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Evolutionary game theory model and its application in the credit market

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ABSTRACT

This paper first reviews the development history of game theory, and find out the difference of evolutionary game theory and classical game theory, and the reason of evolution game theory. Through the static analysis to find a can make the group behavior restrictions issued to a consistent ESS in a strategy, and extended to the dynamic analysis method, to solve more practical problems. And this paper uses the evolution game theory to analyze the problem of lending banks and enterprises, and to find a stable point, that is to choose the best strategy, is also the bank to loan, enterprise can not insist on credit. And for this kind of social disadvantage which the enterprises to credit loans, banks do not adhere to the loan guarantee find the right solutions. It is through government regulation of bank loans to enterprises; let the lending bank and enterprise development to the positive direction.

KEYWORDS

Game theory; Evolutionary game theory; Lending problems.



OVERVIEW OF THE GAME THEORY

In twentieth Century 50, the game theory as a practical theory of knowledge spreading out, but game theory emphasizes the interaction between the individual and the individual, early economists did not attract attention. Until 70 years later, economists gradually realized the importance of the interaction between individuals, so the game theory has been the world's attention. Then the rise of behavioral economics, "rational" concept began to promote, the original game theory vehicle can be slightly improved, evolutionary game theory was born.

Evolutionary game theory, corrected the entire individual traditional game theory can make the best strategy for rational hypothesis. In reality, the individual decisions with their own wisdom, knowledge and ability are related. Is not a simple repeated game can be obtained, because individuals because of its rationality are limited, so the individual can not clear Game Analysis of the whole process. Therefore, the evolutionary game theory emphasizes that the whole process of game is to rely on long-term performance and living standards to achieve, not by reasoning abilities of individual.

MODEL ASSUMPTIONS

- 1) Assume that there is a continuum of members of the group, and the individual can two matching game.
- 2) Assume the public policy for the, gain matrix is.
- 3) Assume that is a discrete time.

EXPLANATION OF SYMBOLS

a_{ij}	An individual with s_i strategy, the opponent by s_j benefit value
$\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n) \in \Delta^{n-1}$	Mixed strategy
$x(t) = (x_1(t), x_2(t), \dots, x_n(t)) \in \Delta^{n-1}$	Group of state
$x_i(t)$	The probability of individual selection strategy
$v(t)$	The total number of groups
$v_i(t)$	Select the s_i membership
$S_b = \{S_{b1}, S_{b2}\}$	Bank strategy
S_{b1}	To guarantee loans
S_{b2}	Do not adhere to guarantee loans
$S_c = \{S_{c1}, S_{c2}\}$	Business strategy
S_{c1}	Adhere to the credit loan
S_{c2}	Don't insist on credit
M_b	The bank did not reach the credit contract and enterprise losses
M_c	The enterprise does not reach the credit contract with the bank 's loss
N_b	Bank and enterprise credit contracts agreed profit
N_c	The enterprise and the bank a credit contract profit
Q	Extra profit of enterprise contract
λ	Discount coefficient
$X_{b1}(t)$	Banks using the probability of S_{b1}
$X_{c1}(t)$	Enterprise using the probability of S_{c1}

ESTABLISHING AND SOLVING OF THE MODEL

Evolution of static analysis method of game theory

Firstly, using static analysis method, the stationary points found in the process of evolution game.

$$\sigma A \sigma' = \sum_{i=1}^n \sigma_i \left[\sum_{j=1}^n a_{ij} \sigma'_j \right] = \sum_{i=1}^n \sum_{j=1}^n a_{ij} \sigma'_j \sigma_i \tag{1}$$

Among them, $\sigma A \sigma'$ said the individual choice of mixed strategy σ , opponents σ' expected payment. So the evolutionary stable strategies in the process can be defined as, a mixed strategy $\sigma \in \Delta^{n-1}$, if $\forall \sigma' \neq \sigma, \exists \varepsilon > 0$ made $0 < \varepsilon \leq \bar{\varepsilon}$

$$\sigma A[(1-\varepsilon)\sigma + \varepsilon\sigma'] > \sigma' A[(1-\varepsilon)\sigma + \varepsilon\sigma'] \tag{2}$$

So $\sigma \in \Delta^{n-1}$ is said to be stable strategy.

The evolutionary game theory of classical game theory assumes the correct equivalent to refine on Nash equilibrium, therefore, let σ be an ESS of a random matching and game, available

$$\forall \sigma' \in \Sigma, \sigma A \sigma' \geq \sigma' A \sigma' \tag{3}$$

$$\forall \sigma' \in \Sigma (\sigma' \neq \sigma), \sigma A \sigma' > \sigma' A \sigma' \tag{4}$$

Similarly, if a particular strategy $A \sigma \in \Sigma$ satisfies (3) and (4), he belongs to an ESS.

Dynamic analysis of game theory

The above knowable, evolutionary game theory is correct and complete rationality hypothesis, and that the individual is limited rational game. This paper has found a way to make the group behavior restrictions issued to a consistent ESS in a strategy, but in the stationary state as the premise. But it can't solve the dynamic state of ESS. Therefore this paper introduces a dynamic analysis in the evolution process.

