Fetal electrocardiogram extraction combining with wavelet de-noising and blind source separation algorithm

Chen Yu*, Wen Xinling, Liu Zhaoyu
Zhengzhou Institute of Aeronautical Industry Management, Zhengzhou Henan 450015, (CHINA)
E-mail: chenyu3440@gmail.com

ABSTRACT

In practical application, the weak signal of fetal electrocardiogram (FECG) is obtained through the surface electrodes placed upon the maternal abdominal with all kinds of other signals, such as the maternal electrocardiogram (MECG), work frequency jamming, and noise signal, etc, therefore, the application of FECG extraction algorithm in noise mixture circumstance has the practical significance and important clinical value. Independent Component Analysis (ICA) is a new blind source separation technology developed in recent years. And blind source separation (BSS) algorithm based on ICA technology has been widely applied to the medical field. This paper adopted a kind of popular BSS algorithm of Fast ICA combining with the wavelet de-noising algorithm to separate the FECG signal. Through the experiment, the MECG, FECG and noise can be separation from the mixture signal, and after the de-noising to the FECG signal, we can extract more clear signal effectively.

KEYWORDS

ICA; Fast ICA; FECG; Wavelet de-noising; MECG.
INTRODUCTION

FECG contains important information about fetal health. Therefore, early obtaining FECG signals is conducive to timely discover fetal hypoxia and umbilical cord entanglement, pregnancy or pathological conditions during delivery, and early taking measures to ensure fetal health, reduce the morbidity and mortality of the perinatal fetus. However, obtained FECG signals through the maternal contain a variety of interference, such as mother electrocardiogram (MECG) signal, power frequency, breathing and muscle power, etc[1].

At present, the FECG signal extraction or separation method is varied, including filtering method, the adaptive noise cancellation technology and ICA technology, etc[2,3,4]. And the ICA technology is considered to be the best approach to realize FECG signal collection. Based on the characteristics of multi-resolution wavelet transform multi-scale, combination with the wavelet analysis and blind source separation algorithm[5], using the blind source separation technology of Fast ICA algorithm[6] to complete the extraction of FECG signals with noise signal, the result demonstrated the isolated FECG signal is more pure than only using Fast ICA algorithm, which largely improves the effect of blind source separation algorithm, effectively extracts FECG component, this method can realize early detection of fetal disease and provide theoretical basis for measures taken.

ICA METHOD

ICA technology is initially used to solve the problem of cocktail party voice mixture. Because many persons’ voices aliasing, required to let the voices separated one by one. The aim of ICA technology is to get independent component of source signal only by observation mixture signals. Figure 1 shows the ICA model structure[7].

Assuming \( x(t) = [x_1(t), x_2(t), ..., x_n(t)]^T \) is a \( n \) dimension random observation mixture signal, there is \( m \) numbers of source signal \( s(t) = [s_1(t), s_2(t), ..., s_m(t)]^T \), the observation value of \( x_i(t) \) is random sampling. \( X = (x_1, x_2, ..., x_n)^T \) is a \( n \) dimension vector of random observation mixture signal. \( S = (s_1, s_2, ..., s_m)^T \) is \( m \) dimension vector of unknown source signal, then the ICA linear model can be presented as formula (1).

\[
X = AS = \sum_{j=1}^{m} a_j s_j(t), \; i=1,2,...,m
\]  

In formula (1), \( s_j(t) \) is an independent component, \( A=(a_1, a_2, ..., a_m) \) is a \( m \times n \) full rank mixture matrix. Each observation data \( x_i(t) \) is obtained by different linear weighted value \( a_j \) of the independent source \( s_j(t) \). The mixture matrix \( A \) is also an unknown matrix, the information can be obtained by the observation of random vector \( X \). Without restriction limitation, only use \( X \) to estimate \( S \) and \( A \), there are many more equation solution. And in some limitation of ICA, according to the statistic property of \( X \), it gives the only solution, and realize the equation of independent component of the extraction. An important assumption in ICA is the requirements of independence character for all unknown source signals. In ICA model, each of source signals needs be an independent value, and obey the non-Gaussian distribution. In addition, in order to simplify the mathematical model, we assume the unknown mixture matrix \( A \) has a square formation of \( m=n \). So, the purpose of the ICA is to find a transformation matrix, an transform \( X \) to get \( n \) output vector \( Y \).

\[
Y = WX = WAS
\]  

The projection coefficient \( s_1, s_2, ..., s_n \) is independent. If make \( Y = WX \), in ICA algorithm, the goal is to find a optimal matrix \( W \), which making output \( y_j \) statistical independence, namely each other mutual information of \( Y \) for zero. At this time, \( W^{-1} = [s_1, s_2, ..., s_n] \) is the ICA coordinate system of the linear description model.
Now, assuming that the distribution of data vector \( x \) is formed in accordance with the data model \( X=AS \). Then, the estimation of the independent component can be realized by looking for appropriate linear combination of the mixture variable, this kind of transformation can be represented as:

\[
s = A^{-1}x
\]  

(3)

Obviously, if there is no further assumptions and constraints, the estimation problem of the basic ICA model is impossible realized.

