Scientific Reviews & Chemical Communications

Sci. Revs. Chem. Commun.: 3(1), 2013, 94-99
ISSN 2277-2669

FILAMENTATION OF A UNIFORM LASER BEAM PROPAGATING IN A HOMOGENEOUS COLLISIONAL PLASMA INCLUDING THE EFFECT OF THERMAL CONDUCTION

GHANSHYAM*

Department of Physics, B. I. T. Sindri, DHANBAD – 828123 (Jharkhand) INDIA

(*Received : 09.01.2013; Revised : 18.01.2013; Accepted : 21.01.2013*)

ABSTRACT

In this paper we have studied the filamentation of laser beams in plasmas where both collisional and thermal-conduction losses are present simultaneously. A uniform intensity laser beam propagating through collisional plasma is unstable to transverse perturbations, and breaks up into filaments. An optimum value of $q_\perp$ of the perturbation is required for a maximum growth rate. A uniform plane wave does not cause redistribution of the carriers. However, as a result of perturbation in the intensity distribution along the wave, electrons do become redistributed. It is noted that the spatial growth rate is a monotonically increasing and saturating function of the incident intensity of the beam.

Key words: Filamentation, Uniform laser beam, Homogeneous collisional plasm, Thermal conduction.

INTRODUCTION

It has been shown in recent years that a high amplitude electromagnetic beam propagating in plasma is unstable to small-amplitude perturbations[1-3]. This instability causes the breaking of the beam into filaments and is known as filamentation instability[1-15]. On the time scale $t > \tau_h$ (which is more relevant to laser – plasma interactions), where τ_h is the heating time of electrons, the nonlinearity arises through nonuniform heating and redistribution of electrons[4]. The understanding of filamentation of laser light may be important to the success of laser fusion. In the long scale length plasmas envisioned for reactor targets, local intensity hot spots caused by self-focusing or laser light filamentation can drive the plasma above parametric instability thresholds. These instabilities tend to be saturated by the creation of super thermal electrons[5]. The hot electrons can penetrate deeply into the pellet, heating the interior, making high compressions difficult. Directly driven targets require very uniform driving pressures. Filamentation could spoil this uniformity, making large compressions difficult. The laser light absorption, penetration, and conversion to X rays could also be affected by self-focusing and filamentation. The earlier investigations of filamentation of laser beams on a long time scale are restricted to large-scale perturbations where the thermal conduction effects may be neglected[6,7]. But in the cases of real interest one is much more concerned about the growth of small-scale perturbations where thermal conduction could play a dominant role in determining the energy dissipation of electrons. The relative size of perturbations depends on the ratio m_i/m, since beam radius r_o is generally of the same order as electron mean free path λ_m. In this paper, we have studied the filamentation of laser beams

Available online at www.sadgurupublications.com
*Author for correspondence; E-mail: ghanshyam123@rediffmail.com

in plasmas where both collisional and thermal-conduction losses are present simultaneously. The variation of maximum growth rate with the intensity of the main beam has been investigated.

EXPERIMENTAL

Growth rate

Let us consider the propagation of a plane uniform laser beam in collisional plasma along the z-axis,

$$\vec{E} = \vec{A}_o (r, z) \exp [-i (\omega t - k_0 z)], \quad ...(1)$$

$$k_o = (\omega/c) \left(1 - \frac{\omega_{po}^2}{\omega^2}\right)^{\frac{1}{2}}, \quad ...(2)$$

$$\omega_{po}^2 = 4\pi n_o e^2 / m \quad ...(3)$$

and ω, ω_{po}, c, -e, m and n_o are the frequency of the main beam, the unperturbed plasma frequency of the medium, the velocity of the medium, the velocity of light, the electron charge, the electron mass and the unperturbed concentration of the plasma respectively. In the presence of the field (1), the electrons acquire drift velocity in accordance with the momentum balance equation.

$$m \frac{\partial \vec{v}}{\partial t} = -e \vec{E} - m\nu_{ei} \vec{v} \quad ...(4)$$

where ν is the electron collision frequency. Expressing the variation of $\vec{v}$ as exp [-i(ωt-kz)], we obtain, in the limit $\omega^2 >> \nu_{ei}^2$,

$$\vec{v} = \frac{e\vec{E}}{im\omega}\left(1 - \frac{i\nu_{ei}}{\omega}\right) \quad ...(5)$$

