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FILAMENTATION INSTABILITY OF LASER BEAM IN INHOMOGENEOUS RELATIVISTIC PLASMA

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ABSTRACT

In the present investigation, the analytical investigation of nonlinear propagation of intense electromagnetic waves through under dense inhomogeneous plasmas, taking into account the relativistic nonlinearity, is presented. The relativistic ponderomotive force is shown to have a major effect on nonlinear dynamics of the propagation of intense electromagnetic waves. It is seen that a plane wave of uniform intensity becomes unstable and gets filamented in the presence of transverse density fluctuation in the plasma. For a linear density profile the amplitude of the filament varies with z as an Airy's function. The growth rate increases with transverse wave vector of the perturbation. The characteristic growth length decreases with the size of perturbation and the ratio of expansion velocity to sound velocity. It increases with the angle laser k vector makes with the density gradient.

Key words: Filamentation, Laser beam, Inhomogeneous, Relativistic plasma.

INTRODUCTION

Filamentation instability is an important nonlinear process having implications on energy deposition and transport in long pulse laser plasma experiments[1-11]. It may also influence parametric processes, e.g., stimulated Raman scattering and two Plasmon decay[2,3]. The analytical theories of filamentation instability are largely applicable to plasma without a velocity of mass flow, while in laser-plasma experiments one is often encountered with expanding plasmas having large flow velocities. Short et al.[1] have studied the filamentation of a laser beam in expanding plasma where ponderomotive force mechanism prevails. Andreev et al.[2] reported results of filamentation instability in expanding inhomogeneous plasma. The effects of flowing plasma on thermal and ponderomotive light filamentation are examined by Schmitt[3]. Ghanshyam et al.[9] reported the results of filamentation instability of an electromagnetic wave in flowing magnetized homogeneous plasma. Sodha et al.[10] have studied self-focusing instability in ionospheric plasma with thermal conduction. Growth of a ring ripple on a Gaussian electromagnetic beam in a plasma with relativistic - ponderomotive nonlinearity is examined by Sodha et al.[11] The filamentation instability causes perturbations in the intensity profile of an incident laser beam to grow in amplitude, resulting beam into intense filaments. Filamentation is produced by perturbations or non-uniformities in the laser beam that cause (or are caused by) local changes in the refractive index of a medium. In laser plasmas there are a variety of mechanisms that give rise to an intensity-dependent refraction index and produce filaments. The

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number of electrons that are produced via ionization at a particular point of the beam path strongly depends on the prevailing local intensity. As a consequence, any intensity variation across the beam profile would give rise to a spatially varying index of refraction with an excess of electrons around the beam axis, which would lead to defocusing because of the lensing effect associated with it. In addition the diffraction of the beam leads to a defocusing effect independent of density. A counteracting process is the relativistically induced self-focusing due to the electron mass increase in high intensity regions and the expulsion of electrons from these regions by the ponderomotive force. These factors lead to a positive focusing effect which becomes stronger as the laser beam decreases in diameter and becomes more intense.

In this paper, we present a model calculation of filamentation instability in collisionless relativistic plasma having a stationary but non-uniform velocity profile. The nonlinearity arises through the relativistic ponderomotive force[8], which tends to push the plasma away from the regions of higher wave intensity. The process of plasma redistribution depends sensitively on the angle, the ponderomotive force makes with the flow velocity and on the ratio of flow velocity to the acoustic speed v_b/c_s.

We have studied the filamentation instability of a plane uniform laser beam in inhomogeneous plasma and obtained the growth rate. The case of homogeneous plasma has not been considered because one can always move to a frame of reference where plasma is stationary and the earlier results of filamentation instability theory are applicable. We have also discussed the obtained results.

Instability anslysis

Consider non-uniform plasma with density gradient $\nabla n_o \parallel \hat{z}$ and flow velocity $\vec{v}_b(z)$ along $-\hat{z}$ (cf. Fig. 1). In the quasi-steady state $(\partial n_o/\partial t) = 0$, hence $n_o v_b$ = constant (cf. equation of continuity). Using n_o $(dv_b/dz) = -v_b\,(dn_o/dz)$, the equation of motion for the plasma fluid can be cast as -

