Excitation of surface plasmons in multilayer structures containing alternate \( \varepsilon \)-negative and the \( \mu \)-negative metamaterials

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**ABSTRACT**

The localized surface waves at the interface between a semi-infinite periodic multilayer structure containing alternate \( \varepsilon \)-negative (ENG) and the \( \mu \)-negative (MNG) layers and a uniform right-handed material and also a uniform left-handed metamaterial have been investigated. We demonstrated that in the presence of uniform right-handed materials, this system can support TE and TM-polarized surface waves depending on the relative thicknesses of the ENG and the MNG layers. But, in the presence of uniform left-handed metamaterials, the supposed structure only support TM-polarized surface waves. These localized modes can have two different transverse structure related to the order of the ENG and the MNG layers; one with a hump at the interface between uniform material and the cap layer and the other one with a hump at the interface between the cap layer and the periodic multilayer structure. © 2015 Trade Science Inc. - INDIA

**KEYWORDS**

Surface plasmons;  
Left-handed metamaterials;  
Single-negative metamaterials.

**INTRODUCTION**

Recently, metamaterials as specially engineered media with unconventional response functions has attracted a great deal of attention. Particularly, media in which both of the material parameters, permittivity (\( \varepsilon \)) and permeability (\( \mu \)), can attain negative real parts in a certain frequency band, have been studied by numerous groups (see e.g., [1]). When both material parameters possess negative real parts, such double-negative (DNG) media can support wave propagation and exhibit the unusual phenomenon of negative refraction. Besides the DNG materials, we can also have materials in which only one of the two material parameters \( \varepsilon \) and \( \mu \) is negative [2]. These so-called single negative (SNG) materials support evanescent wave in order to maintain positive definite energy density. Because \( \varepsilon \) and \( \mu \) are frequency dependent, only within a certain frequency range we have \( \varepsilon < 0 \) and \( \mu > 0 \) (epsilon-negative) or \( \varepsilon > 0 \) and \( \mu < 0 \) (mu-negative), which is called the SNG frequency range. The DNG and SNG metamaterials, formed by embedding arrays of metallic split-ring resonators and wires in a host medium [2], have been successfully constructed in the microwave regime by several groups, and some of their unusual properties (e.g., negative refraction) have been experimentally demonstrated [11]. All these artificial composites (including DNG and SNG materials) have exhibited special features in photonic crystals (PCs) [3–6]. The essential property of PCs is the photonic band gap (PBG) structure originated from the consequence of Bragg scattering. Such a Bragg gap in conventional PCs is strongly
dependent on the lattice constant, and the incident angle and polarization of light, and is affected by disorder of devices. Jiang et al.\cite{5} found that the ENG-MNG multilayered periodic structures can possess a new type of photonic band gap (called the effective zero-phase gap), which is distinct from the Bragg gap but similar to the zero average index gap\cite{6–8}. Such a zero-phase gap is an omnidirectional band gap and is insensitive to the incident angles and polarizations of the incident light\cite{9–11}.

The excitation of surface waves (SWs) has recently been proposed\cite{12–14} as a way to efficiently inject light into a PC waveguide, or to extract a focused beam from a channel. SWs on a 1DPC were observed almost 30 years ago\cite{15,16}. The basic theory was developed at that time by Yariv and Yeh\cite{17,18}. The effect of varying the thickness of the termination layer has been measured experimentally\cite{19} and a sensor based on the properties of SWs has been proposed and demonstrated\cite{20}. In parallel, numerical calculations for SWs in the band gaps have been performed\cite{21–27}.

