Dynamics principle-based analysis and teaching on basketball shooting act

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ABSTRACT

The paper firstly makes research on kinematical equation after basketball shooting and gets that basketball flight distance and ball initial speed are in direct proportion, and ball flight distance will change as athlete release angle changes, when player shooting release angle is 45°, ball flight distance is the farthest. Subsequently analyze athlete shooting instant arms dynamics principle; it gets shooting moment each joint speed relations. And then from Lagrange equation, it establishes constraint particle dynamics equation, solves basketball momentum is up to athlete shooting instant wrist joint momentum, and gets when basketball player shoots, player’s release angle should let arms and horizontal line included angle to remain around 45°-50°, when ball is about to enter into rim, its included angle with horizontal line should be 50°-55° that is most proper. Now, player’s right shoulder joint and horizontal line instantaneous angle is 142°.

KEYWORDS

Kinematic principle; Rotational inertia; Basketball shooting; Dynamical equation; Physiology and anatomy.

INTRODUCTION

Basketball is one of the earliest American competitive events, it is originated from 1890s that was created by a young teacher in Field at that time and he was James Naismith. And inspiration that he created basketball was from local children favorite activity that shot the ball into peach basket, while subsequently he compiled basketball such sports event, which solved students in school couldn’t take outside training in cold winter such difficulty at that time. In the beginning basketball stipulated one team appeared five people in field, recorded basketball shot into rim as one score, and at that time rim was the one with bottom closed, every time shooting, they should take ball out by ladder, subsequently it improved bamboo rim with bottom closed as live bottom iron rim. In 1892, it regulated that holding ball running was not allowed in the field, and rude acts also forbidden as well as others thirteen rules, while till 1893, basketball then added backboard and net, and it was introduced to China in 1910.

Basketball as a kind of team competitive event, mainly scoring ways is roughly divided into three kinds, shooting, layup and dunk. Shooting is player’s uppermost scoring way in the field, and shooting has very high requirements on player’s technique and team tactics; layup is favorite shot way by most players, because layup hit rate is very high and layup act is full of artistic feelings; and dunk is the biggest defensive difficulty shot way, it is up to player holds ball and jumps, then directly dunks it into rim. With technological progress, basketball has also been rapidly developed, nowadays players’ heights are becoming higher and higher, basketball technique is also improving, which
lets player attack and defense technique to be also con-
stantly improved. The paper will research on basket-
ball shooting act, analyzes its techniques and finds out
correct shooting postures.

MODEL ESTABLISHMENT AND SOLUTION

Kinematics analysis when basketball is shot

After basketball player shooting basketball, the bas-
ketball will make parabolic motion in the air under grav-
ity effects, by kinematical principle, it is clear that when
the ball makes parabolic motions, it will have a initial
horizontal speed \( V_0 \cos \alpha \) and ball horizontal direc-
tion can be regarded to make constant motion, vertical di-
rection initial speed is \( V_0 \sin \alpha \), and regard it makes
upcast motions. Therefore basketball kinematical equa-
tion is:

\[
X = V_0 \cos \alpha \ t \\
H - H_0 = V_0 \sin \alpha \ t - \frac{1}{2} g t^2
\]

\( V_0, \alpha \) are respectively basketball initial speed and
angle after player shooting the ball, rim height is \( H \),
release height is \( H_0 \), basketball flight journey is using \( X \)
to express. Combine with above formula; it can get bas-
ketball flight journey and initial speed as well as release
angle and other factors relationship

\[
X = \frac{V_0^2 \sin \alpha \cos \alpha + V_0^2 \cos \alpha \sqrt{V_0^2 \sin^2 \alpha - 2g(H - H_0)^2}}{g}
\]

According to above formula, it can get that basket-
ball flight distance will increase with initial speed \( V_0 \)
increases, and it can get basketball shot moment differ-
ent release angles and flight journey relations, as Figure
1 show.

By above Figure 1, it is clear that when ball initial
speed is defined, ball flight distance will change with
player release angle changes, and when player shoot-
ing release angle is \( 45^\circ \), ball flight distance is the far-
thest.

Basketball player shooting moment dynamics
analysis

Now it can regard basketball player shooting used
arms big arm and small arm as two rigid bodies with
different volumes, and establish \( T, T_1 \) and \( T_3 \) points
into three freedom degree model, as Figure 2 show.

\( T, T_1 \) and \( T_3 \) points are respectively basketball
player shoulder joint, elbow joint and wrist joint, big
arm and small arm anatomical angles are \( \varepsilon_1, \varepsilon_2, L_1, L_2 \)
are lengths of arms. Set that \( T \) point and \( T_1 \) point
trivector is \( \lambda_1, \lambda_2 \), which is basketball player shooting
arms big arm and small arm actual angular speed,

\( \varepsilon_2 \) is \( T_1 \) point speed, that: \( \varepsilon_2 = \lambda_2 - \lambda_1 \).

