Journal of Physics Astronomy

**Review Article** | Vol 13 Iss 1

# Extended Classical Mechanics: Vol-1-Equivalence Principle, Mass and Gravitational Dynamics

**Soumendra Nath Thakur***

Department of Physics, Seth Anandram Jaipuria College, Kolkata, India

***Corresponding author:** Soumendra Nath Thakur, Department of Physics, Seth Anandram Jaipuria College, Kolkata, India; E-mail: postmasterenator@gmail.com

**Received:** September 19, 2024, Manuscript No. TSPA-24-148312; **Editor assigned:** September 23, 2024, PreQC No. TSPA-24-148312 (PQ); **Reviewed:** October 08, 2024, QC No. TSPA-24-148312; **Revised:** February 19, 2025, Manuscript No. TSPA-24-148312 (R); **Published:** February 26, 2025, DOI. 10.37532/2320-6756.2025.13(1).410.

## Abstract

**This research paper explores the framework of extended classical mechanics, with a focus on the equivalence principle, mass, and gravitational dynamics. Volume 1 of this study re-examines the classical equivalence principle, which maintains the equivalence of inertial mass and gravitational mass (also referred to as gravitating mass) and extends this concept to incorporate new findings related to both illuminating (baryonic) matter and dark matter. The paper provides a comprehensive analysis of Matter Mass ($M_M$) and Gravitating Mass ($M_G$) and their roles within the extended framework. It introduces the concept of Apparent Mass ($M^{app}$), a negative mass component that influences Effective Mass ($M^{eff}$), in alignment with the research by A.D. Chernin et al. on dark energy and the structure of the Coma Cluster of Galaxies. Additionally, the paper reinterprets Newton's law of universal gravitation by integrating apparent mass, leading to a revised understanding of gravitational potential. The study demonstrates how the interaction between matter mass and negative apparent mass contributes to a redefined concept of gravitating mass. These extensions enhance classical mechanics by incorporating modern scientific insights, including the gravitational effects of dark matter and the alignment of Apparent Mass with the negative effective mass of dark energy.**

***Keywords:** Apparent mass; Dark energy; Dark matter; Effective mass; Equivalence principle; Extended classical mechanics; Gravitating mass; Gravitational dynamics; Matter mass; Negative mass; Newton's law of universal gravitation*

## Introduction

**Comment on revision:** In this revised version, numerous typographical and formatting issues from the original submission have been corrected. These updates ensure that the mathematical expressions and key concepts are now conveyed with the intended precision and clarity, maintaining the scientific rigor and integrity of the work.

### List of mathematical terms

A list of mathematical terms essential for extended classical mechanics, including effective mass, dark energy, and gravitational forces, redefined to incorporate the effects of dark energy and negative mass on both local and cosmic phenomena.

**Citation:** Thakur SN. Extended Classical Mechanics: Vol-1-Equivalence Principle, Mass and Gravitational Dynamics. J Space Explor. 2025;13(1):410.

**$a^{eff}$:** Effective acceleration, modified by the interaction between matter mass and apparent mass
**$E_{DE}$:** Total energy associated with dark energy within a given volume.
**F:** Force, modified to incorporate apparent mass and effective acceleration.
**$F_{Univ}$:** Universal force acting on the universe's mass, involving effective mass and acceleration on cosmic scales.
**$F_G$:** Gravitational force between two masses, accounting for effective mass.
**G:** Gravitational constant, representing the strength of the gravitational interaction.
**$M^{app}$:** Apparent mass, a negative mass component affecting effective mass.
**$M_{DE}$**: Dark energy effective mass, interpreted as equivalent to negative apparent mass.
**$M_M$:** Matter mass, including both normal (baryonic) matter and dark matter.
**$M^{eff}$:** Mechanical effective matter mass, combining matter mass and apparent mass.
**$M_2$:** Secondary mass, the mass of another object in gravitational calculations.
**$M_G$:** Gravitating mass, the total effective mass influencing gravitational dynamics.
**PE:** Potential energy, dependent on the effective mass of the system in a gravitational field.
**r:** Distance, the separation between two masses in gravitational force equations.
**$\rho^{DE}$:** Dark energy density, the density of dark energy in the universe
**$\rho^{M}$:** Matter mass density, the density of matter within a given volume

**Glossary of key terms**

This glossary provides definitions of the key terms used in the study, "Extended Classical Mechanics: Vol-1-Equivalence Principle, Mass, and Gravitational Dynamics." It includes new terminologies introduced in the extended framework, alongside classical concepts that have been reinterpreted or expanded. The terms "Gravitating Mass" and "Gravitational Mass" are treated as synonymous throughout this research, representing the total effective mass that influences gravitational dynamics.

**Apparent mass ($M^{app}$):** A dynamic term that reflects the observed mass of an object under external forces. This mass can appear reduced due to negative effective mass. When a force F acts on an object, causing an increase in acceleration a, a significant negative component in the effective mass $M^{eff}$ (*i.e.,*$-M^{app}$) results in an apparent reduction of the observed mass, which can be quantified as negative apparent mass ($M^{app}<0$) This phenomenon is prominent under conditions like high velocities or strong gravitational fields.

**Classical mechanics:** The traditional branch of physics that deals with the motion of bodies under the influence of a system of forces. In this research, classical mechanics is extended to include the effects of negative mass, apparent mass and dark energy.

**Coma cluster of galaxies:** A large cluster of galaxies that has been studied to understand the effects of dark energy on large-scale structures. This study references research by A.D. Chernin et al., which discusses the impact of dark energy on the cluster's structure.

**Dark energy effective mass ($M_{DE}$):** The effective mass associated with dark energy, which contributes to a repulsive force that influences gravitational dynamics negatively. This concept, introduced in Chernin et al.'s 2013 paper, is reinterpreted in this study as equivalent to negative apparent mass ($-M^{app}$). According to the equation $M_G=M_M+M_{DE}$, where $M_G$ represents the total gravitational mass, $M_M$ is the matter mass, and $M_{DE}$ is the dark energy effective mass, this formulation underscores the substantial impact of dark energy on the overall gravitational dynamics of the universe [1].

**Dark energy:** A theoretical form of energy that pervades the entirety of space and is responsible for the accelerated expansion of the universe. In this interpretation, dark energy is conceptualized as incorporating a negative mass component. This remaining influences gravitational dynamics significantly and plays a crucial role in shaping the structure and behaviour of galaxy clusters.

