Experimental research on the hydraulic power take-off system of a two-body floating-point wave energy absorber

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ABSTRACT

This paper presents our recent study on the Dragon I which is a two-body floating-point absorber (FPA) wave energy conversion (WEC) device. The device operates in heave and generates energy from the relative motion between the two bodies. In order to solve the instability of power output, we use a hydraulic system which has the energy storage effect as power generation system. A mathematical model of the accumulator was established without considering the impact of the connecting pipeline; In addition, a series of land experiment was conducted based on the prototype machine, further verify the stabilizing effect of the accumulator to the periodic motion generated power; The resulted showed that the accumulator can achieve the function of “peak clipping fill valley”, and the effects could significantly influence the power output.

KEYWORDS

Wave energy; Modelling; Hydraulics; Gas accumulator; Floating-point absorber.
INTRODUCTION

Ocean waves are a huge, largely untapped energy resource, and the potential for extracting energy from waves is considerable. Despite being discussed in patents since the late 18th century[1], modern research into harnessing energy from waves was stimulated by the emerging oil crisis of the 1970s[2,3]. With global attention now being drawn to climate change and the rising level of CO2, the focus on generating electricity from renewable sources is once again an important area of research.

To convert wave energy into useful power, a wide variety of WEC designs have been proposed. These include oscillating water columns, bottom-hinged pitching devices, floating pitching devices, overtopping devices, and point absorbers. The history and the status of the technology development of WEC systems were comprehensively reviewed in Refs.[4,5]. The point absorber is regarded as one of the simplest WEC devices, and the energy capturing of the device is designed to be at its maximum when the system is close to resonance. A two-body floating-point absorber (FPA) system named Dragon I is the focus of this paper, which converts wave energy into electrical power based on the relative motion between the two bodies.

In this paper, we presented the hydrodynamic analysis and a linear mechanical mode of Dragon-I; the frequency domain analysis of the system under the conditions of regular waves had been completed; the working principle of the hydraulic system was described in detail; a mathematical model of the accumulator in the hydraulic system was established without considering the impact of the connecting pipe; Finally, the terrestrial experiment for the entire system verified the regulatory role of the accumulator.

![Figure 1: Schematic representation of the Dragon I](image)

MECHANICAL MODELS

The hydrodynamics of Dragon I

The buoy heaves along the vertical direction (x direction, horizontal plane: x=0), when it is considered as a simple single-degree-of-freedom model. The control equation of the body oscillations is

\[ m \ddot{x} = f_h(x) + f_m(x) \]  \hspace{1cm} (1)

In which, \( m \) is the weight of the buoy, \( \ddot{x} \) is the acceleration, \( f_h \) is the wave force, \( f_m \) is the force used to generate electricity (the PTO force). If the amplitude of the wave and the buoy is small (which meets the requirements of the linear system), usually the wave force can be seen as follows

\[ f_h = f_d + f_r + f_h^s \]  \hspace{1cm} (2)

In which, \( f_d \) is the diffraction force, \( f_r \) is the radial force, \( f_h^s \) is the hydrostatic pressure (\( f_h^s = 0 \), when x=0). In a linear system, it can be expressed that \( f_h^s = -\rho g \dot{x}_s \), where \( \rho \) is the density of the sea water, g is the gravitational acceleration and \( s \) is the cross-sectional area of the buoy in static water.

Frequency domain analysis

The diffraction force is a sinusoidal function of time \( \dot{x} \) when the frequency of the regular wave is \( \omega \). Besides we supposed that the power take off system was linear, so the PTO force could be represented as

\[ f_m = -Kx - C\dot{x} \]  \hspace{1cm} (3)

Where, \( -K \) represents the action of the spring and \( -C \) represents the damping which is associated with the energy absorption. (\( K \) and \( C \) is constant)

In this way, the system can be seen as completely linear. When the incident wave is regular, the related displacement and force are sinusoidal functions of time \( t \), and they can be expressed as:
\[ x(t) = R e(x_0 e^{j\omega t}), \quad f_R(t) = Re(F_R e^{j\omega t}) \]  

(4)

In which, \( x_0, F_R \) are complex amplitude, \( Re(\cdot) \) is the real part. Since the system is linear, \( F_R \) is proportional to the height of the incident wave \( x_0 \) and expressed as

\[ |F_R| = |\Gamma(\omega)|x_0 \]

(5)

Where, \( \Gamma(\omega) \) is the exciting force coefficient.

