International Journal of Chemical Sciences

International Journal of Chemical Sciences
ISSN: 0972-768 X

Research | Vol 15 Iss 1

D-Radius and D-Diameter of Some Families of Graphs

Reddy Babu D[1*] and Varma PLN[2]

[1]Department of Mathematics, K.L. University, Vaddeswaram522502, A.P, India

[2]Department of Science and Humanities, V. F. S. T. R. University, Vadalamudi, Guntur, India

[*]**Corresponding author:** Reddy Babu D, Department of Mathematics, K.L. University, Vaddeswaram522502, A.P, India, Tel: +919441752543; E-mail: reddybabu17@gmail.com

Received: March 20, 2017; **Accepted:** April 10, 2017; **Published:** April 12, 2017

Abstract

The D-distance between vertices of a graph is obtained by considering the path lengths and as well as the degrees of vertices present on the path. In this article, we study the D-radius and D-diameter of some families of graphs with respect to D-distance. 2000 Mathematics Subject Classification: 05C12.

Keywords: D-distance; D-radius; D-diameter

Introduction

By a graph G=(V, E), we mean a finite undirected graph without loops and multiple edges. The concept of distance is one of the important concepts in study of graphs. It is used in isomorphism testing, graph operations, Hamilton city problems, extremal problems on connectivity and diameter, convexity in graphs etc. Distance is the basis of many concepts of symmetry in graphs [1].

In an earlier article authors introduced the concept of D-distance [2], by considering not only path length between vertices, but also the degrees of all vertices present in a path while defining the D-distance.

Preliminaries

Throughout this article, by a graph G(V, E) or simply G, we mean a non-trivial, finite, undirected graph without multiple edges and loops. Further all graphs we consider are connected.

Definition 2.1: If u, v are vertices of a connected graph G the D-length

P of a u-v path S is defined as l(s)=d(u, v)+deg(u)+deg(v)+ deg(w) where sum runs over all intermediate vertices w of s.

Definition 2.2: (D-distance). The $D-distance$ $d^D(u,v)$ between two vertices u,v of a connected graph G is defined

Citation: Reddy Babu D, Varma LNP. D-Radius and D-Diameter of Some Families of Graphs. Int J Chem Sci. 2017;15(1):116.

as $d^D(u,v)=\min\{l(s)\}$ if u,v are distinct and $d^D(u,v)=0$ if $u=v$, where the minimum is taken over all $u-v$ paths s in G.

Definition 2.3: The *D-eccentricity* of any vertex v, $e^D(v)$, is defined as the maximum distance from v to any other vertex, i.e., $e^D(v)=\max\{d^D(u,v):u\in V(G)\}$

Definition 2.4: Any vertex u for which $d^D(u,v)=e^D(v)$ is called D *eccentric* vertex of v. Further, a vertex u is said to be D eccentric vertex of G if it is the $D-$ eccentric vertex of some vertex.

Definition 2.5: The D*-radius,* denoted by $r^D(G)$, is the minimum D-eccentricity among all vertices of G i.e., $r^D(G)=\min\{e^D(v):v\in V(G)\}$. Similarly the D*-diameter,* $d^D(G)$, is the maximum *D*-eccentricity among all vertices of G.

Definition 2.6: The D*-center* of G, $C^D(G)$, is the sub graph induced by the set of all vertices of minimum $D-$ *eccentricity.* A graph is called D*-self-centered* if $C^D(G)=G$ or equivalently $r^D(G)=d^D(G)$. Similarly, the set of all vertices of maximum D-eccentricity is the *D-periphery* of G.

D-radius and D-diameter of families of graphs

There are some graphs for which D-radius and D-diameter are same, i.e., they are D-self-centered.

Theorem 3.1: *For Complete graph,* K_n*, on* n *vertices* $(n\geq 3)$ *we have* $r^D(K_n)=d^D(K_n)=2n-1.$

Proof: In complete graph K_n, degree of each vertex is $n-1$. Thus D-distance between any two vertices is $1+n-1+n-1=2n-1$. Thus D-eccentricity of every vertex is $2n-1$. Hence D-radius and D-diameter of K_n are equal and is equal to $2n-1$.

