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## Estimating the bubble size distribution characteristics in foam images

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## Abstract

This paper proposes a statistic model for estimating the bubble size distribution in foam images based on uniform scanning and edge analysis. It also presents the algorithm for calculating the bubble size distribution of circular bubbles as well as oval bubbles, complete with a weighted statistical algorithm of bubble size distribution under the condition of multi-directioned uniform scanning. The simulation results indicate that the widths of the bubble's section lines and bubble sizes are identically distributed, and the estimated number of bubbles in the foam image is similar to the actual value. © 2014 Trade Science Inc. - INDIA

# **K**EYWORDS

Ege detection; Fam image; Uiform scanning; Fature recognition.

## INTRODUCTION

Foam images are widely used in such many fields as chemical industry, metallurgical operation, water conservancy, shipping and nuclear industry, etc.<sup>[1-4]</sup>. It is a very important measure to judge the manufacturing state in the related industries by automatically detecting the bubble features and classifying them by using computeraided image processing technology so as to realize the industrial procedure control and to optimize the decision-making<sup>[5-7]</sup>.

There are two main existing methods used to analyze the foam size distribution. One is based on image segmentation. Yang Chunhua et al have researched on a foam image segmentation technology and made a quantitative description of the foam shape plus its size features, which offers the data support for the flotation process<sup>[8]</sup>. Shao Jianbin et al applied watershed method

to segment the foam images but couldn't effectively solve the segmentation problem<sup>[9]</sup>. Zeng Rong et al applied foam edge scanning method to segment the images, resulting in a problem of under-segmentation<sup>[10]</sup>. Such methods enjoy a high recognition rate of image features, but its algorithm is complex and the real-time performance is not good. The other major type of method is based on foam feature determination. Bartolacci used gray level concurrence matrix and wavelet analysis to extract foam patterns, and applied the least square method to establish an empirical model of the concentrate ore grades, thus realizing a feedback control over the flotation process<sup>[11]</sup>. Moolman proposed a method to obtain the average size of the copper ore flotation foam based on image color analysis and fast Fourier transform algorithm, but the method cannot be used to match the flotation conditions<sup>[12]</sup>. These feature determination algorithms are relatively simple, with a good

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real-time performance; but the calculation is rough with bigger errors, thus lacking determination of the essential foam characteristics.

In response to the deficiency of the existing methods to determine the bubble size features, this paper proposes an algorithm of determining the bubble size features based on statistics through an edge scanning of the bubbles to establish a bubble combination model which is applied to estimate the number of bubbles as well as their size distribution by using statistical approaches.

## BUBBLE EDGE SCANNING OF THE FOAM IMAGE

Flotation process is a method of separation widely used in the wastewater treatment and mineral processing industries. An efficient method to automtic monitoring flotation process is to capture its foam image on time. Figure 1 shows the alluminum flotation process with image sensor. A foam image consists of a number of bubbles in a background, and Figure 2 shows a typical foam image of alluminum flotation process. Generally, workers determine condition of alluminum flotation process by estimating the distribution of bubble size and number<sup>[13]</sup>.

The foam image is scanned between even intervals from a certain angle to produce a number of scan lines, each of which might penetrate various bubbles. At the same time, a bubble might be penetrated by various scan lines, with its width in direct proportion to the number of scan lines. If the scanning interval is smaller than the width of the smallest bubble, then a bubble must be crossed by at least a scan line according to the smallest drawer principle.

Figure 3 shows a frame of typical foam image, which is greyfied and then evenly scanned vertically.

The scan line goes across different bubbles, and the number as well as the widths of bubble section lines can be calculated through a bubble edge detection of scan lines.



Figure 1 : Aluminum flotation process with image sensor



Figure 2 : A typical foam image



Figure 3 : Vertical scan lines of the foam image

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## STATISTIC MODEL FOR CALCULATING THE BUBBLE SIZE DISTRIBUTION OF FOAM IMAGES

The shapes of bubbles mainly fall into two categories— circular or oval. We can establish a statistical model for calculating the bubble size distribution according to bubble section line widths and their number which are scanned evenly in different directions.