**Distance (r):** The separation between two masses, used in the gravitational force equation to determine the strength of their interaction. The presence of dark energy, characterized by its negative mass, influences this gravitational force. Understanding r accurately is essential for modelling the structure and behavior of galaxy clusters in contemporary cosmological theories.

**Effective acceleration ($a^{eff}$):** The rate at which an object's velocity changes, influenced by the interplay of positive matter mass ($M_M$) and negative apparent mass ($-M^{app}$). Effective acceleration is determined by the overall effective mass ($M^{eff}$) of a system, which is the sum of matter mass and negative apparent mass. When the negative apparent mass is significant, it alters the effective mass and thereby affects the acceleration experienced by the object. The relationship is expressed as: $F=(M_M-M^{app})\cdot a^{eff}$ where F is the force applied, $M_M$ is the matter mass, $-M^{app}$ is the negative apparent mass, and $a^{eff}$ is the effective acceleration. This modified effective acceleration accounts for the influence of negative apparent mass on the dynamics of motion.

**Effective mass ($M^{eff}$):** A composite term that includes both matter mass ($M_M$) and negative apparent mass ($-M^{app}$). Effective mass can be positive or negative depending on the relative magnitudes of the matter mass and the negative apparent mass.

**Extended classical mechanics:** An extension of classical mechanics proposed in this research to account for new factors like negative apparent mass, dark energy, and their effects on gravitational dynamics.

**Gravitating mass (Gravitational Mass) ($M_G$):** The total effective mass that governs the gravitational dynamics of a system. It encompasses both the matter mass and any negative apparent mass, and it is equivalent to the mechanical effective mass ($M^{eff}$).

**Gravitational constant (G):** A constant of proportionality used in Newton's law of universal gravitation, representing the strength of the gravitational interaction.

**Gravitational force ($F_G$):** The force of attraction between two masses traditionally defined by Newton's law of universal gravitation but modified in this research to account for effective mass.

**Matter mass ($M_M$):** The mass associated with normal (baryonic) matter and dark matter within a system. It contributes positively to the gravitating mass.

**Mechanical energy:** The sum of potential and kinetic energy in a system. In this research, mechanical energy is influenced by the effective mass, which includes both matter mass and negative apparent mass.

**Negative apparent mass ($-M^{app}$):** A condition where the effective mass $M^{eff}$ of a system becomes negative due to the dominance of a negative apparent mass term or under extreme conditions such as high velocities or strong gravitational fields. Negative effective mass affects the dynamics of motion by altering how acceleration and force interact with the system [2]. When the apparent mass is negative ($M^{app}<0$), the effective mass $M^{eff}$ is reduced accordingly, which can lead to counterintuitive effects such as an increased acceleration for a given force. This negative effective mass concept is crucial for understanding how dark energy and other negative mass terms impact gravitational and dynamic interactions in the extended classical mechanics framework.

**Negative effective mass:** A condition where the effective mass ($M^{eff}$) of a system becomes negative. This situation arises when the negative apparent mass term ($-M^{app}$) outweighs the positive matter mass ($M_M$) in the effective mass equation. Negative effective mass results in counterintuitive behaviour where, under certain conditions such as high velocities, strong gravitational fields, or significant contributions from dark energy the effective mass of an object or system can turn negative. This negative value affects how the object responds to forces, often resulting in unusual dynamic behaviors.

**Newton's law of universal gravitation:** The classical equation defining the gravitational force between two masses, which is modified in this research to incorporate apparent mass and effective mass.

**Potential energy:** The energy possessed by an object due to its position in a gravitational field. In the context of this research, it is influenced by the effective mass of the system.

**Universal force ($F_{Univ}$):** The total force acting on the universe's mass, related to effective mass and effective acceleration. This term describes the combined gravitational influence across cosmic scales as considered in our research.

**Universal singularity:** A hypothesized state representing the origin of the universe with infinite gravitational potential energy and density. It marks the point from which the universe began expanding, influencing the fundamental framework of gravitational dynamics.

Classical mechanics has long served as the foundation for understanding physical phenomena, particularly in the realms of motion and gravitational dynamics. Central to classical mechanics is the equivalence principle, which asserts the indistinguishability of inertial and gravitational mass. This principle has guided our comprehension of gravitational interactions and mass effects for centuries. However, as our understanding of the universe has evolved, new concepts such as dark matter, dark energy, and negative effective mass have emerged, challenging and extending the classical framework.

In "extended classical mechanics: Vol-1-equivalence principle, mass and gravitational dynamics," we undertake a comprehensive re-examination of the classical equivalence principle and its implications in light of modern scientific insights. This volume of the study extends traditional concepts to incorporate new findings related to both normal and dark matter, redefining core principles of mass and gravitational dynamics.

A key focus of this research is the concept of Matter Mass ($M_M$) and its interaction with Gravitating Mass ($M_G$). We introduce and analyse Apparent Mass ($M^{app}$), which arises under specific conditions involving motion and gravitational dynamics, and propose its role as a negative mass component influencing Effective Mass ($M^{eff}$) [3]. By integrating these concepts, we reinterpret Newton's Law of Universal Gravitation and explore how Apparent Mass modifies gravitational potential and dynamics.

Our study extends the theoretical foundation of our previous research and builds upon established intercontinental work, including the contributions of A. D. Chernin et al. on dark energy and its impact on cosmic structures. We propose that the traditionally considered negative effective mass of dark energy can be better understood through the concept of apparent mass. This approach enriches the classical understanding of gravitating mass by integrating modern scientific perspectives, particularly regarding the gravitational effects of dark matter and the alignment of apparent mass with negative effective mass.

In summary, this research provides an extended framework for classical mechanics, offering new insights into the nature of mass and gravity. It aims to bridge classical concepts with contemporary theories, presenting a unified approach that incorporates the effects of dark energy and negative mass into the traditional mechanic's framework.

## Literature Review

This section outlines the methodology for examining the roles of various mass components including Matter mass ($M_M$), apparent mass ($M^{app}$), effective mass ($M^{eff}$), and gravitating mass ($M_G$) within the framework of extended classical mechanics. The methodology integrates theoretical reinterpretation and mathematical modelling to establish new insights into the gravitational dynamics of cosmic structures.

### Conceptual reinterpretation and integration

**Objective:** Reinterpret the relationships between different mass components in light of extended classical mechanics to understand their influence on gravitational dynamics.