Radial force \( f_R \) can be conveniently represented as

\[ f_R(t) = -A(\omega)x_0 - B(\omega)x \]

(6)

Where, \( A(\omega) \) is the added mass coefficient, \( B(\omega) \) is the damping coefficient of the radiation. As the heaving motion of the buoy is along the vertical symmetrical axis, the exciting force coefficient and the damping coefficient of the radiation can be expressed each other as follow based on the literature\(^{[6,7]}\).

\[ \Gamma(\omega) = \left( \frac{2g^2 B(\omega)}{a^2} \right) \]

(7)

The added mass coefficient \( A(\omega) \) and the damping coefficient of the radiation \( B(\omega) \) are related to the geometrical characteristic of the buoy and the frequency \( \omega \), and they can be calculated by commercial software WAMIT.

Under the condition of linear system, the control equation of the body oscillations is

\[ (m + A)\ddot{x} + (B + C)\dot{x} + (\rho g S + k)x = F_R e^{j\omega t} \]

(8)

We can take \( x = x_0 e^{j\omega t} \) into Eq.8:

\[ x_0 = \frac{-\omega^2 (m + A) + j\omega (B + C) + \rho g S + k}{\omega^2 (m + A) + j\omega (B + C) + \rho g S + k} \]

(9)

HYDRAULIC SYSTEM

As shown in Figure 2, the high pressure oil generated from the relative motion between the piston and the cylinder flows out from the left side through two group of the one-way valve, and the oil is absorbed from the tank trough the one-way valve on the right. The energy accumulator is used to "peak clipping fill valley", so the motor can move smoothly. The safety valve controls the highest working pressure of the system, so that the motor and pipeline are protected. The throttle regulates the flow of the motor to steady by rotation speed of the motor.

When the cylinder rod moves downward relative to the cylinder, the volume of the hydraulic cylinder's up cavity becomes larger and the pressure comes down. A differential pressure from the atmospheric pressure is generated. When the differential pressure rises to the opening pressure of one-way valve 2, the one-way valve will be open, and the hydraulic oil will go into the up cavity of the hydraulic cylinder. We call it the oil suction. At the same time, the volume of the hydraulic cylinder's down cavity becomes smaller and the oil pressure comes up. When the pressure rises to the opening pressure of the oil suction one-way valve 3, high-pressure oil drives the hydraulic motor 10 through the one-way valve 3.

Finally the generator 12 works with the hydraulic motor and generates electricity. When the cylinder rod moves upward relative to the cylinder, the working process is contrary to the former, the down cavity absorbs oil through the one-way valve 2 and discharges, the down cavity discharge oil through the one-way valve 3. The oil drives hydraulic motor 10, and turns the generator 12 to produce electricity. The role of the accumulator 6 is to "peak clipping fill valley cut": when the high pressure oil comes out from the hydraulic cylinder, the accumulator can store part of the energy, and also reduce the impact on the power system from the instantaneous high pressure; In the same way, when the output of the hydraulic cylinder is less and the pressure of the oil is smaller, the accumulator can release some energy and maintain power systems working properly. So that the use of accumulator makes the power system work steadily and also improves the rate of the energy utilization.

WAVE ENERGY CONVERTER
Most of wave energy conversion devices adopt the hydraulic cylinder as the power output device, and the relative movement between two shocking floating body drives the relative movement between the hydraulic cylinder piston and cylinder. In particular cases, the hydraulic cylinder piston or cylinder is fixed at the bottom of the ocean or the coast with only one shocking floating body. Assume one of the simplest case that floating body has only one degree of freedom-heaving along x axis of the Cartesian coordinate system.