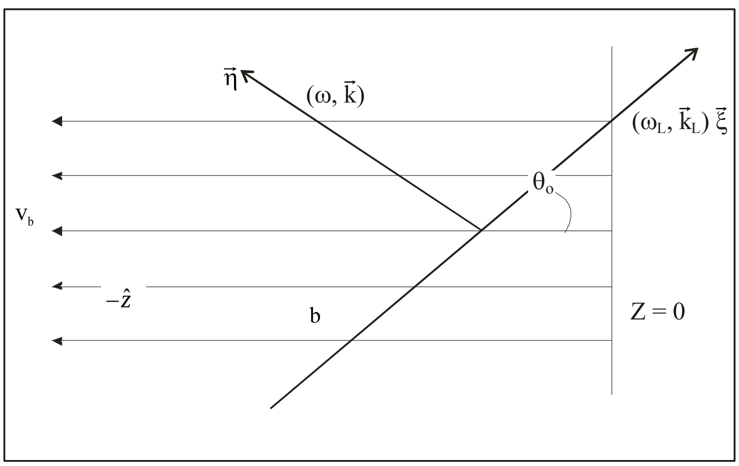

Fig. 1: Schematic of electromagnetic wave propagating in a inhomogeneously expanding plasma at an angle θ_o with density gradient $\nabla n_o \parallel -\hat{z}$ and flow velocity $\vec{v}_b(z) \parallel -\hat{z}$. ξ axis aligns along $\vec{k}_L$ and $\vec{\eta}$ axis is perpendicular to it. $\vec{k} \perp \vec{k}_L$ and $\vec{k} \parallel \vec{\eta}$

$$m_i v_b \frac{\partial v_b}{\partial z} = -\frac{1}{n_o}\frac{d}{dz}\,[n_o(T_e + T_i)], \qquad \ldots(1)$$

$$= -v_b\,\frac{d}{dz}\left(\frac{T_e + T_i}{V_b}\right). \qquad \ldots(2)$$

Eq. (2) leads

$$m_i v_b + \frac{T_e + T_i}{v_b} = \text{constant}, \qquad \ldots(3)$$

where m_i and T_i are the ionic mass and temperature, and T_e is the electron temperature. For a linear density profile.

$$n_o = n_o^o \left[1 + \frac{z}{L}\right], \qquad \ldots(4)$$

$$v_b = \frac{v_b(z=0)}{\left[1 + \frac{z}{L}\right]}$$

$$T_e + T_i = \frac{m_i c_s^2(0)}{\left[1 + \frac{z}{L}\right]} + \frac{m_i v_b^2(0)}{\left[1 + \frac{z}{L}\right]^2} \frac{z}{L}, \qquad \ldots(5)$$

Where $c_s^2(0) = \frac{(T_e + T_i)_{z=o}}{m_i}$, and L is the density scale length. Let an electromagnetic pump wave propagate through such plasma:

$$\vec{E}_L = \hat{y} E_L^{/}\, e^{-\left(\omega_L t - \int \vec{k}_L . d\vec{x}\right)}, \qquad \ldots(6)$$

where

$$k_L = \frac{\omega_L}{c}\left[1 - \frac{\omega_p^2}{\omega_L^2}\right]^{1/2},$$

$$\omega_p^2 = \frac{4\pi n e^2}{\gamma m_o \omega_L{}^2}$$

$$\gamma = \left(1 + \frac{e^2 A_o^2}{m_o^2 \omega_L{}^2 c^2}\right)^{\frac{1}{2}}$$

$\vec{k}_L$ is in the x-z plane and $E_L^{/}$ is real. It is useful to consider the coordinate system so that ξ axis aligns along $\vec{k}_L$ and $\vec{\eta}$ axis is perpendicular to it, in the x-z plane (cf. Fig. 1). Strictly speaking the pump wave would undergo refraction as it propagates and ray trajectories would be curved; however, in the underdense region such effects are not important, hence ignored here. The pump wave imparts oscillatory velocity to the electrons:

$$\vec{v}_L = \frac{e\vec{E}_L}{im\gamma\omega_L}, \qquad \ldots(7)$$

where –e and m are electron charge and mass. We perturb this equilibrium by a density perturbation

$$n = n^{/}(x,z) \exp\left[-i\left(\omega t - \int \vec{k} . d\vec{x}\right)\right],$$

which couples with the pump wave to produce two electromagnetic sidebands,

$$\vec{E}_j = \hat{y}E_j^{/}(x,z)e^{-i\left(\omega_j t-\int \vec{k}_j.d\vec{x}\right)} \;;\; j = 1,2, \qquad \text{…(8)}$$

where

$\omega_{1,2} = \bar{\omega} \mp \omega_L$, $\vec{k}_{1,2} = \vec{\bar{k}} \mp \vec{k}_L$,

The linear response of electrons to the sideband waves is -

$$\vec{v}_j = \frac{e\vec{E}_j}{im\gamma\omega_j} \;;\; j = 1, 2 \qquad \text{…(9)}$$