Besides this, the electrons absorb energy absorb energy from the wave at the rate of $-e\vec{E}.\vec{v}$. Whose time average is -

$$-\frac{1}{2} e\vec{E}^*. \vec{v} = \frac{e^2 A_o A_o^*}{2m\omega^2} \quad ...(6)$$

In the steady state the rate of energy gain must balance with the rate of energy loss through collisions and thermal conduction. Hence

$$-\nabla.\left(\frac{\chi}{n}\nabla T_e\right) + \frac{3}{2}\delta\nu_{ei}(T_e - T_o) = \frac{e^2\nu_{ei}A_o A_o^*}{2m\omega^2} \quad ...(7)$$

where

$$\frac{\chi}{n} = \frac{v_{th}^2}{\nu_{ei}} \quad ...(8)$$

$\delta = 2\ m/m_i$ is the fraction of excess energy lost per electron-ion energy exchange collision, T_e is the nonlinear field – dependent electron temperature and $\upsilon_{th} = (2T_o/m)^{\frac{1}{2}}$ is the electron thermal speed. For $\nu_{ei} r_o^2 / v_{th}^2 < (\delta\nu)^{-1}$ thermal conduction is important, and we solve the energy-balance equation in the perturbation approximation. For a beam of finite extent we express -

$$T_e = T_o + \Delta T_e$$

where $\Delta T_e << T_o$. Then Eq. (7) can be recast as -

$$\nabla^2(\nabla T_e) - \frac{3}{2}\frac{\delta \nu^2}{v_{th}^2}(\Delta T_e) = -\frac{e^2 \nu_{ei}^2}{2m\omega^2 v_{th}^2}\left|A_o\right|^2 \qquad ...(9)$$

Now we perturb the beam by a perturbation -

$$A_1(x,z)\exp[-i(\omega t - kz)] \qquad ...(10)$$

where A_1 (x,z) is not necessarily a slowing varying function of space variables. The total electric vector of the laser may now be written as -

$$\vec{E} = \left[\vec{E}_o + \vec{A}_1(x,z)e^{-i(\omega t - kz)}\right] \qquad ...(11)$$

where A_o is the amplitude in the absence of fluctuations (polarized in the y direction) and A_1 is the amplitude of the fluctuations, which is a spatially slowly varying function. The combined effect of these two fields is to heat the electrons and exert a pressure-gradient force, causing redistribution of plasma via ambipolar diffusion. The nonlinear field-dependent electron temperature T_e in the steady state may be obtained by solving Eq. (9) only the x dependence of A_1 is known. Taking $A_1 \propto e^{iq.x}$ with $q_{\|} << q_{\perp}$, where $q = q_{\perp} + q_{\|}$ is the scale length of the perturbation (the subscripts $\|$ and $\perp$ referring to components parallel and perpendicular to the z direction), T_e may be written as -

$$T_e - T_0 = \frac{e^2\left[A_0.\left(A_1 + A_1^*\right) + A_0^2\right]}{3m\omega^2\delta^{/}} \qquad ...(12)$$

Where

$$\delta^{/} = \delta + \frac{2}{3}\frac{q^2 v_{th}^2}{3v_{ei}^2} \qquad ...(13)$$

As a result of non-uniformity in heating, the plasma is redistributed so that -

$$n(T_e + T_0) = n_0(T_{e0} + T_0), \qquad ...(14)$$

where

$$T_{e0} = T_0 + \frac{e^2 A_0^2}{3m\omega^2\delta^{/}} \qquad ...(15)$$

Using Eq. (12), (13) and (15) in (14), the modified electron density may be written as -

$$n = n_0\left[1 - \frac{e^2 A_0.\left(A_1 + A_1^*\right)}{3T_0 m\omega^2\delta^{/}\left(2 + e^2 A_0^2/3m\omega^2 T_0\delta^{/}\right)}\right]. \qquad ...(16)$$

