. Then F is a subsemigroup of T. Infact: $a, b, c \in F$. If $abc \in T \setminus F$, since $T \setminus F$ is prime, $a \in T \setminus F$ or $b \in T \setminus F$ or $ca \in T \setminus F$. It is impossible. Thus we have $abc \in F$. Let $a, b, c \in T;$ and $abc \in F$. If $a \in T \setminus F$ then, since $T \setminus F$ is an ideal of T; $abc \in T \setminus F$. It is impossible. If $a \in T \setminus F$ then, since $T \setminus F$ is an ideal of T; $abc \in T \setminus F$. It is impossible. If $b \in T \setminus F$ then, since $T \setminus F$ is an ideal of T; $abc \in T \setminus F$. It is impossible. If $c \in T \setminus F$, similarly $abc \in T \setminus F$. It is impossible. Thus, we have $a \in F;$ $b \in F$ and $c \in F$. $\therefore$ F is a filter of T.

Now we introduce the notion of a ternary filter of T generated by *A*.

Definition 2.8: Let T be a ternary semigroup and A be a nonempty subset of *T*. The smallest filter of $T \subseteq A$ is said to be a ternary filter of T generated by A and is symbolized by $F_t(A)$.

Theorem 2.9: The ternary filter of a ternary semigroup T generated by a nonempty subset A of T is the intersection of all ternary filters of $T \subseteq A$.

Proof: Let Δ be the set of all ternary filter of $T \subseteq A$. Since T itself is a ternary filter of $T \subseteq A$, $T \in \Delta$. So $\Delta \neq \phi$. Let $F^* = \bigcap_{\alpha \in \Delta} F_\alpha$. Since $A \subseteq F \quad \forall\ F \in \Delta$, $A \subseteq F^*$. So $F^* \neq \phi$. By theorem 2.4, F^* is a ternary filter of T. Let K be a ternary filter of $T \subseteq A$. Clearly $A \subseteq K$ and K is a ternary filter of T. Therefore F^* is the smallest ternary filter of $T \subseteq A$ and F^* is the ternary filter of T generated by *A*.

We now introduce the notion of a principal ternary filter of a ternary semigroup.

Definition 2.10: A ternary filter F of a ternary semigroup T is known as a principal filter provided F is a ternary filter generated by $\{a\}$ for some $a \in T$. It is symbolized by $F_t(a)$.

Example 2.11: As example 2.2; T is a ternary semigroup and $F_t(x) = \{x\}$; $F_t(y) = \{y\}$; $F_t(z) = \{z\}$ are all the principal ternary filters of the ternary semigroup T.

Corollary 2.12: Let T be a ternary semigroup and $\alpha \in T$. Then $F_t\,(a)$ is the least ternary of T $\subseteq \{a\}$.

Proof: For every $\alpha \in T$, the intersection of all ternary filter containing $\{a\}$ is again a ternary filter and thus the least ternary filter containing $\{a\}$.

We now introduce the notion of a semilattice congruence on T.

Let T be a ternary semigroup and I be a primeideal of T. We define a relation R_I on T as follows. $R_I = \{(a, b, c) / a, b, c \in I$ or $a, b, c \notin I\}$. Then R_I is a semilattice congruence on T. We denote $N(a)$ the filter of T generated by a ($\alpha \in T$). We symbolized by N the equivalence relation on M defined $N = \{(a, b, c) / N(a) = N(b) = N(c)\}$.

Theorem 2.13: Let T be a ternary semigroup. The succeeding statement hold true:

$$N = \bigcap \left\{ R_I \ / \ I \in I\left(T\right) \right\}$$

where is the set of primeideals T.

Proof: Let $(a, b, c) \in N$ and $I \in I\,(T)$. Let $(a, b, c) \notin R_I$. Then $a \notin I$ and $b, c \notin I$ or $a \in I$ and $b, c \notin I$. Let $a \notin I$ and $b, c \in I$. Then $\phi \neq T \setminus I \subseteq T$ and $a \notin T \setminus I$. Since $TT \setminus (T \setminus I) = I$, $TT \setminus (T \setminus I)$ is a primeideal of T. By the Lemma (T \ I) is a filter of T. Since $a \in T \setminus I$, we have $N(a) \subseteq T \setminus I$ and thus $b, c \in T \setminus I$. It is impossible. Similarly from $a \notin I$ and $b, c \in I$ we get a contradiction. Thus we have $N \subseteq \bigcap\limits_{I \in I(T)} R_I$. Conversely, let $(a, b, c) \in R_I$ for all $I \in I(T)$. If $\left(a, b, c\right) \notin N$, then $a \notin N(b)$ or $b \notin N(b)$ or $c \notin N(b)$. Infact, if $a \in N(b)$, $b \in N(c)$ and $c \in N(a)$, then $N(a) \subseteq N(b)$, $N(b) \subseteq N(c)$ and $N(c) \subseteq N(a)$ and so $\left(a, b, c\right) \notin N$. Let $a \notin N(b)$. Then $a \in T \setminus N(b)$ and thus $T \setminus N(b) \neq \phi$. Since $N(b)$ is a filter of T, by the lemma, $T \setminus N(b)$ is a primeideal of T. Therefore we have $T \setminus N(b) \in I\left(T\right)$, $a \in T \setminus N(b)$ and $b, c \notin T \setminus N(b)$, i.e $T \setminus N(b) \in I\left(T\right)$ and $\left(a, b, c\right) \notin R_{T \setminus N(b)}$ we get a contradiction. Similarly, from $b, c \notin N(a)$, we have a contradiction.

CONCLUSION

This concept is used in filters of chemistry, physical chemistry, electronics, etc.

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