The sum of a number of members of S_i in the t moment should equal the total number and member of the group members of the product probability selection strategy for S_i , i.e.

$$v_i(t) = v(t)x_i(t) \tag{5}$$

A member of the group can only survive for a time, then each member leaves depend on the number of offspring in their respective adaptability. The total number of members, choose strategy S_i at $t+1$ moment for

$$v_i(t+1) = v_i(t) \left[\sum_{j=1}^n a_{ij} x_j(t) \right]_{(i=1,2,\dots,n)} \tag{6}$$

$$x_i(t+1) = \frac{v_i(t+1)}{v(t+1)} = \frac{x_i(t)v(t) \left[\sum_{j=1}^n a_{ij} x_j(t) \right]}{\sum_{u=1}^n x_u(t)v(t) \left[\sum_{j=1}^n a_{uj} x_j(t) \right]} \tag{7}$$

$$\frac{\Delta x_i(t)}{x_i(t)} = \frac{x_i(t+1) - x_i(t)}{x_i(t)} = \frac{\sum_{j=1}^n a_{ij} x_j(t) - x(t)Ax(t)}{x_i(t)Ax(t)} \tag{8}$$

From the formula available the probability of any strategy S_i change rate equal to $\sum_{j=1}^n a_{ij} x_j(t)$ minus

$$x(t)Ax(t), \text{ among them } x(t)Ax(t) = \sum_{u=1}^n x_u(t)v(t)[\sum_{j=1}^n a_{uj} x_j(t)].$$

If you set t as the time parameter is continuous, let $\theta \in (0,1]$ denote the duration of each time period of t , then

$$\begin{aligned} \square \frac{x_i(t)}{x_i(t)} &= \frac{1}{x_i(t)} \lim_{\theta \rightarrow 0} \frac{x_i(t+\theta) - x_i(t)}{\theta} \\ &= \sum_{j=1}^n a_{ij} x_j(t) - x(t)Ax(t), t \geq 0, (i = 1, 2, \dots, n) \end{aligned} \tag{9}$$

In the formula $x_i(t) \neq 0$, $x_i(t) = 0$ said group members selection probability S_i strategy for 0, so the type (9) adapted into

$$\square \dot{x}_i(t) = x_i(t) [\sum_{j=1}^n a_{ij} x_j(t) - x(t)Ax(t)], t \geq 0, (i = 1, 2, \dots, n) \tag{10}$$

APPLICATION OF EVOLUTIONARY GAME THEORY MODEL

After the economic crisis, the economy of our country has a large influence, resulting in the supply and demand of funds of enterprises in our country is out of balance. On the other hand, due to the lack of reliable credit standards, the bank has refused to lend May. Evolutionary game theory is able to clear carries on the analysis to the enterprise and bank lending. Enterprises to the bank loan guarantees and credit loans. In order to reduce the cost of loans, most enterprises choose credit loans, and banks to reduce their risk, more inclined to guarantee loans. Because of the differences, it leads to the game. It need long time to evolution and adjustment to reach a mutual interest best lending. The paper will use the evolutionary game theory to analyze this. Figure 1 is a game tree banks and enterprises.

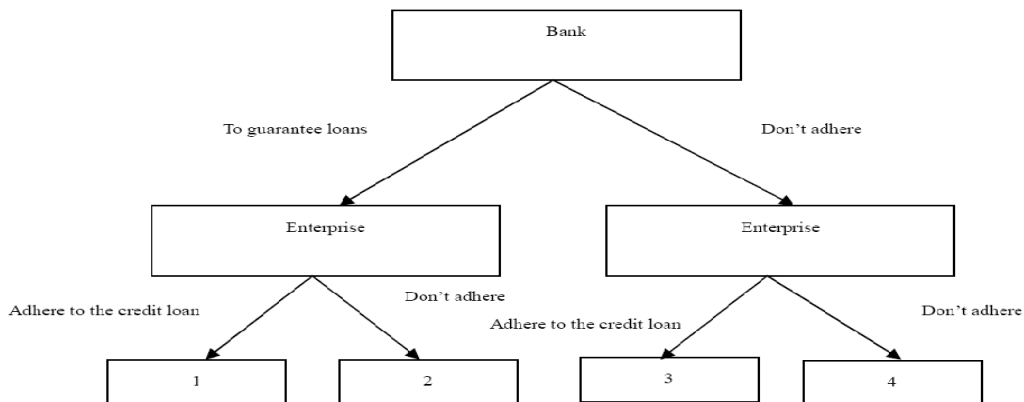


Figure 1 : The game tree banks and enterprises

TABLE 1 is the gain matrix about bank and enterprise game.

TABLE 1 : The gain matrix about bank and enterprise game.

		Bank	
		S_{b1}	S_{b2}
Enterprise	S_{c1}	$-M_c, -M_b$	$N_c\lambda + Q, N_b - Q$
	S_{c2}	$N_c\lambda, N_b$	$N_c\lambda + \frac{Q}{2}, N_b - \frac{Q}{2}$

From the table we can see, when the credit enterprise hope, the hidden danger of default. When the bank and enterprises do not want to credit loans, the possibility of corporate default is reducing.