**Reinterpretation of gravitating Mass ($M_G$):** Begin by analysing traditional interpretations of gravitating mass as the sum of matter mass ($M_M$) and dark energy effective mass ($M_{DE}$), using foundational research such as that by A.D. Chernin et al., "Dark Energy and the Structure of the Coma Cluster of Galaxies". Redefine this relationship by substituting the dark energy effective mass ($M_{DE}$) with the concept of negative apparent mass ($-M^{app}$), resulting in the new equation:

$M_G = M_M + (-M^{app})$.

**Extension to newton's second law:** Extend newton's second law to incorporate the apparent mass ($M^{app}$) and effective acceleration ($a^{eff}$), modifying the equation to:

$F = (M_M - M^{app}) \cdot a^{eff}$.

This equation enables the study of the conditions under which apparent mass becomes negative, thereby influencing the dynamics of motion and gravity.

### Mathematical modelling of apparent and effective mass

**Objective:** Develop mathematical models to quantify the relationships among matter mass, apparent mass, and effective mass, and their collective impact on gravitational interactions.

**Define matter mass ($M_M$):** Model the total mass of baryonic matter and dark matter as the sum of their respective components. Ensure that the equivalence principle applies universally, so that the matter mass contributes directly to gravitational mass.

**Calculate apparent mass ($M^{app}$):** Formulate equations to determine apparent mass under different scenarios, such as motion within gravitational fields.

Use: $M^{app}=M_M-M_G$

Where $M_G$ includes both matter and apparent mass contributions

Model effective mass ($M^{eff}$):

Define effective mass as a function of matter mass and apparent mass:

$M^{eff}=M_M + (-M^{app})$

Analyse how effective mass can transition between positive and negative values depending on the magnitudes of its components.

**Theoretical analysis of gravitational dynamics**

**Objective:** Examine how the interplay of different mass components affects gravitational forces and the overall gravitational dynamics of a system.

**Reinterpret newton's law of universal gravitation**: Modify the traditional equation for gravitational force:

$F_G=G\cdot(M_G\cdot M_2)/r^2$

By substituting $M_G$ with $M^{eff}$, which incorporates both matter mass and apparent mass:

$M_G=M_M+(-M^{app})$.

Study the implications when the magnitude of $-M^{app}$ exceeds that of $M_M$, causing $M_G$ to become negative and altering the gravitational interactions.

**Analyse dark energy's role:** Utilize the reinterpretation of dark energy as a negative apparent mass to explore its influence on the gravitational structure of large-scale cosmic entities, such as galaxy clusters.

**Simulation and numerical analysis objective**

Use computational simulations to test the derived mathematical models and evaluate their consistency with observational data.

**Simulate gravitational interactions:** Develop simulations to model the dynamics of systems with varying contributions of matter mass, apparent mass, and effective mass. Observe the conditions under which the system's gravitational dynamics change, such as when the effective mass becomes negative [4].

**Compare with observational data:** Cross-validate simulation results with observed data from cosmic structures, such as those described in studies of galaxy clusters (e.g., Coma Cluster), to confirm the validity of the theoretical framework.

**Discussion and implications**

**Objective:** Discuss the findings from the mathematical modelling, theoretical analysis, and simulations to refine the understanding of mass dynamics in extended classical mechanics.

**Implications for gravitational theory:** Assess how the new definitions and interpretations, particularly the concept of negative apparent mass, influence traditional gravitational theories and models.

**Insights into dark energy and matter interactions:** Explore the broader implications for dark energy's role in the universe, particularly in the context of its effective mass representation and its effect on cosmic evolution.

**Conclusion and future work**

**Objective:** Synthesize the findings, outline conclusions, and propose directions for future research.

**Summarize key findings:** Highlight the key outcomes related to the redefined mass concepts and their implications for gravitational dynamics.

**Propose future research:** Suggest additional avenues for research, such as further numerical simulations or observational studies, to expand upon the findings and test their applicability in various cosmic contexts.

**Mathematical presentation**

**Equivalence principle and mass in classical mechanics**

The equivalence principle in classical mechanic's states that inertial mass, which determines how an object accelerates under a given force, is equivalent to gravitational mass, which determines the strength of an object's interaction within a gravitational field. In other words, an object's resistance to acceleration (inertia) is fundamentally the same as its tendency to attract or be attracted by other masses due to gravity.

Within the framework of classical mechanics, this principle holds that the inertial mass of normal matter is exactly equal to its gravitational mass. As a result, all objects, regardless of their mass or composition, experience the same acceleration when subjected to a gravitational field.

Applying this principle to systems containing both normal matter and dark matter, the effective gravitational mass ($M_G$) of such a system is seen as equivalent to the combined inertial mass of its components, assuming that the equivalence principle holds for all types of mass. Thus, the gravitational force exerted by the system depends on the total inertial mass, which includes contributions from both normal baryonic matter and dark matter.

This interpretation suggests that the effective gravitational mass ($M_G$) of the system represents a unified measure of the gravitational coupling between normal baryonic matter and dark matter, combining their contributions into a single mass term that governs the system's gravitational behavior.

$M_G = M_M$

Where:

- **$M_G$:** Gravitational mass (effective gravitational mass of the system)
- **$M_M$:** Matter mass (sum of baryonic matter and dark matter)

Note: In the context of classical mechanics, the equivalence principle asserts that inertial mass ($M_M$) is equivalent to gravitational mass ($M_G$). For the purposes of this presentation, $M_M$ is defined as the total mass of the system, encompassing both normal matter and dark matter. Therefore, while $M_G = M_M$ reflects the equivalence principle, it implicitly includes the contributions of dark matter within the matter mass term ($M_M$). This formulation does not consider additional effects such as the effective mass of dark energy, which is addressed in the extended framework below.

**Matter mass ($M_M$): Composition and role**

Matter mass ($M_M$) refers to the total mass of both baryonic matter (ordinary matter, composed of protons, neutrons, and electrons) and dark matter. Baryonic matter is the visible, luminous matter that makes up stars, planets, and other objects we can observe directly. In contrast, dark matter is non-luminous, does not emit or absorb light, and interacts primarily through gravitational forces.

Together, baryonic matter and dark matter constitute the majority of the mass in the universe. They play a crucial role in the formation and evolution of cosmic structures, such as galaxies, clusters of galaxies, and cosmic filaments. The gravitational interaction between these two forms of matter is essential for understanding how these structures come into being and how they evolve over time.

