Epidemic modeling using data from the 2001-2002 measles outbreak in Venezuela

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ABSTRACT

We present a methodology that is based on a well-known epidemiological model (the SIR model) and which will lead to a critical parameter that can be later employed to analyze the actual occurrence of an epidemic. The parameter can be used to specify whether an epidemic is currently in existence and that a mass vaccination campaign should be commenced or if it were just a misinterpretation of the data and a mass vaccination campaign would not be justified. As a confirming example, weekly data from the outbreak of measles that occurred in Venezuela during 2001-2002 is examined using this technique.

INTRODUCTION

Among the many diseases that have had deleterious effects on human kind over the centuries, measles is certainly one of the worst. According to historical records, epidemics of measles have occurred quite frequently. Herein we suggest an analytical technique that is based upon the well-known SIR model to analyze epidemics and find that it could possibly lead to a predictive parameter which can be used to indicate whether a local or global immunization campaign should be commenced. In the SIR model, S stands for the fraction of the population that is susceptible to acquiring the disease, I stands for the fraction of the population that is infected by the disease, and R stands for the fraction of the population that has recovered from the disease.

Recalling that despite of substantial worldwide efforts to eradicate measles, 39.9 million cases and 777,000 deaths, primarily among young children in Africa and Southeast Asia, occurred during 2000[1]. Currently there are significant outbreaks of measles at the present time. For example, Ukraine has recently reported 10,386 cases media has reported 1286 cases according to recently published data in the CDC news[2]. In Venezuela, the first documented outbreak of measles occurred in 1993-1994 and involved 38,000 cases and 124 deaths[3-5]. Because of this outbreak, a national vaccination campaign for children from 6 months to 14 years old was conducted throughout the country. A measure of the success of the campaign is that only one documented death attributable to measles occurred during 1995, 4 cases in 1998 and none in 1999. However, an outbreak of 22 cases involving preschool-age children occurred during 2000. In 2001-2002, another outbreak occurred, the first case being an adult male (age 39) in Falcón. This individual had previously visited Europe in August 2001 which at that time was enduring a measles
epidemic. His brother subsequently contracted the disease in September 2001. The disease spread to the city of Maracaibo and later to Falcón, Maracaibo, Trujillo, Mérida, Lara and other localities\(^5\). Using data from the Venezuelan epidemic and analyzing it with the model introduced by Kermack and McKendrick\(^7\), a new parameter will be obtained that could have important predictive consequences. A mathematical analysis of this model has also appeared\(^8\).\(^9\)\(^10\).

A problem facing an epidemiologist is to identify potential outbreaks occurring in localized regions in a given geographic area before the disease becomes a national epidemic or a worldwide pandemic. Therefore, the proposed methodology has been derived that allows one to determine whether an outbreak is an epidemic or not according to the resolution of the differential equations of the SIR model.

**MATERIALS AND METHODS**

The SIR model is defined by the following equations\(^11\):

\[
\begin{align*}
\frac{dS}{dt} &= -\beta SI \\
\frac{dI}{dt} &= \beta SI - \nu I \\
\frac{dR}{dt} &= \nu I
\end{align*}
\]

(1)

The coefficient \(\beta\) represents the rate of infection of the susceptible population and \(\nu\) represents the rate of recovery of the infected population. In a particular epidemic, it is reasonable to assume that the total population \(N\) would have a constant value. Accordingly one can write that, \(S = N - I - R\). This expression assumes that no changes in the birth or death rates are included in the analysis. Therefore, it is possible to reduce the three equations to the following two equations

\[
\begin{align*}
\frac{dR}{dt} &= \nu I \\
\frac{dI}{dt} &= \beta(N - I - R)I - \nu I
\end{align*}
\]

(2)

These two equations can be combined to yield

\[
-\frac{\beta}{\nu} \left( \frac{dR}{dt} \right)^2 = \frac{d}{dt} \left[ \frac{1}{2} \beta R^2 - (\beta N - \nu) R \right]
\]

(3)

As has been noted before\(^13\), we also makes the assumption that the left-hand side of this equation is small and can be set equal to 0. This assumption is in agreement with the supposition that the rate of removal of the recovered population is small in comparison with the total recovered population. Under this assumption, equation (3) may be integrated and rewritten as:

\[
\frac{dR}{dt} + \frac{1}{2} \beta R^2 - (\beta N - \nu) R = 0
\]

(4)

The constant of integration is equal to zero because the coefficients are equal to zero before the onset of the epidemic. This equality holds because we only consider the time at which the infectious process begins. The solution of (4) is\(^13\)

\[
R = \frac{\lambda}{\beta} [1 + \tanh(\lambda(t - t_0))]
\]

(5)

The rate of removal per unit time is

\[
\frac{dR}{dt} = \frac{\lambda^2}{\beta} \sech^2 \left( \lambda(t - t_0) \right)
\]

(6)

Therefore, it is possible to examine the infected population using the equation

\[
\sech^2 \left( \lambda(t - t_0) \right)
\]

(7)

where \(a = \lambda^2/\beta\).