**FAST ICA ALGORITHM**

Negative entropy maximization criterion of the fast fixed-point algorithm is usually referred to as fast independent component analysis (Fast ICA) algorithm. Considering the basic ICA model expressed by formula (2) satisfies the constraint conditions. Usually, accurately get the matrix \( A^{-1} \) in formula (3) is not possible, and can only get the estimation of matrix \( A \) and source vector \( s \), so, the formula (3) can be changed into follow formula (4):

\[
y = Wz
\]  

(4)

In the formula (4), \( n \) dimension column vector \( z \) can be obtained by the centralized and bleaching pre-treatment. \( n \) dimension columns vector \( y \) represents the source vector \( s \) estimates. Now consider a single independent component estimation problem in \( y_i = w_i^Tz \cdot w_i^T \) in formula \( y_i = w_i^Tz \) is the \( i \)th line for the estimation matrix, \( y_i \) is the \( i \)th of component. Thus, the problem of independent component calculation can be included into the \( w_i \)’s optimal solution problem under the maximization criterion of objective function. The objective function is show as formula (5).

\[
J(w_i)=[E\{G(w_i^Tz)\} - E\{G(v)\}]^2
\]  

(5)

The maximum of approximate negative entropy is usually gained in \( E\{G(w_i^Tz)\} \), and based on the Lagrange conditions, \( E\{G(w_i^Tz)\} \) can get the extreme value under the constraint conditions of \( E\{G(w_i^Tz)\} = \| w_i \|^2 = 1 \), which is in those pints who make the gradient of Lagrange multiplier type be zero.

\[
E\{zg(w_i^Tz)\} + \beta w_i = 0
\]  

(6)

By using Newton's method to solve this equation, let \( F \) be the left side of the formula (6), we can obtain its gradient as follows:

\[
\frac{\partial F}{\partial w_i} = E\{zz^Tg'(w_i^Tz)\} + \beta I
\]  

(7)

Because

\[
E\{zz^Tg'(w_i^Tz)\} \approx E\{zz^T\}E\{g'(w_i^Tz)\} = E\{g'(w_i^Tz)\}I
\]  

(6)

We can get approximate Newton iteration formula:

\[
w_i \leftarrow w_i - \frac{E\{zg(w_i^Tz)\} + \beta w_i}{E\{g'(w_i^Tz)\} + \beta}
\]  

(7)

Simplifying the approximate Newton iterative algorithm, we can get the formula (8).

\[
w_i \leftarrow E\{zg(w_i^Tz)\} - E\{g'(w_i^Tz)\}w_i
\]  

(8)

This is the fixed-point iteration formula corresponding to the Fast ICA algorithm. We make it extend to the estimation of \( n \) numbers’ independent component, and can get Fast ICA calculation steps as follows: (a) To centralize
observation data $x$, and make its mean value be zero; (b) To make the data bleach; (c) To choose the number of component needed estimated $m$, and $m \leq n$; (d) To choose the initial value $w_i$ with unit norm; (e) $w_i \leftarrow E\{g(w_i^T z)\} - E\{g(w_i^T z)w_i\}$, among it, the process of choosing function $g$ is same as upon single independent component estimation algorithm; (f) To orthogonal the vector $w_i$ according to the formula: $w_i \leftarrow w_i - \frac{1}{N} \sum_{j=1}^{N} (w_j^T w_i)w_j$; (g) Standardized treatment $w_i \leftarrow w_i/\|w_i\|$; (h) If $w_i$ has not convergence, return to the step f; (i) Set $i \leftarrow i+1$, if $i \leq m$, then return step d, or, the calculation process end, during the iterative process, the mathematical expectation can be used by the average value of sample$^{[8]}$.

**WAVELET DE-NOISING**

Wavelet analysis in pure mathematics theory has gained development, at the same time, it also got rapid development in the engineering application research, especially in signal processing, image processing, pattern recognition, quantum physics and nonlinear science, etc, it has been widely used, and is considered to be scientific tools and methods for breakthroughs in recent years$^{[5]}$.