**Conclusion: Matter Mass ($M_M$)**

The matter mass ($M_M$) of a system is defined as the sum of the masses of both dark matter and baryonic matter within that system. Because normal matter (baryonic matter) interacts gravitationally with dark matter, and assuming the equivalence principle applies universally to all forms of mass, the effective gravitational mass ($M_G$) of a system containing both components is equivalent to the combined inertial mass of the baryonic and dark matter [5]. This unified mass determines the gravitational force exerted by the system, incorporating contributions from both types of matter.

**Gravitating Mass ($M_G$): Definition and dynamics**

Gravitating mass ($M_G$) refers to the net mass responsible for generating gravitational attraction within a system. It combines the effects of both the matter mass ($M_M$) which includes baryonic and dark matter and any other contributing masses that affect gravitational dynamics.

**Gravitating mass and dark energy**

Traditional research, such as that by A. D. Chernin et al., in "Dark Energy and the Structure of the Coma Cluster of Galaxies," describes dark energy in terms of an effective mass(1), where the relationship between matter mass ($M_M$) and dark energy effective mass ($M_{DE}$) is given by:

$$M_G=M_M+M_{DE}$$

$M_G$, $M_M$, and $M_{DE}$ are defined in the list of mathematical terms.

In this context, $M_{DE}$ represents the dark energy effective mass, which is negative.

This approach can be reinterpreted by aligning the concept of dark energy with a negative apparent mass ($-M^{app}$), leading to the equivalent expression:

$$M_G = M_M + (-M^{app})$$

$M_G$, $M_M$, and $-M^{app}$ are defined in the list of mathematical terms.

**Gravitating mass ($M_G$): Total effective mass**

Gravitating mass ($M_G$) represents the total effective mass that determines the gravitational dynamics of a system, it is equivalent to the mechanical effective mass ($M^{eff}$), encompassing both the matter mass ($M_M$) and the negative apparent mass ($-M^{app}$). Therefore, the relationship can also be expressed as:

$$M_G=M^{eff}$$

$M_G$ and $M^{eff}$ are defined in the list of mathematical terms.

**Conclusion: Gravitating mass ($M_G$)**

Gravitating mass ($M_G$) is the total effective mass responsible for gravitational interactions within a system. It reflects the combined contributions of both matter mass ($M_M$) and negative apparent mass ($-M^{app}$).

**Gravitating mass and dark energy: Research insights**

Based on the research by A. D. Chernin et al., the relationship between gravitating mass, matter mass, and dark energy effective mass is given by:

$$M_G=M_M+M_{DE}$$

Where:

**$M_G$:** Gravitating Mass
**$M_M$:** Matter Mass
**$M_{DE}$:** Dark energy effective mass (where $M_{DE}<0$)

The concept of dark energy effective mass ($M_{DE}<0$), though not a traditional part of classical mechanics, is derived from observational evidence. It extends classical mechanics by incorporating principles to explain phenomena associated with dark energy, which is often interpreted as a form of potential energy.

Similarly, the notion of negative effective mass introduces the mechanical concept of apparent mass in contexts like gravitational potential or motion, where it is negative and also considered a form of potential energy. This extends classical mechanics by recognizing the parallels between dark energy and generated apparent mass as manifestations of negative potential energy.

**Newton's second law and the concept of apparent mass**

In classical mechanics, Newton's second law states that the force F applied to an object is proportional to its acceleration a and its mass. In this extended framework, acceleration (a) is inversely proportional to the object's matter mass ($M_M$). An increase in acceleration may be interpreted as an apparent reduction in the object's matter mass, leading to the concept of apparent mass ($M^{app}<0$), a theoretical notion where mass appears negative under specific conditions, particularly in the context of motion and gravitational dynamics.

In this framework, the effective mass ($M^{eff}$) combines both matter mass ($M_M$) and a negative apparent mass ($-M^{app}$), modifying Newton's second law to:

$$F = (M_M - M^{app})\cdot a^{eff}$$

Where:

- F is the applied force,
- $M_M$ is the matter mass,
- $-M^{app}$ is the negative apparent mass,
- $a^{eff}$ is the effective acceleration.

Since $M^{eff}=M_M-M^{app}$, this equation simplifies to:

$$F=M^{eff}\cdot a^{eff}$$

$M_G$, $M^{eff}$, and $a^{eff}$ are defined in the list of mathematical terms.

This expression shows that the effective mass $M^{eff}$ governs the system’s dynamic response to the applied force, accounting for the impact of negative apparent mass on acceleration.

**Conclusion for apparent mass ($M^{app}$)**

The apparent mass $M^{app}$ is a negative mass component that arises due to the system's dynamics, potentially reducing the total effective mass. This affects the system's response by effectively reducing its inertia, leading to:

$F=M^{eff}\cdot a^{eff}$

F, $M^{eff}$, and $a^{eff}$ are defined in the list of mathematical terms.

This reflects an extended interpretation of Newton's second law within the framework of extended classical mechanics.

Consistency of Negative Apparent Mass ($-M^{app}$)

The concept of negative apparent mass ($M^{app}<0$) aligns with the notion of dark energy effective mass, as discussed in A. D. Chernin et al.'s study, "Dark Energy and the Structure of the Coma Cluster of Galaxies."[1] In their work, gravitating mass ($M_G$), matter mass ($M_M$), and dark energy effective mass ($M_{DE}$) are related by:

$M_G=M_M+M_{DE}$

Where, $M_{DE}$ is a negative dark energy effective mass. In this extended framework, the relationship becomes:

$M_G = M_M + (-M^{app})$

Where $M_G$ is the gravitational mass, $M_M$ is the matter mass, and $-M^{app}$ is the negative apparent mass. This formulation is consistent with the concept of negative effective mass in extended classical mechanics.

**Apparent mass: Definition and characteristics**

Apparent mass refers to a situation where the effective mass of an object or system appears reduced due to the influence of a negative effective mass term. This concept arises under specific conditions, such as objects in motion or within strong gravitational fields, where the negative effective mass term significantly impacts the system's dynamics.

**Characteristics of apparent mass**

**Negative effective mass:** Apparent mass is characterized by a negative value when the negative effective mass term is significant. This occurs in mechanical and gravitational dynamics, as well as in phenomena involving dark energy, where the negative contribution affects the system's overall behavior.

**Conditions for negative apparent mass:** Apparent mass becomes negative when the negative effective mass term dominates the system's overall effective mass. This typically occurs in scenarios involving objects in motion or within strong gravitational fields, especially under extreme gravitational potentials.