When basketball player is shooting, basketball
shooting initial speed has relations with wrist shooting
moment instantaneous speed, and \( T_3 \) angular speed is
affected by shooting arms big arm and small arm angu-
lar speed as well as \( T_1 \) angular speed. So \( T_3 \) speed is
correlated to its relative speed and \( T_1 \) speed, expression is:\[ V(T_1)_G = \varepsilon_1 \times D_1 = \varepsilon_1 \times D_1, \]
\[ V(T_1)_L = \varepsilon_2 \times D_2. \]

\( V(T_1)_G \) is \( T_1 \) actual speed vector, \( V(T_1)_L \) is \( T_3 \) point relative to \( T_1 \) point speed, \( D_1 \) is \( T \) to \( T_1 \) position vector, \( D_2 \) is \( T_1 \) to \( T_3 \) position vector. According to vector theorem, it solves \( L_1 \) and \( L_2 \) partial motions to \( T \) impact:
\[ T_{3G} = \varepsilon_1 \times D_1 + \varepsilon_2 \times D_2 + \varepsilon_1 \times D_3 + \varepsilon_2 \times \varepsilon_1 \times D_3, \]
Simplify and get:\[ T_{3G} = T_{3G} \times \varepsilon_1 + \varepsilon_2 \times D_2. \]

\( T_{3G} \) is \( T_3 \) point position vector in reference system, \( T \) point leads to \( T_1 \) point generate speed as \( \dot{\varepsilon}_1 \times D_1 \), \( \varepsilon_2 \times D_2 \) is \( T_1 \) point leads to \( T_3 \) point generated speed. For Figure 2 model \( T \) and \( T_1 \) point angle as well as \( T_3 \) point position relations, it decomposes and writes as:
\[ \begin{align*}
    x_p &= D_1 \cos \varepsilon_1 + D_2 \cos(\varepsilon_1 + \varepsilon_2) \\
    y_p &= D_1 \sin \varepsilon_1 + D_2 \sin(\varepsilon_1 + \varepsilon_2) \\
    z_p &= D_1 \cos \varepsilon_1 + D_2 \sin(\varepsilon_1 + \varepsilon_2)
\end{align*} \]

Then make differential with \( T \) point and \( T_1 \) point angle, relation with \( T_3 \) point position vector can be derived from above formula and get:
\[
\begin{pmatrix}
    dX \\
    dY \\
    dZ
\end{pmatrix} =
\begin{pmatrix}
    \frac{\partial X(e_1,e_2)}{\partial e_1} de_1 + \frac{\partial X(e_1,e_2)}{\partial e_2} de_2 \\
    \frac{\partial Y(e_1,e_2)}{\partial e_1} de_1 + \frac{\partial Y(e_1,e_2)}{\partial e_2} de_2 \\
    \frac{\partial Z(e_1,e_2)}{\partial e_1} de_1 + \frac{\partial Z(e_1,e_2)}{\partial e_2} de_2
\end{pmatrix}
\]

Convert into matrix form as:
\[
\begin{pmatrix}
    dX \\
    dY \\
    dZ
\end{pmatrix} =
\begin{pmatrix}
    \frac{\partial X(e_1,e_2)}{\partial e_1} & \frac{\partial Y(e_1,e_2)}{\partial e_1} & \frac{\partial Z(e_1,e_2)}{\partial e_1} \\
    \frac{\partial X(e_1,e_2)}{\partial e_2} & \frac{\partial Y(e_1,e_2)}{\partial e_2} & \frac{\partial Z(e_1,e_2)}{\partial e_2}
\end{pmatrix}
\begin{pmatrix}
    de_1 \\
    de_2
\end{pmatrix}
\]

By matrix property and vector product method, write above formula as:\[ d\text{T}_{3G} = \dot{\text{W}} \times d\text{e}, \]
from which \( \dot{\text{W}} \) is:
\[
\dot{\text{W}} = \begin{pmatrix}
    \frac{\partial X}{\partial e_1} & \frac{\partial X}{\partial e_2} \\
    \frac{\partial Y}{\partial e_1} & \frac{\partial Y}{\partial e_2} \\
    \frac{\partial Z}{\partial e_1} & \frac{\partial Z}{\partial e_2}
\end{pmatrix}
\]