In this paper, we study the properties of linear SWs at the interface between uniform right-handed (RHM) materials (and LHM medium) of refractive index, $n_0 = \sqrt{\varepsilon_0 \mu_0}$ and a semi-infinite 1DPCs containing SNG materials. We assume that each cell of PCs consists of the ENG-MNG or the MNG-ENG layers with the thickness $d_i$, relative permittivity $\varepsilon_i$ and permeability $\mu_i$ ($i = 1, 2$) (see Figure 1). Now, we suppose that relative permittivity and permeability in the ENG materials are given by\cite{3}

$$\varepsilon_i = 1 - \frac{\omega_p^2}{\omega^2}, \quad \mu_i = a,$$

(1)
and those in the MNG materials are given by

\[ \varepsilon_i = b, \quad \mu_i = 1 - \frac{\omega_{\text{mp}}^2}{\omega^2}, \]

(2)

where \( \omega \) is the angular frequency, \( \omega_{\text{ep}} \) and \( \omega_{\text{mp}} \) are the electronic plasma frequency and the magnetic plasma frequency, respectively. The frequency \( \omega \) in the above equations is measured in \( 10^9 \) rad/s. Here both positive constants \( a \) and \( b \) are assumed to be \( a=b=3.0 \). Both \( \omega_{\text{ep}} \) and \( \omega_{\text{mp}} \) are set to be \( 10 \times 10^9 \) rad/s [5]. In the frequency range of \( \omega < \omega_{\text{ep}}, \omega_{\text{mp}} \), either \( \varepsilon \) or \( \mu \) is negative in each layer, that is, to say we have SNG materials. So, in the SNG frequency range of \( \omega \), we have \( \varepsilon_i < 0, \mu_i > 0 \), and \( \varepsilon_2 > 0, \mu_2 < 0 \) for the periodic structure consists of the ENG-MNG layers. While, for a periodic structure consists of the MNG-ENG layers \( \varepsilon_1 > 0, \mu_1 < 0 \), and \( \varepsilon_2 < 0, \mu_2 > 0 \). Hence, in each layer the refractive index is a pure imaginary number and the electromagnetic fields are evanescent. As shown in Figure 1, the crystal is capped by a layer of the same material but different width, \( d_c \) as an adjusting parameter. We consider the propagation of TE-polarized waves described by [24]

\[ E = E_z(z)e^{i \phi - \varepsilon_i z}, \]

\[ H = (H_z(z)e^{i \phi - \mu_i z}), \]

(3)

with the electric field \( E \) in the y direction (the dielectric layers are in the x-y plane and the z direction is normal to the interface of each layer) (see Figure 1). Here \( k = \omega/c \) is the vacuum wave number.

Surface modes correspond to localized solutions with the field decaying from the interface in both directions. In the left-side homogeneous medium (semi-infinite LHM or RHM) \( z < d_c \) the fields are decaying provided \( \beta > \varepsilon_0 \mu_0 \). In the right-side periodic structure, the waves are the Bloch modes,

\[ E(z) = \psi(z) \exp(i \kappa_B z), \]

(4)

where \( \kappa_B \) is the Bloch wave number, and \( \psi(z) \) is the Bloch function which is periodic with the period of the photonic structure. In the periodic structure the waves will be decaying provided that \( \kappa_B \) is complex, and this condition defines the spectral gaps in a finite PC. For the calculation of the Bloch modes, we use the well-known transfer matrix method [18]. To find the SMs, we take solutions of Eq. (4) in a homogeneous medium and the Bloch modes in the periodic materials and satisfy the conditions of continuity of the tangential components of the electric and magnetic fields at the interface between the homogeneous medium and periodic structure [28]. In this way, we can obtain the exact dispersion relation \( \omega = \omega(\beta) \) for TE-polarized SWs by numerically solving the following dispersion condition for the SMs:

\[ \begin{align*}
q_i / \mu_i = -i & \lambda - A - B \overline{\lambda - A + B} \\
\end{align*} \]

(5)

Here

\[ \lambda = \frac{A + D}{2} \pm \sqrt{\left(\frac{A + D}{2}\right)^2 - 1}, \]

\[ A = e^{i \phi} (\cosh(k, d) + 1/2(x + 1/x) \sinh(k, d)), \]

\[ B = 1/2 e^{i \phi} \sinh(k, d)(x - 1/x) \sinh(k, d), \]

\[ D = e^{i \phi} (\cosh(k, d) - 1/2(x + 1/x) \sinh(k, d)), \]

(6)

where \( k_1 = k/\sqrt{\beta^2 - \varepsilon_i \mu_1} \),

\[ k_2 = k/\sqrt{\beta^2 - \varepsilon_2 \mu_2}, \]

\[ q_0 = \sqrt{\beta^2 - n_0^2}, \]

and \( x = k_2 / \mu_2, k_1 / \mu_1 \).