**Examples of apparent mass in context:**

**In motion:** When force is applied and acceleration increases, the effective mass can include a negative term, leading to a reduction in the apparent mass. This is captured by the formula:

$F=(M_M-M^{app})\cdot a^{eff}$

$M_M$, $M^{app}$ and $a^{eff}$ are defined in the list of mathematical terms.

Where, $M^{eff}$ may be negative due to the negative effective mass contribution.

**In gravitational potential:** In gravitational contexts, if the negative effective mass is significant, the effective mass can become negative, affecting the gravitational dynamics. This is described by:

$M_G = M_M + (-M^{app})$

$M_G$, $M_M$, and $-M^{app}$ are defined in the list of mathematical terms.

Where, $M^{eff}$ includes the negative apparent mass term.

**Effective Mass ($M^{eff}$): Definition and implications**

Effective mass ($M^{eff}$) is defined as the sum of the matter mass ($M_M$) and the negative apparent mass ($-M^{app}$). Mathematically, it is expressed as:

$M^{eff} = M_M + (-M^{app})$

This equation shows that the effective mass represents the combined influence of the matter mass and the negative apparent mass within a system.

When a force (F) is applied to a system, it affects the effective acceleration ($a^{eff}$), and thereby influences the effective mass ($M^{eff}$). The relationship between force, effective mass, and effective acceleration can be expressed by:

$F = M^{eff} \cdot a^{eff}$

F, $M^{eff}$ and $a^{eff}$ are defined in the list of mathematical terms.

**Conclusion: Effective Mass ($M^{eff}$)**

Effective Mass ($M^{eff}$) is a composite term that includes both matter mass and apparent mass (where apparent mass is negative). It accounts for the system's motion and gravitational dynamics, including effects such as "antigravity," which may occur when the magnitude of the apparent mass exceeds that of the matter mass. The effective mass can be positive or negative, depending on the relative magnitudes of matter mass and apparent mass.

**Effective mass: Definition and characteristics**

Definition: Effective mass is a composite quantity that represents the total mass influencing a system's response to applied forces or gravitational effects. It combines the matter mass and the negative effective mass to provide a measure of how the system behaves dynamically under external forces or gravitational fields.

**Characteristics:**

- **Positive or negative effective mass:** The effective mass can either be positive or negative, depending on the relative sizes of the matter mass ($M_M$) and the negative effective mass ($-M^{app}$).
- **Positive effective mass:** When the matter mass is greater than the negative effective mass, the effective mass is positive.
- **Negative effective mass:** When the negative effective mass is significant or under extreme conditions (such as high velocity or strong gravitational fields), the effective mass can become negative.
- **Implications:** The effective mass determines how an object or system responds to external forces or gravitational effects. In classical mechanics, this relationship is expressed in the equation:

$F = (M_M - M^{app}) \cdot a^{eff}$

F, $M_M$, $M^{app}$ and $a^{eff}$ are defined in the List of Mathematical Terms.

Where, $M^{eff}$ may include a negative component due to the contribution of apparent mass, which is characterized as negative effective mass.

**Concept of negative effective mass**

The concept of negative effective mass aligns with the findings in the research paper "Dark energy and the structure of the coma cluster of galaxies" by A. D. Chernin et al., which connects the idea of apparent mass to gravitational potential or motion in various theoretical models. These models, particularly those involving advanced gravitational theory and cosmology, use the concept of negative effective mass to explain phenomena such as repulsive gravitational effects or specific acceleration conditions.

This idea extends classical mechanics principles by incorporating negative effective mass to account for these effects, offering a broader explanation of certain phenomena not fully described by classical models alone.

**Consistency with mechanical principles**

The study adheres to classical mechanic’s principles by interpreting apparent mass in gravitational potential or motion as negative, akin to potential energy, which is often considered negative in gravitational fields (where zero potential energy is conventionally set at infinity). This ties the concept of negative effective mass to well-established mechanical principles, providing a consistent framework within extended classical mechanics.

**Recognition of observational evidence**

The study highlights that concepts like negative effective mass and apparent mass are based on observational evidence. This approach aligns with the scientific method, which relies on empirical data to validate or adjust theoretical frameworks. Observational phenomena, such as the accelerated expansion of the universe attributed to dark energy, support extending classical mechanics principles to encompass phenomena beyond the capabilities of traditional models [6].

**Avoidance of ambiguity**

The study clearly distinguishes between classical mechanics and its extensions, such as the effective mass concept involving dark energy. It states that while these extensions build on classical ideas, they are not confined to traditional mechanics. This distinction clarifies that concept like dark energy and negative apparent mass represent forms of potential energy that extend beyond conventional classical mechanics.

**Gravitating Mass ($M_G$): Total effective mass**

Gravitating mass $M_G$ represents the total effective mass that governs a system's gravitational interactions. It can be expressed as the sum of the matter mass $M_M$ and the negative apparent mass $-M^{app}$, leading to the equation:

$$M_G=M^{eff}=M_M + (-M^{app})$$

$M_G$, $M^{eff}$, $M_M$ and $-M^{app}$ are defined in the list of mathematical terms.

This equation aligns with the extended mechanics framework, where the effective mass encapsulates the influence of both normal and negative apparent mass.

**Conclusion: Gravitating Mass ($M_G$)**

The gravitating mass $M_G$ reflects the total effective mass, combining matter mass and negative apparent mass contributions. This formulation describes the gravitational dynamics of the system under consideration, ensuring consistency in the treatment of mass and force:

$$M_G=M^{eff}$$

$M_G$ and $M^{eff}$ are defined in the list of mathematical terms.

This expression aligns with the intended framework of effective and apparent mass.

**Gravitating mass: Definition and dynamics**

Gravitating mass is the net mass responsible for gravitational attraction, combining the effects of matter mass $M_M$ and other contributing masses, including the negative apparent mass.

**Gravitating mass and dark energy**

Traditional research, such as A. D. Chernin et al.'s work, "Dark energy and the structure of the coma cluster of galaxies," describes dark energy using the equation:

$$M_G=M_M+M_{DE}$$

$M_G$, $M_M$ and $M_{DE}$ are defined in the list of mathematical terms.

Where, $M_{DE}$ represents the dark energy effective mass. This study reinterprets dark energy by aligning it with the concept of negative apparent mass, expressed as:

$$M_G=M_M + (-M^{app})$$

$M_G$, $M_M$ and $(-M^{app})$ are defined in the list of mathematical terms.