Figure 1 shows the schematic representation of the system. Denote by \( P_u \), \( P_d \), \( P_1 \) and \( P_2 \) the pressure of the up side and down side of the hydraulic cylinder, the high side and low side of the hydraulic motor, respectively. At first, analysis of the situation when the piston moves upward with \( P_u \equiv P_1 \) and \( P_d \equiv P_2 \) the volume flow rate can be expressed as \( q = \frac{c d x}{d t} \), where \( c \) refers to the effective area of the hydraulic cylinder and \( x \) is the displacement of the piston. Assume that the process is a one-dimensional flow, then

\[
1 \text{- hydraulic oil container; 2- oil-absorbing one-way valve; 3-oil outlet one-way valve ;4-pressure gage; 5-globe valve; 6-energy accumulator; 7-isolating valve; 8-pressure gage; 9-safety valve; 10-hydraulic motor; 11-throttle valve; 12-generator; 13-hydraulic cylinder}
\]

![Schematic representation of the hydraulic system](image)

Figure 2: Schematic representation of the hydraulic system

\[
p_1 - p_2 = p_2 - p_1 - k_u q^2 - l \frac{dq}{dt}
\]

(10)

Where, \( k_u \) is the pressure loss coefficient caused by friction in circuits and \( l \) is the inertia coefficient when the liquid flows. Similarly, when the piston moves downward, it can be represented as

\[
p_1 - p_2 = p_2 - p_1 - k_d q^2 - l \frac{dq}{dt}
\]

(11)

Regardless of the upward or downward movement of the piston, it can be expressed by the following general formula at \( k_u = k_d = k \).

\[
p_1 - p_2 = \Delta p - k q^2 - l \frac{dq}{dt}
\]

(12)

As shown in Figure 1, the movement of the floating body makes the high pressure oil of hydraulic cylinder into the circuit. Hydraulic motor is driven by the pressure difference between the both side, \( p_1(t) = p_2(t) \), which changed with time flies because of two reasons as follows:(i) the movement of the floating body attached to the piston (ii) the flow of fluid through the hydraulic motor. Simplify the process by ignoring the liquid inertia and pipe pressure loss of hydraulic circuits then we got the Eq.12 \( l \equiv k \equiv 0 \). Therefore, \( |p_1 - p_2| = |p_1 - p_2| \) when the piston moves. (The inertia of the fluid in the hydraulic circuits can be replaced by a quality model, which can be added on the quality of the floating body in Eq.1)

The radial force \( F(t) \) in Eq.4 can be got by the coefficients of the spectrum of waves and the excitation force coefficient \( f(t) \). When the floating body moves, control equation would refer to Eq. 3, where \( f = F \), \( \dot{s} = s(t) \), \( \dot{s} = \frac{d}{dt} \), \( \dot{d} = \frac{d}{dt} \) and \( \rho \) is the effective area of hydraulic cylinder. Sometimes, the velocity of the floating body, changing
over time, will be zero. At this time, the floating body remains still until the hydrodynamic force act on the floating body

\[ f_0(t) = mg + \int_{-a}^{a} L(t - r) \delta(t) \, dr \]

, overcomes the resisting force \( F = F_0 \left( y_1 - y_2 \right) \). Then the hydraulic oil can be pumped to the accumulator.

The instantaneous power of the wave energy converter can be represented as \( P(t) = \phi \left| x(t) \right| \), and the time-average power in \( t_0 \leq t \leq t_1 \) is \( \bar{P} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} P(t) \, dt \). The value of the average power depended on the time interval \( \Delta t = t_f - t_0 \). Practically, in order to ensure error on \( \pm 10\% \), \( \Delta t \) must be less than fifteen minutes.

Within a sufficiently long time, it is generally believed that \( \bar{P} = \bar{P}_m \), where \( \bar{P}_m \) referred to the flow rate through the hydraulic motor. At the same time

**MATHEMATICAL MODEL OF ACCUMULATOR**

Dealing with the mathematical model of the accumulator, we neglected the influence of the connecting line and divided the accumulator into two parts: the gas chamber and the liquid chamber. Then the relationship between the pressure and the flow of the two parts was built to draw the mathematical model completely. In the actual analysis process, we did the stress analysis of the join points-oil chamber between the accumulator and the hydraulic circuit to get the mechanic model of the accumulator. Then according to the relationship of the pressure and flow between different parts, we analysed the mechanic model to get the full mathematical model of the accumulator\[^{[8]}\].