These equations are valid only for the specific condition of \( \varepsilon_1 < 1, \mu_1 > 1, \varepsilon_2 > 1, \mu_2 < 1 \) or \( \varepsilon_1 > 1, \mu_1 < 1, \varepsilon_2 < 1, \mu_2 > 1 \).

**RESULTS AND DISCUSSION**

In the following, we summarize the dispersion properties of the SMs in the second SNG band gaps which are in the microwave range suitable for the SNG metamaterials. In Figure 2, we plotted the dispersion curves of TE-polarized SMs on the plane of \( \omega \) versus the propagation constant \( \beta \) for different thickness of the cap layer (\( d_c \)). Here we used the structure with \( d_{\text{ENG}} = 0.6 \) cm and \( d_{\text{MNG}} = 0.8 \) cm. It is seen that the dispersion curve of TE-polarized SMs for a given \( d_c \) depends on the type of arrangement of SNG layers: ENG-MNG arrangement (Figure 2(a)); MNG-ENG arrangement (Figure 2(b)). Specially, for thick cap layer the difference is obvious (see solid lines in Figure 2). As it is seen from Figure 2(a), we can have nearly omni-directional dispersion curve in the ENG-MNG arrangement, whilst in the MNG-ENG arrangement; disper-
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Figure 2: Dispersion property of the TE-polarized SWs at the second SNG band gap for different thicknesses of the cap layer $d_c = 0.01d_1$ (dotted lines), $d_c = 1d_1$ (dashed lines) and $d_c = 3d_1$ (solid lines) in (a) the ENG-MNG and (b) the MNG-ENG arrangements. The unshaded regions show the first and the second band gaps of the periodic multilayer structure with SNG constituents, while the shaded regions indicate the corresponding pass bands. In the used structure $d_{ENG} = 0.6 \text{ cm}$, $d_{MNG} = 0.8 \text{ cm}$ and $d_{ENG}/d_{MNG} < 1$.

Figure 3: The transverse profile of the SWs vs coordinate z for (a) $d_c = 1d_1$ in the ENG-MNG arrangement, (b) $d_c = 0.01d_1$ in the MNG-ENG. Here $\beta = 1.5$, $\omega = 5 \text{ GHz}$ and the other parameters are same as Figure 2.

Figure 4: Total energy flow of surface Tamm modes vs. $\beta$ in the ENG-MNG. Dotted, dashed, and solid curves show the energy flow of the surface modes for $d_c = 0d_1$, $0.3d_1$, and $0.8d_1$, respectively.
Figure 5: Existence regions for the SMs; the modes exist in the shaded regions for (a) the ENG-MNG arrangement and (b) the MNG-ENG arrangement. The other parameters are the same as the Figure 2.

Figure 6: Dispersion properties of the TM-polarized surface modes for (a) ENG-MNG and (b) MNG-ENG periodic structures. Unshaded regions show the zero-$\phi_{\text{eff}}$ spectral gap of the 1D PC containing SNG materials. Dotted, dashed, and solid curves show the dispersion of the surface modes for $d_c = 0.2d_1$, $0.8d_1$, and $2d_1$, respectively.

From Figure 3, we see that the peak of the electric field is appeared at the interface of the cap layer and the 1DPC in the ENG-MNG arrangement, while in the case of MNG-ENG arrangement the peak of the electric field is located at the interface of the uniform medium and the cap layer. Here, the unusual structure of SMs comes from sharp jumps at the interface of layers resulting from opposite signs of permeability of adjacent layers and decaying evanescent waves in each layer.

By further inspection of dispersion curves we see that the dispersion curves have positive slopes. As the slope of the dispersion curve determines the corresponding group velocity of the mode and the direction of energy flow at the surface, we see that in our structure all modes are forward modes (see Figure 4).