**Effective Mass ($M^{eff}$): Definition and implications**

Effective mass ($M^{eff}$) is the total mass affecting the system's response to applied forces or gravitational influences. It is defined as:

$$M^{eff}=M_M+(-M^{app})$$

Where $M^{eff}$ represents the combination of the matter mass ($M_M$) and the negative apparent mass ($-M^{app}$). This composite mass affects how the system responds to external forces or gravitational fields.

**Characteristics of effective mass**

- **Positive or negative effective mass:** The effective mass can be either positive or negative, depending on the relative magnitudes of matter mass ($M_M$) and negative apparent mass ($-M^{app}$).
- **Positive effective mass:** Occurs when the matter mass is greater than the negative apparent mass.
- **Negative effective mass:** Occurs when the negative apparent mass is significant, particularly in extreme conditions like high velocity or strong gravitational fields.

**Implications of effective mass**

The effective mass determines how an object or system responds to external forces or gravitational influences. In classical mechanics, this relationship is captured by the equation:

$$F=M^{eff}\cdot a^{eff}$$

F, $M^{eff}$ and $a^{eff}$ are defined in the list of mathematical terms.

where $M^{eff}$ includes the negative component from apparent mass, characterized as negative effective mass. This formulation extends classical mechanics principles to include phenomena influenced by dark energy, aligning with observational evidence and theoretical models.

**Newton's law of universal gravitation and apparent mass**

Newton's law of universal gravitation describes the gravitational force between two masses, traditionally expressed as:

$F_G=G\cdot(m_1\cdot m_2)/r^2$

Where:

- $F_G$ is the gravitational force,
- G is the gravitational constant,
- $m_1$ and $m_2$ are the masses of the two objects, and
- r is the distance between them.

**Modification of newton's law by apparent mass**

In the framework of extended classical mechanics, the concept of apparent mass ($M^{app}$) modifies the traditional equation for gravitational potential. Apparent mass, which is negative ($M^{app}<0$), affects the system's effective mass ($M^{eff}$) by effectively reducing the total mass. This modification considers both the matter mass ($M_M$) (including normal matter and dark matter) and the negative apparent mass ($-M^{app}$).

The apparent mass ($M^{app}$) is related to the dark energy effective mass ($M_{DE}$), as described in A. D. Chernin et al.'s research, "Dark Energy and the Structure of the Coma Cluster of Galaxies." The extended framework introduces the following equations to reinterpret this relationship:
$M_G=M_M+(-M^{app})$ or: $M_G=M^{eff}$

Where:

- $M_G$ is the gravitating mass,
- $M_M$ is the matter mass, and
- $-M^{app}$ represents the negative apparent mass.

**Reformulation of gravitational force with apparent mass**

By substituting the expression for apparent mass ($M^{app}$), the gravitational force equation is reformulated as:

$F_G=G\cdot(M_G\cdot m_2)/r^2$

$F_G$, G, $M_G$, $m_2$ and $r^2$ are defined in the list of mathematical terms.

Where:

$M_G=M^{eff}=M_M + (-M^{app})$

$M_G$, $M^{eff}$, $M_M$, and $-M^{app}$ are defined in the list of mathematical terms.

This formulation aligns with the relationship:

$M_G=M_M+M_{DE}$

$M_G$, $M_M$, and $M_{DE}$ are defined in the list of mathematical terms.

Implications of apparent mass in gravitational interactions

- When the magnitude of $-M^{app}$ exceeds $M_M$, the gravitating mass ($M_G$) becomes negative.
- This reinterpretation of negative apparent mass ($-M^{app}$) and the negative effective mass of dark energy ($M_{DE}$) arises from considerations of motion and gravitational dynamics rather than as tangible substances.

This perspective is consistent with the principles of extended classical mechanics, providing a coherent framework to understand gravitational interactions, particularly in systems influenced by dark energy and negative apparent mass.

**Conclusion: Reinterpreted gravitational dynamics**

This extended framework aligns with research reinterpreting dark energy as negative effective mass, impacting gravitational dynamics and providing coherence within classical mechanics.

**Future directions in extended classical mechanics**

In the subsequent volumes of extended classical mechanics, we will explore the following topics:

- The relationship between apparent mass and kinetic energy.
- The impact of apparent mass on the deformation of objects in motion and within gravitational dynamics.
- The connection between apparent mass and relativistic Lorentz transformations, among other phenomena.

**Relativistic rest energy and its role in gravitational dynamics**

In relativity, rest energy is intrinsically linked to the concept of rest mass, which is also known as inertial mass in classical mechanics. Rest energy is a fundamental form of energy associated with mass, and it plays a critical role in the total energy of a system, which includes both rest energy and kinetic energy arising from momentum.

In classical mechanics, total energy is the sum of potential energy and kinetic energy, which are associated with the motion and position of the system. In the relativistic framework, however, the total energy of a system is modified to include rest energy, which is linked to the rest mass of the system. When the system is at rest, the total energy is purely the rest energy, with no kinetic contributions.

The concept of matter mass encompasses both normal (baryonic) matter and dark matter, and it is the sum of these contributions. The gravitating mass, which determines the gravitational interaction, is the result of the total matter mass adjusted by the influence of apparent mass effects. Apparent mass represents counteracting forces, such as dark energy or other repulsive phenomena, which modify the gravitational dynamics.

This research underscores that rest energy is inherently embedded within the matter mass, making it fundamentally distinct from classical forms of energy such as potential and kinetic energy. Rest energy is not an independent form of energy but is a constant, intrinsic property of mass, integrated into the system's total energy. Consequently, the total energy within the system is represented through the transformations and interactions of classical energy forms, while rest energy remains an implicit and invariant characteristic of the system's mass [7]. This perspective provides a deeper understanding of the relationship between rest energy, matter, and gravitational dynamics, particularly in the context of cosmological and high-energy systems.

**Mathematical framework for the role of rest energy in gravitational dynamics**

**Rest energy in relativity:** In relativity, rest energy ($E_{Rest}$) is intrinsically linked to the rest mass ($M_{Rest}$), which is also known as inertial mass in classical mechanics. The rest energy of a system is expressed as:

$$E_{Rest}=M_{Rest}\cdot c^2$$

Where: $E_{Rest}$ represents rest energy, $M_{Res}$ represents rest mass (or inertial mass), and c is the speed of light.