Analysing the force on the oil and gas cavity, we did the following assumption: For each of the different sea conditions, the time-average temperature of the gas in the accumulator could keep same with the surrounding and we could thought of that it was an isothermal process. Compared with the gas, the oil compressibility could be ignored. The oil flowed in the accumulator could be considered as laminar flow.

\[ p_0 v_0^b = p_1 v_1 c^R = \text{constant} \tag{13} \]

We take \( k=1 \) in the process of the isothermal change. When suffered from the pressure of oil cavity, the seal nitrogen gas in the accumulator skins could be simplified as a gas spring-damper model shown in Figure 3, as be compressed in one direction. Based on the analyses of the system the chamber stress model of the accumulator is:

\[ (p_f - p_a) \frac{dA}{d} = \kappa_a \frac{F_c}{A} + c_a \frac{1}{A} V_a \tag{14} \]

Where, \( p_f \) is the oil pressure of the accumulator, \( \kappa_a \) is the gas stiffness coefficient of the skin at any time and \( c_a \) is the damping coefficient of the gas. Set the flow of the accumulator’s import to \( q \), so the change of the flow and the volume change in the oil cavity chamber satisfy \( q = -\frac{dV}{dt} \), which implies that

\[ q = -\frac{P}{c_a} \tag{15} \]

In which the minus sign indicates that the volume of the chamber changes contrary to the oil flow. We take that into the formula above and after the Laplace transform we can get:

\[ (p_f(s) - p_a(s)) \frac{dA}{d} = -\left( \frac{\kappa_a}{c_a s + \frac{c_a}{A}} \right) q(s) \tag{16} \]

The stress analysis of the oil in the accumulator’s oil cavity is carried out: Assumed that the area at the bottom of the bag is equal to the accumulator’s cross section and express it as \( A_0 \); the quality of the oil cavity is \( m_a \); ignore the elastic modulus of the oil, so that the force balance equation of quality of the part is:

\[ (p - p_a) m_a = \frac{\frac{dV}{dt} + A_0 \frac{dP}{dt} + c_a \frac{dV}{dt} + \kappa_a V_a}{A} \tag{17} \]

Where, \( P \) is the pressure of the oil near the oil inlet. When the impact of the connecting line is neglected, \( P \) can be seen as the pressure of the system and \( \frac{dV}{dt} \) is approximate to the oil cavity damping coefficient. We can get the mathematical model for the output of the accumulator’s chamber volume after the Laplace transform
\[G(s) = \frac{V_a(s)}{P_a(s)} = \frac{A_a^2}{m_a s^2 + (B_a + C_a)s + \left(k_a + \frac{k_d A_a^2}{l_a}ight)}\]

(18)

Where, \(P_a\) and \(V_a\) respectively indicates as the initial air pressure and the volume of the accumulator. Then the formula can be reorganized as:

\[G(s) = \frac{A_a^2}{k_a + \frac{k_d A_a^2}{l_a}} \cdot \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}\]

(19)

Where, \(P_c\) and \(V_t\) respectively means the pressure and the volume of the accumulator at any time, \(\omega_n\) is the undamped natural frequency of the accumulator and \(\omega_n = \sqrt{\frac{k_a}{m_a}}\), \(k_a\) is the equivalent spring coefficient of the accumulator’s model and \(\xi = \frac{(B_a + C_a)}{2\sqrt{k_d m_a}}\). \(\xi\) is the gas chamber-oil chamber’s equivalent damping ratio and

It can be seen from the formula above that when the impact of the connecting line is neglected the performance of the accumulator is mainly about the air volume, the oil damping, the air pressure and the cross-section area.

**EXPERIMENT**

In order to verify the role of the accumulator, we did a series of terrestrial experiment using the prototype machines. We used the bridge crane to drag the buoy which drove the hydraulic cylinder piston upward slowly, then maintained a certain traction to the buoy and made the buoy come down slowly relied on the weight of the buoy. Before the experiment it is necessary to drag the float to move up and down several times to guarantee that the accumulator is full of high pressure oil. When the experiment began, we recorded the time that the buoy went up, down and the generator worked when the buoy stopped and also the change of the pressure of the system in the above process. The average pressure is shown in TABLE 2. We used the resistance whose rated power consumption is 25 kW, and the opening pressure of the accumulator was set to 6Mpa. Here was the process: moved up and down two times and then stopped and the schedule was 1.2m. The data acquisition was used to record the single-phase and three-phase total voltage, current and power during the terrestrial experiment. The data is shown in Figure 4, Figure 5, Figure 6 and Figure 7.