The type of arrangement of the structure not only affects the dispersion curves of SMs, but also it changes the existence regions of TE-polarized SMs. To demonstrate this, we plotted the existence regions of the SMs at the second SNG band gaps on the parameter plane $(d_c, \beta)$ in Figure 5 in which the shaded areas show the existence regions of the TE-polarized SMs. From Figure 5 we can find that the considered structure can support SMs for almost every $\beta$ and $d_c$ in the ENG-MNG arrangement. But, the existence region of SMs is restricted to a narrow domain of incident angles for relatively large $d_c$ in the MNG-ENG arrangement. The similar discussions can be offered for TM wave.

Up to now, we discussed the property of polarized SMs in the considered structure with RHM layer. Then, we theoretically study SWs that can be excited at the interfaces between a semi-infinite uniform LHM medium and a semi-infinite 1DPC containing two types of single-negative materials (ENG-MNG and MNG-ENG). As we know, the Tamm states exist in the gaps of the PBG spectrum (unshaded regions in Figure 5).
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Figure 7: Total energy flow of surface Tamm modes vs. $\beta$ in the (a) ENG-MNG and (b) MNG-ENG periodic structures for different $d_c$. Dotted, dashed, and solid curves show the energy flow of the surface modes for $d_c = 0.2d_1$, $0.8d_1$, and $2d_1$, respectively.

Figure 8: The dependence of the zero-$\Phi_{eff}$ PBG and surface modes on the ratio of $d_2/d_1$ for (a) ENG-MNG (b) MNG-ENG structures. The dotted and solid lines correspond to the surface modes with $d_c = 0.2d_1$ and $d_c = 1.5d_1$, respectively. Here, $\beta = 1.9$, and the other parameters are the same as the Figure 1.

Since our studies show that we have only TM-polarized SWs for proposed structure we turn our attention to the TM-polarized SWs and present dispersion characteristic of SWs for different values of the cap layer thickness $d_c$. For ENG-MNG and MNG-ENG multi-layered structures in Figures 6(a) and 6(b), respectively. It is necessary to remember that in MNG-ENG structure the position of ENG and MNG layers in ENG-MNG structure is exchanged with the same geometry and physical parameters. As one can see from Figure 6, there are different dispersion curves for different values of $d_c$, which describe a possibility to control the dispersion properties of SWs by adjusting $d_c$. Corresponding values of $d_c$ for dotted, dashed, and solid curves of dispersion are $d_c = 0.2d_1$, $0.8d_1$, and $2d_1$, respectively. Moreover, Figure 6 shows that, in ENG-MNG structure, there are no limitation on the existence region of TM surface modes for large $d_c$, whilst in the case of MNG-ENG structure the existence region of TM surface modes decreases by increasing the thickness of cap layer $d_c$ (see dashed and solid lines in Figure 6(b)).

Energy flow of surface Tamm states for ENG-MNG and MNG-ENG structure has the same behavior as the negative values. To demonstrate this, in Figure 7, we plot total energy flow as a function of the wave number $\beta$. We see from Figure 7 that all surface modes in ENG-MNG and MNG-ENG periodic structure have negative energy flow, thus they are backward for different values of cap layer thicknesses $d_c$.

In Figure 8, we study the dependence of surface modes on the ratio of the thicknesses of two SNG layers ($d_2/d_1$) for (a) ENG-MNG (b) MNG-ENG structures. The dotted and solid lines correspond to the surface modes with $d_c = 0.2d_1$ and $d_c = 1.5d_1$, respec-
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But, in the presence of uniform left-handed metamaterials, the supposed structure only support TM-polarized surface waves. These SMs depending on the arrangement of the layers of PC (MNG-ENG or ENG-MNG) can have two different transverse structures of field at the second SNG band gap of the PC. In the ENG-MNG arrangement the peak of SMs are located at the interface between the cap layer and the photonic crystal. But, in the MNG-ENG arrangement, the peak of SMs is located at the interface between uniform material and the cap layer. We have demonstrated that in the presence of a LHM layer there are only backward TM-polarized waves, while in the presence of a RHM layer there are forward waves. More interestingly, we demonstrated that the occurrence of the TE-polarized and the TM-polarized surface waves depends on the ratio of the thicknesses of the ENG layer to the MNG layer.

REFERENCES