The pump and sideband waves exert a low frequency ponderomotive force[7], on the electrons

$$\vec{F}_p = -\frac{1}{2}\nabla\left[\frac{e^2}{m\ \gamma\omega_L^2}\vec{E}_L.\left(\vec{E}_1 + \vec{E}_2\right)\right], \qquad \text{…(10)}$$

causing ambipolar diffusion of the plasma. The velocity of mass motion $\vec{v}$ is governed by the linearized equation of momentum balance,

$$m_i\left(\frac{\partial\vec{v}}{\partial t} + v_b\frac{\partial\vec{v}}{\partial z} + \hat{z}v_z\frac{d}{dz}v_b\right) = \vec{F}_p - \frac{1}{n_o}\nabla\left[\left(T_e + T_i\right)n\right] - \frac{n}{n_o^2}\nabla\left[\left(T_e + T_i\right)n_o\right] \qquad \text{…(11)}$$

For kL > 1, Eq. (11) can be solved to obtain -

$$\vec{v} = \frac{i\left(\frac{1}{2}\nabla\frac{\left[e^2\vec{E}_L.\left(\vec{E}_1 + \vec{E}_2\right)\right]}{mm_i\omega_L^2\gamma}\right) + \frac{c_s^2\nabla n}{n_o}}{\left(\vec{k}.\vec{v}_b - \omega\right)}, \qquad \text{…(12)}$$

where

$$c_s^2 = \frac{\left(T_e + T_i\right)_z}{m_i}$$

Using Eq. (12) in the linearized equation of continuity,

$$\frac{\partial n}{\partial t} + \nabla.\left(n_o\vec{v} + n\vec{v}_b\right) = 0 \qquad \text{…(13)}$$

one obtains the density perturbation

$$n \cong -\frac{1}{2}\frac{n_o e^2\vec{E}_L.\left(\vec{E}_1 + \vec{E}_2\right)}{mm_i\omega_L^2 c_s^2\gamma}\left(1 + \frac{\left(\vec{k}.\vec{v}_b - \omega\right)^2}{k^2c_s^2}\right) \qquad \text{…(14)}$$

The nonlinear current density driving the electromagnetic sidebands can be written as

$$\vec{J}_{1,2} = \vec{J}_{1,2}^{L} + \vec{J}_{1,2}^{NL},$$

where

$$\vec{J}_{1,2}^{L} = -n_o e\vec{v}_{1,2}$$

and

$$\vec{J}^{NL}_{1,2} = \frac{1}{2} ne\left(\vec{v}^{*}_{L}, \vec{v}_{L}\right)$$

Using n_o^o *and* $J_{1,2}^{NL}$ in the wave equation one obtains –

$$\nabla^2 E_1 + \frac{\omega_1^2}{c^2} E_1 - \frac{\omega_p^2(0)}{c^2\gamma}\left(1+\frac{z}{L}\right) E_1 + \frac{2\pi\omega_1 e^2 E_L^* n}{mc^2\omega_L\gamma} = 0, \quad \ldots(15)$$

$$\nabla^2 E_2 + \frac{\omega_2^2}{c^2} E_2 - \frac{\omega_p^2(0)}{c^2\gamma}\left(1+\frac{z}{L}\right) E_2 + \frac{2\pi\omega_2 e^2 E_L^* n}{mc^2\omega_L\gamma} = 0, \quad \ldots(16)$$

Where $\omega_p^2(0) = 4\pi n_o^o e^2/m$. It is implied here that over a growth length of filamentation instability the direction of $\vec{k}_L$ is not significantly changed (i.e., growth length < $L\frac{\omega_L^2}{\omega_p^2}$). We chose, $\vec{k} \perp \vec{k}_L$ i.e., $\vec{k} \| \vec{\eta}$, as is convenient to a filamentation instability and restrict ourselves to the spatial growth only, i.e., take ω = 0. Then Eqs. (15) and (16), on using Eq. (8) and assuming $\partial E_1^{/} / \partial\xi < k < k_L$ take the form:

$$-2ik_L \frac{\partial}{\partial\xi} E_1^{/} - k^2 E_1^{/} + \frac{\omega_p^2(\xi\cos\theta_o)}{2c^2\gamma}\frac{v_L^2}{v_e^2}\frac{1}{\left(1-\frac{(\vec{k}.\vec{v}_b)^2}{k^2c_s^2}\right)} E_1^{/}$$

$$= -\frac{\omega_p^2(\xi\cos\theta_o)}{2c^2\gamma}\frac{v_L^2}{v_e^2}\frac{1}{\left(1-\frac{(\vec{k}.\vec{v}_b)^2}{k^2c_s^2}\right)} E_2^{/} \quad \ldots(17)$$