#### Statistical model for scanning circular bubbles

Suppose the smallest diameter of the recognizable bubble is  $2 \cdot e$ .

According to the diameters, the bubbles are classified into "m" types with the diameter of each type of bubbles being

$$(i+2) \cdot e > R_i \ge (i+1) \cdot e \tag{1}$$

Where  $i = 1, 2, \dots, m$ .

Similarly, we classify the evenly scanned bubble section lines into "m" types, and the width of every type of section line is:

$$(i+2) \cdot e > L_i \ge (i+1) \cdot e \tag{2}$$

Where i = 1, 2, ..., m.

Figure 4 shows the situation of type "i" circular bubbles being scanned vertically. The height of the rectangular  $A_j B_j C_j D_j$  (*j*=1, 2, 3, 4,...)is  $(j+1) \cdot e$ , so the smallest section line width crossing the rectangular is

$$(j+1) \cdot e$$
, i.e., the section line is of the type  $k(k \ge j)$ ,



Figure 4 : Vertical scanning of the circular bubble

and the rectangular width is just the effective width of ttype  $k(k \ge j)$  section lines being scanned vertically.

If  $D_{ii}$  (*i* = 1, 2, ..., *m*; *j* = 1, 2, ..., *i*) is set as the ef-

fective width of  $k(k \ge j)$  section lines of type "i" bubbles, then

$$D_{ij} = \sqrt{(i+1)^2 - (j+1)^2} \cdot e$$
 (3)

Where  $i = 1, 2, \dots, m; j = 1, 2, \dots, i$ .

 $D_{ii} \approx 0$ , but we set  $D_{ii} = 0$  for a convenient calculation.

If  $P_{ij}$  ( $i = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, i$ ) is set as the effective width of type "j" section lines of type "i" bubbles being scanned, then

$$\begin{cases} P_{i1} = D_{i1} - D_{i2} = \left(\sqrt{(i+1)^2 - 4} - \sqrt{(i+1)^2 - 9}\right) \cdot e \\ \dots \\ P_{ij} = D_{i,j-1} - D_{ij} = \left(\sqrt{(i+1)^2 - j^2} - \sqrt{(i+1)^2 - (j+1)^2}\right) \cdot e \\ \dots \\ P_{ii} = D_{i,i-1} - D_{ii} = \sqrt{2 \cdot i + 1} \cdot e \end{cases}$$
(4)

Where  $i = 1, 2, \dots, m; j = 1, 2, \dots, i$ .

If the scanning density is set as *d*, then when an type "i" bubble is scanned, the expected times of the width of the section line being the same as that of type "j" section line will be:

$$\begin{cases} E_{i1} = \frac{P_{i1}}{d} = \left(\sqrt{(i+1)^2 - 4} - \sqrt{(i+1)^2 - 9}\right) \cdot \frac{e}{d} \\ \dots \\ E_{ij} = \frac{P_{ij}}{d} = \left(\sqrt{(i+1)^2 - j^2} - \sqrt{(i+1)^2 - (j+1)^2}\right) \cdot \frac{e}{d} \\ \dots \\ E_{ii} = \frac{P_{ii}}{d} = \sqrt{2 \cdot i + 1} \cdot \frac{e}{d} \end{cases}$$
(5)

Where  $i = 1, 2, \dots, m; j = 1, 2, \dots, i$ .

# Calculation of the size distribution of circular bubbles

Through uniform scanning, a certain number of bubbles will be segmented by different scan lines, or on the other way round, every bubble is approximate to

1323

the combination of several section lines. If the number of bubbles in the foam image *G* is  $U_i$  ( $i = 1, 2, \dots, m$ ), then the number of different types of section lines is  $V_i$  ( $i = 1, 2, \dots, m$ ).so,

$$V_i = \sum_{j=i}^m E_{ij} \cdot U_j$$
(6)
Where  $i = 1, 2, \dots, m$ .