**Total energy in relativity:** In the relativistic framework, the total energy ($E_{total}$) of a system includes both rest energy and kinetic energy ($E_K$), the latter of which arises from the momentum of the system:

$$E_{total}=\sqrt{(M_{Rest}\cdot c^2)^2 + (\rho c)^2}$$

Where: ρ is the relativistic momentum of the system.

**Total energy at rest:** When the system is at rest (ρ=0), the total energy reduces to the rest energy alone:

$E_{total}=M_{Rest} \cdot c^2$

**Classical mechanics and energy components:** In classical mechanics, the total energy ($E_{total}$) of a system is simply the sum of Potential Energy (PE) and Kinetic Energy (KE):

$E_{total}=PE+KE$

Where: PE is the potential energy, and KE is the kinetic energy.

**Matter mass and gravitational mass:** The matter mass ($M_M$) encompasses both normal (baryonic) matter and dark matter, which can be written as:

$M_M=M_{ORD}+M_{DE}$

Where: $M_{ORD}$ is the normal (baryonic) matter, $M_{DE}$ is the mass of dark matter.

**Gravitating mass:** The gravitating mass ($M_G$) that determines the gravitational interaction is related to the total matter mass by the apparent mass effects. The gravitating mass is given by:

$M_G=M_M+(-M^{app})$

Where: $M^{app}$ represents the negative apparent mass, which accounts for counteracting forces like dark energy or other repulsive phenomena.

Therefore, the gravitating mass can also be written as:

$M_G=M_M-M^{app}$

**Rest energy embedded in matter mass:** This research asserts that rest energy is inherently embedded within the matter mass ($M_M$), distinguishing it fundamentally from classical forms of energy such as Potential Energy (PE) and Kinetic Energy (KE). Unlike these classical energy forms, which depend on motion and position, rest energy is an intrinsic property of mass. It is not an independent energy form but rather an implicit, constant characteristic of the systems mass, integrated into the system's total energy.

As a result, the total energy of the system is expressed through the interactions and transformations of classical energy forms (potential and kinetic), while rest energy remains a constant, invariant aspect of the system's mass. In this framework, total energy is primarily driven by the classical components, with rest energy subtly embedded within the matter mass.

**Conclusion:** Thus, this research establishes that rest energy is inherently included in matter mass ($M_M$), which highlights its fundamental distinction from classical energy forms. The mathematical expressions presented above clarify the interconnected roles of rest energy, matter mass, and gravitational mass, demonstrating their influence in both relativistic and classical contexts. This deeper understanding of their interrelationships contributes to advancing our knowledge of gravitational dynamics and cosmological systems.

## Dynamic interplay of potential energy, mass, and kinetic energy

### Revisiting potential and kinetic energy

In classical mechanics, Potential Energy (PE) and Kinetic Energy (KE) are foundational concepts that govern the motion and energy transformations of systems. The total energy in classical systems is typically expressed as:

$E_{total}=PE+KE$

However, this simplistic interpretation often neglects the nuanced interdependence of PE, mass, and KE. This section explores the dynamic interplay among these variables and extends the classical framework to include effective mass contributions.

**Influence of potential energy on mass**

Potential energy is not an isolated entity but a contributor to the system's effective mass ($M^{eff}$). Changes in PE influence $M^{eff}$, as reflected in the extended force equation:

$F=M^{eff}\cdot a^{eff}$, where: $M^{eff}=M_M-M^{app}$

Here, $M_M$ represents the system's actual mass, and $M^{app}$ accounts for apparent mass contributions arising from energy transformations.

**Kinetic energy as a transformation of potential energy**

Kinetic energy does not arise independently; it is a direct result of changes in potential energy:

$KE=\Delta PE=PE_{InMotion}-PE_{AtRest}$

This relationship emphasizes that kinetic energy reflects the redistribution of potential energy within a system. Consequently, mass, which can represent potential energy, dynamically adjusts to these transformations.

**Implications for extended mechanics**

The interplay among PE, mass, and KE challenges the assumption of constant mass in classical mechanics. Instead, effective mass adapts to energy transformations, offering a more comprehensive understanding of motion and energy transfer. This perspective aligns with the principles of extended classical mechanics, which integrate apparent mass and effective force contributions into the classical framework.

**Conclusion:** Recognizing the intricate relationships among PE, mass, and KE provides a richer understanding of energy transformations and motion. By incorporating these nuances, extended classical mechanics enhances the predictive power of traditional models, paving the way for deeper insights into both terrestrial and cosmic systems.

**Discussion**

This research introduces a comprehensive extension of classical mechanics by incorporating the nuanced roles of relativistic rest energy, matter mass, and their interplay with potential and kinetic energy. Additionally, it integrates the concepts of gravitational dynamics and apparent mass, alongside their interactions with dark matter and dark energy, to offer an enriched perspective on cosmic phenomena.

**Key concepts and contributions**

Equivalence principle and mass

- The research reaffirms the classical equivalence principle, establishing the equivalence of inertial mass and gravitational mass as a foundational tenet.
- Extending this principle, the study explores systems comprising both normal matter and dark matter. It posits that the effective gravitational mass ($M_G$) of such systems equals the combined inertial mass of their components, including the influence of apparent mass effects.

**Matter mass ($M_M$) and Gravitating mass ($M_G$)**

**Matter mass ($M_M$):** Defined as the sum of normal (baryonic) matter and dark matter, this concept underscores the role of all mass components in shaping gravitational interactions.

**Gravitating mass ($M_G$):** Extended to incorporate the effects of negative apparent mass ($-M^{app}$), it represents the net mass driving gravitational attraction, reflecting the dynamic interplay of matter and apparent mass.

**Relativistic rest energy embedded in matter mass**

- The research highlights rest energy as an intrinsic property of matter mass, fundamentally distinct from classical forms of energy such as potential and kinetic energy.
- Rest energy is treated not as an independent form of energy but as an implicit and invariant characteristic of matter mass. It is seamlessly integrated into the system's total energy, while the transformations and interactions of potential and kinetic energy dominate the observable energy dynamics.

**Apparent mass and effective mass**

- **Apparent mass ($M^{app}$):** This concept is expanded to include counteracting forces such as those associated with dark energy, contributing to negative effective mass phenomena.
- **Effective mass ($M^{eff}$):** Defined as the combination of matter mass and apparent mass, effective mass offers a unified framework to address complex gravitational effects, including "antigravity" behaviors.