Gain matrix for the bank

$$A_b = \left\{ \begin{matrix} -M_b & N_b \\ N_b - Q & N_b - Q/2 \end{matrix} \right\}$$

Gain matrix for the enterprise

$$A_c = \left\{ \begin{matrix} -M_c & N_c\lambda + Q \\ N_c\lambda & N_c\lambda + Q/2 \end{matrix} \right\}$$

The Dynamic equation is that

$$\dot{X}_{b1}(t) = X_{b1}(t)[1 - X_{b1}(t)]\left[\frac{Q}{2} - (M_b + N_b - \frac{Q}{2})X_{c1}(t)\right] \tag{11}$$

$$\dot{X}_{c1}(t) = X_{c1}(t)[1 - X_{c1}(t)]\left[\frac{Q}{2} - (M_c + N_c - \frac{Q}{2})X_{b1}(t)\right] \tag{12}$$

So, when $X_{b1}(t) = 0, X_{c1}(t) = 0, (0,0), (0,1), (1,0), (1,1)$ are equilibrium points between bank and enterprise. According to the stability matrix to analyzed the local stability of this equilibrium points. Ask $X_{b1}(t)$ for $X_{b1}(t)$ partial derivative, and ask $X_{c1}(t)$ for $X_{c1}(t)$ partial derivative. The matrix is

$$Y = \begin{bmatrix} \frac{\partial \dot{X}_{b1}(t)}{\partial X_{b1}(t)} & \frac{\partial \dot{X}_{b1}(t)}{\partial X_{c1}(t)} \\ \frac{\partial \dot{X}_{c1}(t)}{\partial X_{b1}(t)} & \frac{\partial \dot{X}_{c1}(t)}{\partial X_{c1}(t)} \end{bmatrix} = \begin{bmatrix} (1 - 2X_{b1}(t))\left[-M_c - N_c\lambda - Q - N_c\lambda\right]X_{c1}(t) + N_c\lambda + Q & X_{b1}(t)(1 - X_{b1}(t))(-M_c - N_c\lambda - Q - N_c\lambda) \\ X_{c1}(t)(1 - X_{c1}(t))(-M_b - N_b) & (1 - 2X_{c1}(t))\left[N_b + (-M_b - N_b)X_{b1}(t)\right] \end{bmatrix}$$

TABLE 2 for the local stability of the equilibrium point.

Form the table we can see, (0,0) and (1,1) are unstable points, (0,1) is the Saddle point, and (1,0) is the stable point. Figure 2 is the strategy evolution diagram.

Have above knowable, after a long evolution, gradually to the two convergence strategy. To the point (1,0) said the bank adheres to guarantee loans, enterprise can not insist on credit, more reasonable. To the point (0,1) said companies adhere to credit loans, banks do not adhere to guarantee loans, unreasonable. The reason is that, the enterprises may have suspected breach, and the banks to safeguard

their own interests are likely to refuse to loan, make the enterprise into a crisis. The best solution is to support the bank lending to enterprises by government regulation, let the lending bank and enterprise development to the positive direction.

TABLE 2 : For the local stability of the equilibrium point

the equilibrium point $(X_{b1}(t), X_{c1}(t))$	$\det Y$	$tr Y$	The stability
(0,0)	$(N_c \lambda + Q) \cdot N_b$	$N_c + N_c \lambda + Q$	The unstable point
(1,0)	$(-M_c - N_c \lambda) \cdot -N_b$	$-M_c - N_c \lambda - N_b$	stable point
(0,1)	$-(N_c \lambda + Q) \cdot N_b$	$-N_c \lambda - Q + N_b$	Saddle point
(1,1)	$(M_c + N_c \lambda) \cdot M_b$	$M_c + N_c \lambda + M_b$	The unstable point

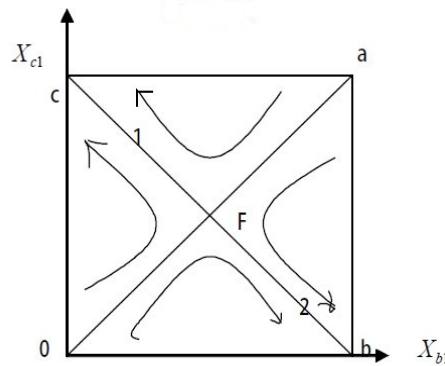


Figure 2 : The strategy evolution diagram

CONCLUSION

This paper first reviews the development history of game theory, and find out the difference of evolutionary game theory and classical game theory, and the reason of evolution game theory. Through the static analysis to find a can make the group behavior restrictions issued to a consistent ESS in a strategy, and extended to the dynamic analysis method, to solve more practical problems. And this paper uses the evolution game theory to analyze the problem of lending banks and enterprises, and to find a stable point, that is to choose the best strategy, is also the bank to loan, enterprise can not insist on credit. And for this kind of social disadvantage which the enterprises to credit loans, banks do not adhere to the loan guarantee find the right solutions. It is through government regulation of bank loans to enterprises; let the lending bank and enterprise development to the positive direction.

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