The dielectric constant of the plasma may be written as -

$$\in = \in_0 + \in_1 A_0.\left(A_1 + A_1^*\right), \qquad ...(17)$$

where

$$\in_1 = \frac{\omega_{po}^2}{\omega^2}\frac{\alpha P}{2 + \alpha P A_0^2} \qquad ...(18)$$

$$\mathrm{P} = \frac{1}{1 + 2q^2 v_{th}^2 / 3v_{ei}^2 \boldsymbol{\delta}} \quad ...(19)$$

$$\alpha = \frac{e^2}{3m\boldsymbol{\omega}^2 T_0 \boldsymbol{\delta}} \quad ...(20)$$

Substituting for E from Eq. (11) into the wave equation, using ∇. (∈E) = 0 and linearizing in A_1, we obtain the following equation for A_1:

$$2\mathrm{ik}\frac{\partial A_1}{\partial z} + \frac{\partial^2 A_1}{\partial x^2} + \frac{\boldsymbol{\omega}^2}{c} \in_1 A_0^2\left(A_1 + A_1^*\right) = 0 \quad ...(21)$$

Expressing $A_1 = A_{1r} + iA_{1i}$ and separating real and imaginary parts, we have

$$2\mathrm{k}\frac{\partial A_{1r}}{\partial z} + \frac{\partial^2 A_{1i}}{\partial x^2} = 0,$$

$$-2\mathrm{k}\ \frac{\partial A_{1i}}{\partial z} + \frac{\partial^2 A_{1r}}{\partial x^2} + \frac{2k^2 \in_1}{\in_0} A_0^2 A_{1r} = 0 \quad ...(22)$$

Taking A_{1r} and A_{1i} to be proportional to exp $[i(q_\perp x+q_\parallel z)]$, Eq. (22) straight away yields the dispersion relation

$$\mathrm{q}_\parallel = -\frac{iq_\perp}{2k}\left(2k^2 \frac{\in_1 A_0^2}{\in_0} - q_\perp^2\right)^{\frac{1}{2}} \quad ...(23)$$

Instability occurs when,

$$q_\perp^2 << 2k^2 \frac{\in_1 A_0^2}{\in_0} \quad ...(24)$$

The growth rate of the perturbation is -

$$\Gamma = \mathrm{iq}_\parallel = \frac{q_\perp}{2k}\left(2k^2 \frac{\in_1 A_0^2}{\in_0} - q_\perp^2\right)^{\frac{1}{2}} \quad ...(25)$$

The condition for maximum growth rate, $\partial q_\parallel/\partial q_\perp=0$, gives the optimum value of $q_\perp$ as -

$$q_{opt}^2 \approx \frac{k^2 \in_1 A_0^2}{\in_0} \quad ...(26)$$

and the maximum growth rate becomes -

$$\Gamma_{\max} \approx \frac{k^2}{2\in_0} \in_1 A_0^2$$

$$\approx \frac{k}{2\in_0}\frac{\omega_{po}^2}{\omega^2}\frac{\alpha A_0^2 P}{2+\alpha A_0^2 P} \qquad ...(27)$$

The spatial separation of striations of order $(q_{opt})^{-1}$, from (26), is given by -

$$\lambda_\perp \approx \frac{\lambda}{2\pi}\left\{\frac{\in_0}{\omega_{po}^2/\omega^2\left[\alpha A_0^2 P/\left(2+\alpha A_0^2 P\right)\right]}\right\}^{\frac{1}{2}} \qquad ...(28)$$

where $\lambda = 2\pi/k$ is the wavelength of the laser beam. The growth length $R_g \equiv \Gamma_{max}^{-1}$ can be written as -

$$R_g \approx \frac{2}{q_{opt}}\left(\frac{\in_0}{\in_1 A_0^2}\right)^{\frac{1}{2}} \qquad ...(29)$$