A similar equation could be obtained for $E_2^{/}$ by replacing $E_1^{/}$ by $E_2^{/}, E_2^{/}, by\, E_1^{/}$ and $\vec{k}_L$ by $-\vec{k}_L$ in Eq. (17),

$$2ik_L \frac{\partial}{\partial\xi} E_2^{/} - k^2 E_2^{/} + \frac{\omega_p^2(\xi\cos\theta_o)}{2c^2\gamma}\frac{v_L^2}{v_e^2}\frac{1}{\left(1-\frac{(\vec{k}.\vec{v}_b)^2}{k^2c_s^2}\right)} E_2^{/}$$

$$= -\frac{\omega_p^2(\xi\cos\theta_o)}{2c^2\gamma}\frac{v_L^2}{v_e^2}\frac{1}{\left(1-\frac{(\vec{k}.\vec{v}_b)^2}{k^2c_s^2}\right)} E_1^{/}$$

where

$$v_e^2 = \left[\frac{(T_e+T_i)_z}{m}\right]$$

and θ_o is the angle, $\vec{k}_L$ makes with the z axis. The coupled set can be combined to give a second order differential equation for $E_1^{/}$:

$$\frac{\partial^2 E_1^{/}}{\partial\xi^2} - \frac{k^2}{4k_L^2}\left(\frac{\omega_p^2(\xi\cos\theta_o)}{c^2\gamma}\frac{v_L^2}{v_e^2}\frac{1}{1-\frac{v_b^2}{c_s^2}\sin^2\theta_o} - k^2\right) E_1^{/} = 0. \quad \ldots(18)$$

For the density, temperature, and flow velocity profiles mentioned above, Eq. (18) takes the form

$$\frac{\partial^2 E_1^/}{\partial \xi^{/2}} - \xi^/ E_1^/ = 0 \qquad \text{…(19)}$$

where

$$\xi^/ = \left(\frac{k^2}{4k_L^2} \frac{\omega_p^2(0)}{c^2\gamma} \frac{v_L^2}{v_e^2(0)} \frac{1}{L^/} \cos\theta_o \alpha \right)^{1/3} (\xi_o + \xi),$$

$$v_e^2(0) = \left[\frac{(T_e + T_i)_{z=0}}{m\gamma} \right],$$

$$\xi_o = \frac{\left[\left\{ \frac{\omega_p^2(o)}{c^2\gamma} \right\} \left\{ \frac{v_L^2}{v_e^2(0)} \right\} \alpha - k^2 \right]}{\left[\left\{ \frac{\omega_p^2(0)}{c^2\gamma} \right\} \left\{ \frac{v_L^2}{v_e^2(0)} \right\} \left(\frac{\cos\theta_o}{L^/} \right) \alpha \right]},$$

$$\alpha = \frac{1}{\left[1 - \frac{v_b^2(0)}{c_s^2(0)} \sin^2\theta_o \right]},$$

$$L^/ = \frac{L\left[1 - \frac{3v_b^2(0)\sin^2\theta_o}{c_s^2(0)} \right]}{1 - \frac{v_b^2(0)\sin^2\theta_o}{c_s^2(0)}}$$

and we have assumed $L^/ \partial/\partial\xi > 1$. It may be worthwhile writing Eq. (18) for stationary plasma when the laser beam is propagating the direction of density gradient,

$$\frac{\partial E_1^/}{\partial \xi^2} + \frac{k^2}{4k_L^2} \left[k^2 - \frac{\omega_p^2(0)}{c^2\gamma^3} \frac{v_L^2}{v_e^2(0)} \left(1 + \frac{z}{L} \right) \right] E_1^/ = 0. \qquad \text{…(20)}$$

Eq. (19) is an Airy's equation[43] with two solutions A_i ($\xi^/$) and B_i ($\xi^/$). The second solution $E_1^/ = AB_i$ *($\xi^/$)* represents a rapid spatial growth, faster than an exponential, for $\xi + \xi_o > 0$. The filamentation starts at $\xi = (\xi = -\xi_o)\ \xi_o$. The effective growth length of instability is given by

$$L_{eff} = \left[\left\{ \frac{4L^/\gamma^3}{k^2} \frac{\omega_L^2}{\omega_p^2(0)} \frac{v_e^2(0)}{v_L^2} \right\} \left\{ \frac{1 - \left(\frac{v_b^2(0)}{c_s^2(0)} \right) \sin^2\theta_o}{\cos\theta_o} \right\} \right]^{1/3} \qquad \text{…(21)}$$

L_{eff} goes as γ ,$k^{-2/3}$ and $v_L^{-2/3}$ with the relativistic parameter ,wave number of perturbation and the electron quiver velocity. It decreases with $v_b^2(0)/c_s^2(0)$ and increases with θ_o and γ. Fig. 2 displays the variation of L_{eff} with θ_o for different values of $v_b^2(0)/c_s^2(0)$ for a typical set of Parameters:

$\frac{\omega_p^2(0)}{\omega_L^2} \approx .5, \frac{v_L^2}{v_e^2(0)} \approx .2$, L = 100 μm, and k = 10^4 cm^{-1}.