 $E_{ij}$  Is the known coefficient. We can get the algorithm formula of  $U_i$  from formula (6):

$$\begin{cases}
U_{m} = \frac{V_{m}}{E_{mm}} \\
U_{m-1} = \frac{V_{m-1} - P_{m,m-1} \cdot U_{m}}{E_{m-1,m-1}} \\
\cdots \\
U_{i} = \frac{V_{i} - \sum_{j=i+1}^{m} P_{ij} \cdot U_{j}}{E_{ii}} \\
\cdots \\
U_{1} = \frac{V_{1} - \sum_{j=2}^{m} P_{2j} \cdot U_{j}}{E_{11}}
\end{cases}$$
(7)

It's easy to solve formula (7) so in turn we can get the product  $U_i (i = m, m-1, \dots, 1)$  and it is the mere one. The product of  $U_i$  cannot be negative, or the result would be impractical. Taking into account of the fact that the bubbles are impossible to be standard circular, we should permit an error in the calculation, so that  $U_i$  can have a non-whole-number product. But the error shouldn't be too big, i.e., the value of  $U_i$  can't be too small. Therefore,  $U_i$  should be confined to  $U_i \ge \lambda$ , in which  $\lambda$  refers to the bubble's approximate calculation threshold value in the range[0.7, 0.9]. The process of solving the formula is as follows:

Step 1: suppose i = m.

Step 2: if  $i \le 0$ , then we turn to step 8 and then the process is over.

Step 3: if 
$$\frac{V_i}{E_{ii}} < \lambda$$
, then  $V_{i-1} = V_{i-1} + V_i$ ,  $V_i = 0$ ,  $U_i = 0$ ,  
 $i = i - 1$ , and we go to step 2.



(a) Angle of 0° (b) Angle of 45°



(c) Angle of 90° (d) Angle of 135° Figure 5 : Multi-dimension scanning and edge analysis of a foam image

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Step 2 and 3 will be repeated until  $\frac{V_i}{E_{ii}} \ge \lambda$  or the process is over.

cess is over.

Step 4: suppose  $W = \frac{V_i}{E_{ii}}$ , Step 5: calculate  $W_j = V_j - P_{j+1,j} \cdot W \ (j = 1, 2, \dots, i-1)$ . If

 $W_j < 0$  ( $j = 1, 2, \dots, i-1$ ), then W = W - 1, and we turn to step 7

Step 6: if 
$$U_i = \frac{V_i - \sum_{j=i+1}^m P_{ij} \cdot U_j}{E_{ii}}$$

 $V_j = W_j (j = 1, 2, ..., i - 1), i = i - 1$ , we turn to step 2.

Step 7: if  $W \ge 0$ , then we turn to step 5; otherwise,  $V_{i-1} = V_{i-1} + V_i$ ,  $V_i = 0$ ,  $U_i = 0$ , i = i - 1, we will turn to step 2.

Step 8: process over.

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### Statistical model for scanning oval bubbles

Suppose the shape of bubbles fit general elliptic equations:

$$a \cdot x^{2} + b \cdot x \cdot y + c \cdot y^{2} - 1 = 0$$
 (8)

Suppose the foam image is scanned *m* times with slope  $k_i$  (i = 1, 2,..., m), and  $h_i$  (*i*=1, 2,..., *m*)section lines are obtained. Then

$$x_i = \frac{h_i \cdot d \cdot \sin(k_i + \frac{\pi}{2})}{2} \tag{9}$$

$$y_i = \frac{h_i \cdot d \cdot \cos(k_i + \frac{\pi}{2})}{2} \tag{10}$$

Calculate the elliptic curve by least squares method<sup>[14]</sup>. Let  $\theta$  be the angle between the long axis and the vertical direction (y-axis) and  $\omega$  be eccentricity. Then

$$\theta = \frac{1}{2} \cdot \arctan \frac{b}{a-c} \tag{11}$$

$$\omega = \sqrt{\frac{2}{1 + \frac{a+c}{\sqrt{(a-c)^2 + b^2}}}}$$
(12)

# Statistic model for calculating the size distribution of oval bubbles

If the bubbles are circular, then the total number of bubbles being scanned from any angle will always be the same.