\( \dot{\text{W}} \) is differential relation between current structure node angular displacement and \( T_3 \) point infinitesimal displacement. Input matrix relationship into above formula and can get:
\[ \begin{align*}
    \frac{d\text{T}_{3G}}{dt} &= \dot{\text{W}} \times \frac{d\text{e}}{dt} \\
    \text{or as} \quad \dot{\text{T}}_{3G} &= \text{W}[\varepsilon_1,\varepsilon_2]^T
\end{align*} \]

Input it into \( T_3 \) point relative speed computational formula and can get:
\[ \begin{align*}
    \dot{\varepsilon}_1 &= \text{W} \times \varepsilon_1 \\
    \dot{\varepsilon}_2 &= \text{W} \times \varepsilon_2
\end{align*} \]

**Basketball player shooting moment rotational inertial analysis and teaching**

Basketball player shooting moment act is as Figure 3 show.

Due to player shooting moment, arms are rotating, according to rotational inertia theorem, it can get basketball player overall rotational inertia should be: \( I = \Sigma m_i r_i^2 \)

Among them, \( m_i \) is human body each particle mass, \( r_i \) is player each particle to axis length, human body continuous function is: \( I = \int \int dm = \int \int r^2 dm \)

Therefore basketball player shooting moment arms rotational tensor \( R_c \) is:
\[ R_c = \int \int \rho r^2 \text{e}^{-r \text{r}} r dV \]

Basketball player body any point \( O \) vector expression is \( r = r_1 \text{e}_1 + r_2 \text{e}_2 + r_3 \text{e}_3 \), \( r \times r \) is product of two vectors; unit tensor is: \( \text{e} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \), unit orthogonal curve frame is \( (V; \text{e}_1, \text{e}_2, \text{e}_3) \).
Basketball player shooting instant arms resultant moment vector \( \sum \vec{y}_c \) is: \( \sum \vec{y}_c = R \cdot \vec{X} + \omega \times R \cdot \vec{X} \).

Player arms moment equation in each coordinate axis direction is projection from original moment equation to three-position coordinate system, so player shooting instant arms rotation generated resultant moment \( T_1 \) is:

\[
\begin{align*}
\varphi_1 &= \varphi_1 \cdot I_1 \\
I_1 &= \text{arms rotational inertia. And: } I_1 = \frac{m_i l_i^2}{2}
\end{align*}
\]

Big arm angular accelerated speed \( \varphi_1 \) is:

\[
\dot{\varphi_1} = \frac{d\varphi_1}{dt} = \frac{d^2 X_1}{dt^2}
\]

And small arm angular accelerated speed \( \varphi_2 \) is:

\[
\dot{\varphi_2} = \frac{d\varphi_2}{dt} = \frac{d^2 X_2}{dt^2}
\]

Then by Lagrange equations, establish constraint particle dynamical equation that: \( D = H - P \)

\[
D = H - P
\]

is system kinetic energy \( H \) and potential energy \( P \) difference, system dynamical equation

\[
F = \frac{d}{dt} \left( \frac{\partial D}{\partial \dot{\varphi}_i} - \frac{\partial D}{\partial \varphi_i} \right)
\]

\( i = 1, 2, \ldots, n \)
L₁ and L₂ anatomic angle changing rate arrives at maximum in unit time, and during the period L₂ anatomic angle changing rate is bigger than L₁ angle changing rate. Because in the instant of shooting arms shooting, force will be transferred along L₁ axis to L₂, which causes force loss during transmission process, therefore L₂ angular speed bigger than L₁ angular speed is more beneficial to Ti point acceleration. And according to player shooting instant arms dynamical analysis result, it can get when basketball player shoots, player release angle should let arms and horizontal line included angle to be remained around 45° - 50°, when ball is about to enter into rim, its included angle with horizontal line should be 50° - 55°. Now, player’s right shoulder joint and horizontal line instantaneous angle is 142°. These results provide good guiding for basketball teaching and training.

CONCLUSION

The paper firstly makes research on kinematical equation after basketball shooting and gets that basketball flight distance and ball initial speed are in direct proportion, and ball flight distance will change as athlete release angle changes, when player shooting release angle is 45°, ball flight distance is the farthest. Subsequently analyze athlete shooting instant arms dynamics principle; it gets shooting moment each joint speed relations. Make analysis of its rotational inertia and get player each node momentum relations, then from Lagrange equation, it establishes constraint particle dynamics equation, solves basketball momentum is up to athlete shooting instant wrist joint momentum, and gets when basketball player shoots, player’s release angle should let arms and horizontal line included angle to remain around 45° - 50°, when ball is about to enter into rim, its included angle with horizontal line should be 50° - 55°. Now, player’s right shoulder joint and horizontal line instantaneous angle is 142°. These results provide good guiding for basketball teaching and training.

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