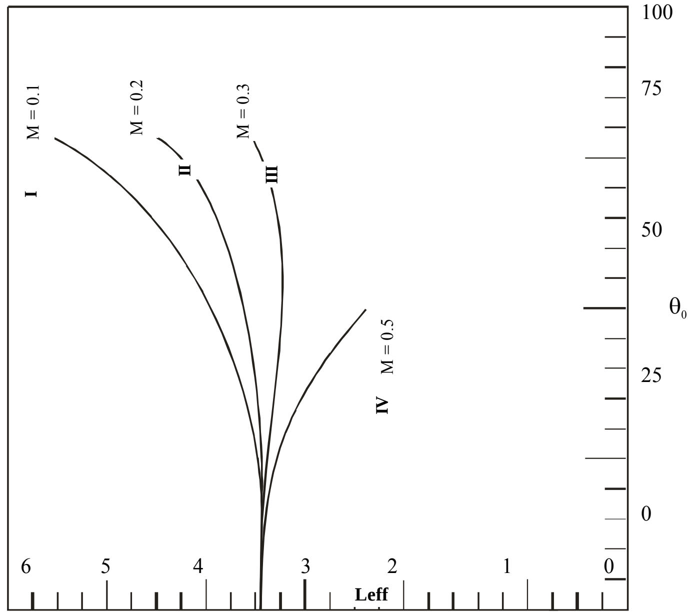

Fig. 2: Variation of effective growth length (L_{eff}) with the angle (θ_o) laser – k vector makes with the density gradient for $\omega_p^2(0)/\omega_L^2 \times v_L^2/v_e^2(0) \approx 10^{-2}$, L = 100 μm and k = 10^4 cm^{-1}, I_{st}, II_{nd}, III_{rd} and IV_{th} curves are for M $\left(= v_b^2(0)/c_s^2(0)\right)$ = 0.1, 0.2, 0.3 and 0.5 respectively

RESULTS AND DISCUSSION

A laser beam propagating at an angle to the flow velocity of a moving plasma is unstable to a transverse density perturbation. For a linear density profile $\omega_p^2 = \omega_p^2$ (0) (1+z/L), the amplitude of the filament varies with z as an Airy's function. In the far underdense region where the density of the plasma low and flow velocity is large, one requires higher power densities to onset a filamentation instability superceding diffraction divergence effects. The perturbation starts to grow beyond $z \geq -z_o$ $(= -\xi_o \cos\theta_o)$ at a rate faster than the exponential. In this process the power flux of the laser builds up around the density minima causing deeper density depressions which in turn attract higher power flux giving rise to the growth of the perturbation. After propagating a distance of a few L_{eff} the density depressions may become quite deep. At this stage the first order perturbation theory fails. However, one would think that as a consequence of enhanced power density in the filaments, the filaments, the filament size would diminish further to nonlinear refraction, until diffraction effects become prominent to offset the nonlinear convergence.

The characteristic growth length of the instability L_{eff} decreases with wave number of perturbation and with the velocity of plasma flow. The characteristic growth length L_{eff} increases slowly with the angle θ_o and increases linearly with γ, it depends on v_b (0). This may be understood as follows. The filamentation is caused by the depletion of the plasma in response to the transverse ponderomotive force. When the plasma flow velocity is not at right angle to the $\vec{k}$ vector of perturbation the plasma, on its way down the density gradient, moves from density maxima to minima periodically. The time of flight from one maximum to a minimum is $\tau_f \sim \dfrac{k^{-1}\cos^{-1}\theta_o}{v_b(0)}$, whereas the time for plasma redistribution via ambipolar diffusion $\tau_d \sim \dfrac{k^{-1}\sin^{-1}\theta_o}{c_s}$, significant density depletion occurs when $\tau_f > \tau_d$, i.e., v_b (0) < c_s tan θ_o. With increasing v_b

(0)/c_s (0) the time available for plasma redistribution decreases, hence, density depletion is diminished, leading to slower growth of the perturbation.

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