If the bubbles are oval, then the total number of bubbles being scanned from different angles will also be different. If the scanning direction is the same as the long oval axis then the total times of bubbles being scanned would be the smallest with a largest average width of the section line while the estimated value of bubbles would be fairly low with a fairly too big average bubble size. If the scanning direction is the same as the short oval axis, the total times of bubbles being scanned would be the largest with a smallest average width of the section line while the estimated number of bubbles would be too high with the bubbles' average size being relatively small.

If the number of bubbles scanned along the long oval axis is  $\mu$ , and the number of bubbles scanned along the short oval axis is  $\nu$ , and the foam image is evenly scanned from different angles for k times, we calculate according to the method suggested in part 3.2 and get the bubble size distribution which is respectively  $\zeta_{ij}$  ( $i = 1, 2, \dots, k$ ,  $j = 1, 2, \dots, m$ ), then  $\mu \leq \zeta_{ij} \leq \nu$  ( $i = 1, 2, \dots, k$ ,  $j = 1, 2, \dots, m$ ). So we can



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get the estimating formula for calculating the number of bubbles in a foam image as is shown below:

$$\overline{\varsigma} = \frac{1}{k} \cdot \sum_{i=1}^{k} \sum_{j=1}^{m} \varsigma_{ij}$$
(13)

The weighted bubble size distribution of the foam image is:

$$\tilde{\varsigma}_{ij} = \frac{\overline{\varsigma}}{k} \cdot \frac{\sum_{i=1}^{k} \varsigma_{ij}}{\sum_{j=1}^{m} \varsigma_{ij}}$$
(14)

## SIMULATED CALCULATION

The foam image in Figure 1 is evenly scanned from directions of  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ , and  $135^\circ$  with a interval of h=10, and the smallest threshold value of the section line's width is d= 10. The edge test results are shown in figure 5.

Which the white point, is the dividing point of section lines. The number of section lines obtained from sub graph (a), (b), (c), (d) are 1383, 1161, 1095, 1218.

With equation (8), (9), (10), (11), (12), we can estimate the fitted oval curve as shown in Figure 6.

The numbers of bubbles obtained from 4 different scanning angles are respectively 72.1351, 58.4746, 58.5100, 71.0316, and the bubble size distribution is shown in Figure 7.

According to the distribution state, we can see that the more big bubbles there are in the foam image, the



more long section lines; while the fewer big bubbles, the fewer long section lines. So the section line's width distribution also reflects the bubble size distribution.

Figure 8 indicates a high degree of similarity between the section line's width distribution curve and the bubble size distribution curve. This turns out to accord with the actual state.



The estimated number of bubbles is 65.0378, which is nearly the same as 66 which represents the artificially-calculated number of the bubbles.

#### CONCLUSION

Multi-dimension even scanning is made on the foam image so as to get the section line width distribution, and the bubble size distribution can be deduced according to statistical principles. The simulation results indicate a high degree of similarity between the section line's width distribution curve and the bubble size distribution curve.

Weighted calculation is made on the bubble size distribution scanned from different directions so as to get the results of the bubble size distribution of foam image with a simulation result indicating the estimated bubble size distribution of the foam image approximates to the actual value.

This paper proposes an accurate algorithm model for calculating the bubble size distribution based on even scanning and edge detection. The amount of calculation is small, while the results are satisfactory. So it can

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be well applied in such related fields as online monitoring and industrial control of the bubble size distribution characteristics of foam images.

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