**TABLE 1**: Experiment parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stroke of Hydraulic Cylinder (m)</td>
<td>1(m)</td>
</tr>
<tr>
<td>Diameter of piston rod</td>
<td>130 (mm)</td>
</tr>
<tr>
<td>Diameter of cylinder</td>
<td>160 (mm)</td>
</tr>
<tr>
<td>Accumulator capacity</td>
<td>40 (L)</td>
</tr>
<tr>
<td>Accumulator pressure</td>
<td>6 (Mpa)</td>
</tr>
<tr>
<td>Gear motor displacement</td>
<td>100 (ml/r)</td>
</tr>
<tr>
<td>Nominal pressure/Max</td>
<td>12.5/20 (Mpa)</td>
</tr>
<tr>
<td>Motor speed/(range) (r/min)</td>
<td>2000/150–2500</td>
</tr>
<tr>
<td>Three-phase alternator rated speed</td>
<td>1500 (r/min)</td>
</tr>
<tr>
<td>Nominal voltage</td>
<td>380 (V)</td>
</tr>
<tr>
<td>Rated power</td>
<td>35 (kW)</td>
</tr>
</tbody>
</table>

The result shows that at the time the hydraulic cylinder changes from rising state to a state of decline the voltage and the current reduces a little and basically remains stable, the power increased gradually as the energy saving of the accumulator, the sustainable time that the generator works rely on the accumulator’s power only when the hydraulic cylinder stops can be up to 31.6s. The effect of the accumulator’s energy storage is obvious while the maximum pressure in the system is only 9Mpa. If the pressure reached to 20Mpa in the actual work, the effect will be more obvious.
TABLE 2: Time and pressure in experiments

<table>
<thead>
<tr>
<th>Motion states</th>
<th>Time (s)</th>
<th>Pressure (Mpa)</th>
<th>Time-average pressure (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upward</td>
<td>18.4</td>
<td>5.5-7</td>
<td>6.5</td>
</tr>
<tr>
<td>Downward</td>
<td>14.7</td>
<td>6.5-7.5</td>
<td>7</td>
</tr>
<tr>
<td>Upward</td>
<td>16.7</td>
<td>7.5-8.5</td>
<td>8</td>
</tr>
<tr>
<td>Downward</td>
<td>12.5</td>
<td>7.5-9</td>
<td>8.5</td>
</tr>
<tr>
<td>After stop motion</td>
<td>31.6</td>
<td>8-5.5</td>
<td>6.75</td>
</tr>
</tbody>
</table>

Figure 4: The voltage of single phase

Figure 5: The electric current of single phase

Figure 6: The power of single phase

Figure 7: The total power of three phase

CONCLUSION

This paper makes a brief introduction to the binary point floating wave power generation device and establishes a simple mathematical model of single degree of freedom under the linear wave conditions. This paper also analyses the hydraulic system of the device and establishes a mathematical model for the accumulator. We also complete a series of land experiments, which indicates that the accumulator makes significant differences to the adjustment of the pressure of the system.

The hydraulic circuit is very important for the whole wave energy conversion device. On one hand, the hydraulic circuit is limited to the maximum load, pressure, available space and equipment cost; On the other hand, the design of the circuit will affect the both sides of the energy conversion chain; the dynamic performance of absorbing water and the performance of the electrical equipment. In order to solve these problems, this paper establishes the relationship between them, which provides design basis for the designers.

Although a relatively simple geometric model has been established (oscillating floating body), this design method can be extended to more complicated situations, such as systems of multi-degree of freedom.

It should be noted that linear wave theory is the basis of the hydrodynamic theory, with which we can extract energy from waves in this paper. Besides, the linear wave theory does not include the practical situations of wave of high size and large vibration of floating body, what’s more, the problem of the actual fluid viscous dissipation is also ignored. In the future, these limitations will be dealt with in effective ways.
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