Under the signal wavelet decomposition, the different scales have different time and frequency resolution. Using multi-resolution analysis method, it equates to a set of filters combination, which can get along with all the information of different frequencies, and also keep the time characteristic of the information at the same time. In general, a one-dimensional signal wavelet de-noising process can divided into the following steps: (a) A one-dimensional wavelet decomposition of signals. Choose a wavelet basic function, and determine the level of the wavelet decomposition of $N$, then the signal is carried out $N$ layers’ wavelet decomposition. (b) The threshold quantization of high frequency coefficient during the wavelet decomposed. Select a threshold to carry out quantization process to each layer from the high frequency coefficient of number from 1 to $N$. (c) A one-dimensional wavelet reconstruction. To the first $N$ layer of low frequency coefficient and the high frequency coefficients form layer of 1 to $N$, we carry out wavelet reconstruction. The de-noising process by wavelet analysis, the most important problem is to chose quantization threshold, usually, we adopt soft-threshold and hard-threshold function to eliminate noise$^{[9]}$. The method of this paper uses the Stein's unbiased likelihood estimation principle to get the adaptive threshold$^{[10]}$.

Assuming $n$ dimensional vector $u=(u_1, u_2, \ldots, u_n)$, $x_i \sim N(u_i, 1)$. let $\hat{u} = \hat{u}(X)$ is a fixed estimate to $u$, Stein introduced an unbiased estimation method to the damage $\|u-u\|^2$, that is to make $\hat{u}(X) = X + g(X)$, among it, $g = (g_i)_{i=1}^n \in \mathbb{R}^n$. Stein got the formula (9) when $g(X)$ is faint differentiable.

$$E \|\hat{u}(X) - u\|^2 = n + E \{\|g(X)\|^2 + 2\nabla_y \cdot g(X)\}$$

(9)

Among the formula (9), $\nabla \cdot y = \sum_{i=0}^{n-1} \frac{\partial g_i}{\partial x_i}$. Now, considering threshold estimator $u_{i}^{(0)} = \eta(x)$, and apply Stein’s conclusion, we can get the formula (10).

$$\text{Sure}(t, X) = n - 2 \cdot I \{\|x\| < t\} + \sum_{i=1}^{n} (\|x_i\| \wedge t)^2$$

(10)

It is an unbiased estimation to the risk $E \|\hat{u}^{(0)}(X) - u\|^2 = E \text{Sure}(t, X)$. Based on upon method, we can get the independent threshold selection method under each scale. After the wavelet transformation to the initial signal, the threshold $t_j$ of this scale can be estimated by the wavelet coefficient $y_{j,k}$.

**SIMULATION ANALYSIS**

The difference of electrical level obtained by the The fetal ecg transmission upon the maternal abdominal recorded is referred to indirection FECG signal. But, by the maternal abdominal measured ecg signal is a mixture of the maternal ecg and the fetal ecg signal, etc. In order to verify the effectiveness and accuracy of algorithm in the role of FECG signal detection, we collected the signal upon the mother's chest and abdomen, the waveform of acquisition signal on both sides respectively is shown in Figure 2.
Assuming that the signal made up of the maternal ecg signal, fetal ecg signal and mixture noise, by using the Fast ICA blind source separation methods to get the signal waveform respectively is shown in Figure 3. The power spectrum after the separation of the waveform signal respectively is shown in Figure 4.

From the Figure (3) and (4) we can see, FECG signal still has part of noise that can not be filtered, we select the position of sampling point from 100 to 900 from the separated FECG signal, which is listed in Figure 5. By multi-scale wavelet decomposition, we can extract the N scale approximate component, use Birge-Massart method to obtain threshold to the one dimensional wavelet, adopt the Stein's unbiased likelihood estimation principle to get an adaptive threshold, through db5 wavelet basis function, and return the signal after noise reduction processing. The wavelet decomposition approximation components are listed in Figure 6.
The approximation components and detail components under each scale respectively is shown as Figure 7(a) and Figure 7(b).

The FECG signal after de-noising is shown as Figure 8.
Comparing with the Figure 8 and Figure 4, we can see, after the FECG signal de-noising, the result of FECG under the Fast ICA blind separation becomes more pure, that is to say the noise has better inhibition, and the quality of the fetal ecg signals got obvious improvement. We can carry out further clinical analysis according to the results.

CONCLUSION

This paper uses blind source separation algorithm of Fast ICA to separate FECG signal from the mixture signal consisting of various components of interference and noise signal. After the blind separation in Fast ICA algorithm, the MECG and FECG signal as well as the noises can be extracted from the initial source signal. But, from the FECG signal obtained through the BSS algorithm, we can find, there are still some noises in it, so, we adopt a kind of wavelet multi-resolution analysis method to filter the noise remained, and finally get the clear FECG signals than before. Through the validation of experiments, the simulation result that combining with the blind source separation algorithm and wavelet de-noising algorithm is better to realize FECG extraction, and the method in this paper can accurately detect the FECG signals from the blind mixture eg signal, which is practical and feasible.

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