The perturbation will not continue to grow indefinitely, as implied by linearized theory, but will saturate with nonlinearity for higher values of αA_0^2. The following parameters have been chosen for calculating the growth rates for uniform laser beams: $\omega = 2 \times 10^{15}$ rad s^{-1} (Nd: YAG laser, $\lambda = 1.06$ μm), $\omega_{po}^2/\omega^2 = 0.5$, $T_0 = 100$ eV, $\nu_{ei} = 10^{13}$ s^{-1}, $\nu_{th} = 6 \times 10^8$ cm s^{-1} and $\alpha A_0^2 = 1$. For these parameters the optimum size (of order q_{opt}^{-1}) of a perturbation turns out to be about 20 μm. The result is displayed in Fig. 1, that shows the variation of the maximum growth rate of filamentation instability inside a collisional plasma with $\alpha\ A_0^2$ *for* $\omega_{po}^2/\omega^2 = 5$. It is interesting to note from the figure that the growth rate is a monotonically increasing and saturating function of the incident intensity of the beam.

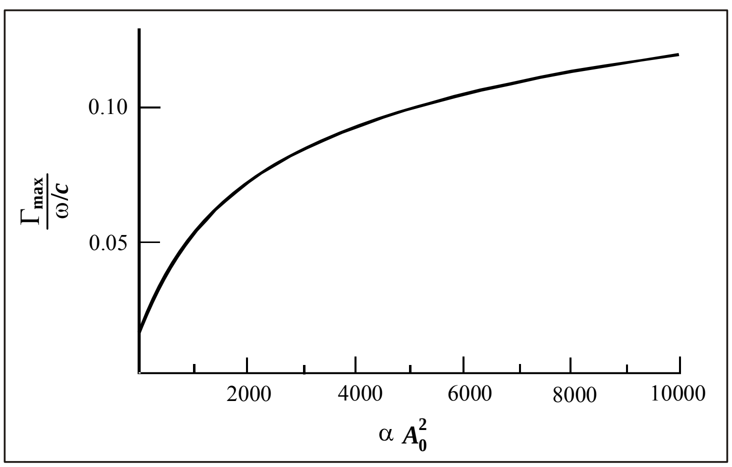

Fig. 1: Dependence of Γ_{max} on αA_0^2 for $\omega_{p0}^2/\omega^2 = 0.5$

CONCLUSION

A plane uniform laser beam of high intensity is seen to be unstable for small-scale fluctuations, i.e., it must break up into filaments in course of its propagation. The growth rate increases with decreasing scale length of perturbation and is seen to be a saturating function of power density of the beam. The growth rate increases with αA_0^2, saturates owing to the fact that nonlinearity increases in the plasma with increasing αA_0^2, and is a saturating function for large power density of the incident beam.

REFERENCES

1. M. S. Sodha, A. K. Ghatak and V. K. Tripathi, Prog. Opt., **13**, 169 (1976).
2. P. K. Kaw, G. Schmidt and T. Wilcon, Phys. Fluids, **16**, 1522 (1973).
3. M. S. Sodha, A. Kumar, V. K. Tripathi and P. K. Kaw, Optoelectronics, **5**, 509 (1973).
4. A. J. Palmer, Phys. Fluids, **14**, 2714 (1971).
5. K. Estabrook, W. L. Kruer and B. F. Lasinski, Phys. Rev. Lett., **45**, 1399 (1980).
6. M. S. Sodha, L. A. Patel and S. C. Kaushik, J. Plasma Phys., **21**, 1 (1979).
7. V. N. Laxmi and V. K. Tripathi, Radio Sci., **32**, 945 (1988).
8. Pallavi Jha, Phys. Scr., **81,** 055504 (2010).
9. Xia Xiongping and Yi Lin, Plasma Sci. Technol., **14**, 1054 (2012).
10. A. J. Schmitt, Phys. Fluids, **31**, 3079 (1988).
11. M. S. Sodha, J. K. Sharma, R. P. Sharma and S. C. Kaushik, J. Appl. Phys., **49**, 599 (1978).
12. M J. Schmitt and R. S. B. Ong, J. Appl. Phys., **54**, 3003 (1983).
13. E. Epperlein, G. J. Richard and A. R. Bell, Phys. Rev. Lett., **61**, 2453 (1988).
14. S. Skupsky, R. W. Short, T. Kessler, R. S. Craxton, S. Letzring and J. M. Soures, J. Appl. Phys., **66**, 3456 (1989).
15. H. Gardner and S. E. Bodner, Phys. Fluids, **29**, 2672 (1986).