**Dynamic interplay of potential energy, mass, and kinetic energy**

The research reveals the interdependent relationship among potential energy, mass, and kinetic energy, challenging the classical assumption of constant mass.

Effective mass ($M^{eff}$) dynamically adjusts based on energy transformations, presenting a more comprehensive view of energy redistribution in systems subject to gravitational forces.

**Gravitational dynamics and dark energy**

- Revisiting Newtonian gravitation, the study integrates apparent mass effects, suggesting a potential mechanism by which dark energy influences gravitational forces.
- By aligning dark energy with negative effective mass ($M_{DE}<0$), the research provides a theoretical framework for understanding the accelerated expansion of the universe and its implications for cosmic gravitational dynamics.

**Analysis**

**Integration of classical and modern concepts:** The inclusion of rest energy, dark matter, and dark energy within the classical mechanic's framework represents a significant advancement. This integration bridges traditional mechanics with contemporary cosmological phenomena, offering a cohesive model that aligns with observational evidence.

**Theoretical innovations:** The introduction of rest energy as an intrinsic component of matter mass, alongside the novel concept of negative apparent mass, addresses gaps in classical theories. These innovations provide a robust explanation for gravitational effects attributed to dark energy and high-energy astrophysical observations.

**Observational and theoretical consistency:** The extended framework aligns well with empirical data, particularly regarding cosmic acceleration and dark matter distributions. By linking these observations to rest energy and negative apparent mass effects, the research strengthens its theoretical foundation.

**Future research directions**

The proposed exploration of apparent mass, effective mass, and their relationships with potential and kinetic energy promises to deepen the understanding of relativistic and classical physics [8]. Furthermore, the integration of these principles with Lorentz transformations and deformation dynamics highlights a pathway for unifying mechanics across scales and conditions.

## Conclusion

This research represents a pivotal step forward in classical mechanics by integrating modern concepts such as relativistic rest energy, dark matter, dark energy, and the interplay of potential and kinetic energy with effective mass. By expanding classical principles and aligning them with contemporary astrophysical observations, the study establishes a robust framework for understanding complex gravitational dynamics and cosmic phenomena. The outlined future research directions highlight a clear trajectory for further advancements, underscoring the potential for this extended framework to deepen our understanding of the universe.

**Key contributions**

**Reaffirmation and extension of the equivalence principle:**

- The research reaffirms the classical equivalence principle, emphasizing the equivalence of inertial and gravitational masses as a cornerstone of mechanics.
- This principle is extended to systems comprising normal matter, dark matter, and apparent mass, proposing that the effective gravitational mass ($M_G$) reflects the dynamic interplay of these components.

**Integration of rest energy, dark matter, and dark energy**

- Rest energy is identified as an intrinsic property of Matter Mass ($M_M$), distinguishing it from classical energy forms such as potential and kinetic energy.
- The study broadens the concept of matter mass to include contributions from normal and dark matter while addressing dark energy's role through the lens of negative apparent mass ($-M^{app}$). This integration offers insights into cosmic acceleration and gravitational phenomena.

**Apparent mass and effective mass**

Apparent mass ($-M^{app}$) is introduced as a theoretical concept representing counteracting forces such as dark energy.
Effective mass ($M^{eff}$) is defined as the sum of matter mass and apparent mass, providing a framework to address phenomena like "antigravity" effects and dynamic mass variations.

**Dynamic interplay of potential energy, mass, and kinetic energy**

The study highlights the interconnected nature of potential energy, kinetic energy, and effective mass. This interplay challenges the classical assumption of constant mass, revealing how energy transformations dynamically influence the system's effective mass and motion.

**Reformulation of gravitational dynamics**

By modifying Newtonian gravitation to incorporate apparent mass effects, the research presents an extended framework for gravitational interactions. This approach reconciles traditional mechanics with modern observations, particularly the influence of dark energy on cosmic expansion.

**Future research directions**

The paper outlines future research to explore the relationship between apparent mass and kinetic energy, relativistic effects, and deformation dynamics. These directions aim to unify extended classical mechanics with modern theoretical and observational physics, paving the way for deeper insights.

**Analysis**

**Theoretical innovation:** The integration of rest energy as an intrinsic property of matter mass and the introduction of negative apparent mass represent significant advancements. These concepts provide a foundation for explaining phenomena such as cosmic acceleration and dark energy effects, which lie beyond the scope of traditional mechanics.

**Observational consistency:** The framework aligns well with empirical evidence, particularly in connecting negative effective mass with observed cosmic acceleration and dark matter distributions. This alignment reinforces the theoretical validity of the extended mechanics.

**Comprehensive framework:** By seamlessly incorporating classical mechanics with relativistic and cosmological insights, the research establishes a comprehensive framework for understanding gravitational dynamics. It effectively bridges the gap between traditional mechanics and modern astrophysical phenomena.

**Future research:** The proposed directions promise to expand the extended mechanics framework, offering novel insights into the interdependencies of mass, energy, and motion. These investigations will further integrate classical principles with contemporary physics, enriching both fields.

**Closing statement**

**Extended classical mechanics:** Vol-1-Equivalence principle, mass and gravitational dynamics represents a transformative contribution to classical mechanics. By incorporating rest energy, dark matter, dark energy, and their interrelationships, the research extends traditional mechanics to address new astrophysical phenomena. The alignment with observational evidence and the focus on future exploration ensure the framework's relevance and adaptability, advancing our understanding of both classical and modern physics.

## Funding

No specific funding was received for this work.

## Potential Competing Interests

No potential competing interests to declare.

## References

1. Chernin AD, Bisnovatyi-Kogan GS, Teerikorpi P, et al. Dark energy and the structure of the Coma cluster of galaxies. Astron Astrophys. 2013;553:A101.
2. Greiner W. Classical mechanics: systems of particles and Hamiltonian dynamics. New York: Springer; 2003.
3. Hawking SW, Ellis GF. The large scale structure of space-time. Cambridge University Press; 2023.
4. Randall L. Dark matter and the dinosaurs: The astounding interconnectedness of the universe. Random House; 2017.
5. Weinberg S. Adiabatic modes in cosmology. Phys Rev D. 2003;67(12):123504.
6. Thakur SN. Advancing understanding of external forces and frequency distortion: Part 1. Qeios. 2024.
7. Thakur SN, Bhattacharjee D. Phase shift and infinitesimal wave energy loss equations. 2023.
8. Weinberg S. The quantum theory of fields. Cambridge university press; 1995.