Using this equation, we examined the published measles data for Venezuela from 2001 to 2002\(^3\)\(^4\). We analyze this data using (7) where \(a\) represents the amplitude of the epidemic equal to \(\lambda^2/\beta\), \(t_0\) represents the time of the maximum of the epidemic, \(\lambda\) is equal to \(\beta N - \nu\) and \(\sech\)\(^2\) is the hyperbolic secant squared. From the data, it is possible to obtain numerical values for the parameters \(\alpha\), \(\lambda\) and \(t_0\).

**RESULTS**

Figure 1 displays temporal evolution on a weekly basis of the number of cases of measles that occurred during the 2001-2002 epidemic in Venezuela. The line represents the best fit of equation (7) with the data from which the following values are obtained. \(a = 190\), \(\lambda = 0.1\) and \(t_0 = 10\). The numerical values obtained from the data are \(a = 201.635\), \(\lambda = 0.188\) at \(t_0 = 10.2\) which corresponds to week 10 of 2001. This model does not provide any indication of subsequent epidemics which may occur significantly later in time.

The \(\lambda\) parameter indicated the growth rate of the disease, a property analogous to Malthusian growth. In addition, we know that \(\lambda = \nu (R_0 - 1)\) and that \(\nu\) is 1/8, the inverse of the average latent period\(^12\)\(^14\). Therefore, \(R_0\) will be equal to \(\lambda/\nu + 1 = 0.188/0.125 + 1 = 2.504\). Because this value is greater than one, it implies that
any member of the population could be infected with measles before being vaccinated. This result indicates that it certainly was justified to perform the mass vaccination in the entire country during 2001 and 2002.

The importance of determining the probability of the occurrence of an epidemic is that this information can help to isolate and possibly eradicate a localized episode. According to the analysis of Figure 1, the measles outbreak in Venezuela during the outbreak 2001-2002 was endemic and should have been eradicated very rapidly.

Velasco et al.[15] analyzed the HIV epidemic and considered the eradication of HIV in terms of the numerical value of the average number of new infections caused by an infected individual $R_0$. If $R_0$ is less than one, it would be possible to eradicate the epidemic. As $R_0$ increases beyond the value of one, the epidemic becomes more severe. The current study found an $R_0$ value of 2.17 for the measles epidemic. From this numerical value, we conclude that it was necessary for the authorities to vaccinate everyone in order to decrease the probability of a measles epidemic in Venezuela.

Published values of $R_0$ vary between 10 and 20[16]. However, the simplicity of the approach taken in this study does not allow the analyst to discriminate between the infected and non-infected populations. In addition, we have made assumptions concerning certain parameters in the derivation of equation (4). For these reasons, the numerical value of $R_0$ determined in this study was lower than the published values. In fact, these comments agree with the observations made by Bjornstad et al.[17] in explaining the episodes of measles that occurred in England.

DEDICATION

This work is dedicated to all those who have died of measles in the world.

CONCLUSIONS

It is important that a country conduct mass immunization campaigns against measles in order to prevent local outbreaks from spreading geographically throughout that particular and possibly reach pandemic proportions into neighboring countries. In order to avoid another pandemic comparable with that which occurred in Europe and because countries are not isolated from each other, it is necessary to develop a monitoring system with real-time geographic information for the early detection of localized episodes of diseases such as measles. The procedure introduced in this paper should provide an additional tool for the epidemiologists as they monitor the spreading of various diseases.

In Venezuela, a measles epidemic occurred in 2001 after the outbreak in Europe. At the present time and under similar conditions, it is important to continuously monitor the onset and progression of the disease in order to prevent an epidemic from occurring later in either the short or the medium terms. For this reason, it is necessary to quantify the parameters of the epidemic of diseases such as measles in order to avoid another possible pandemic.

REFERENCES

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