Theorem 3.2: For cycle graph, C_n, with n vertices we have $r^D(C_n)=d^D(C_n)=\begin{cases}\dfrac{3n+4}{2} & \text{if } n \text{ is even}\\ \dfrac{3n+1}{2} & \text{if } n \text{ is odd.}\end{cases}$

Proof: In cycle graph C_n degree of each vertex is 2. The D-distance between the vertices is given below. We will consider even and odd cases separately.

Case 1: When n is even, the D-distance in C_n are as below.

TABLE 1. **D-distances of cyclic graphs if n is even.**

	v_1	v_2	v_3	$\cdots$	$v_{\frac{n}{2}-1}$	$v_{\frac{n}{2}}$	$v_{\frac{n}{2}+1}$	$v_{\frac{n}{2}+2}$	$v_{\frac{n}{2}+3}$	$\cdots$	v_{n-1}	v_n
v_1	0	5	8	$\cdots$	$\frac{3n-8}{2}$	$\frac{3n-2}{2}$	$\frac{3n+4}{2}$	$\frac{3n-2}{2}$	$\frac{3n-8}{2}$	$\cdots$	8	5
v_2	5	0	5	$\cdots$	$\frac{3n-14}{2}$	$\frac{3n-8}{2}$	$\frac{3n-2}{2}$	$\frac{3n+4}{2}$	$\frac{3n-2}{2}$	$\cdots$	11	8
v_3	8	5	0	$\cdots$	$\frac{3n-20}{2}$	$\frac{3n-14}{2}$	$\frac{3n-8}{2}$	$\frac{3n-2}{2}$	$\frac{3n+4}{2}$	$\cdots$	14	11
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$v_{\frac{n}{2}-1}$	$\frac{3n-8}{2}$	$\frac{3n-14}{2}$	$\frac{3n-20}{2}$	$\cdots$	0	5	8	11	14	$\cdots$	$\frac{3n+4}{2}$	$\frac{3n-2}{2}$
$v_{\frac{n}{2}}$	$\frac{3n-2}{2}$	$\frac{3n-8}{2}$	$\frac{3n-14}{2}$	$\cdots$	5	0	5	8	11	$\cdots$	$\frac{3n-2}{2}$	$\frac{3n+4}{2}$
$v_{\frac{n}{2}+1}$	$\frac{3n+4}{2}$	$\frac{3n-2}{2}$	$\frac{3n-8}{2}$	$\cdots$	8	5	0	5	8	$\cdots$	$\frac{3n-8}{2}$	$\frac{3n-2}{2}$
$v_{\frac{n}{2}+2}$	$\frac{3n-2}{2}$	$\frac{3n+4}{2}$	$\frac{3n-2}{2}$	$\cdots$	11	8	5	0	5	$\cdots$	$\frac{3n-14}{2}$	$\frac{3n-8}{2}$
$v_{\frac{n}{2}+3}$	$\frac{3n-8}{2}$	$\frac{3n-2}{2}$	$\frac{3n+4}{2}$	$\cdots$	14	11	8	5	0	$\cdots$	$\frac{3n-20}{2}$	$\frac{3n-14}{2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
v_{n-1}	8	11	14	$\cdots$	$\frac{3n+4}{2}$	$\frac{3n-2}{2}$	$\frac{3n-8}{2}$	$\frac{3n-14}{2}$	$\frac{3n-20}{2}$	$\cdots$	0	5
v_n	5	8	11	$\cdots$	$\frac{3n-2}{2}$	$\frac{3n+4}{2}$	$\frac{3n-2}{2}$	$\frac{3n-8}{2}$	$\frac{3n-14}{2}$	$\cdots$	5	0

From the above table, we see that the D-eccentricity of every vertex is $\frac{3n+4}{2}$. Hence the D- radius and D-diameter is same, if n is even.

Case 2: When n is odd, the D-distance in C_n are as below

TABLE 2. **D-distances of cyclic graphs if n is odd.**

	v_1	v_2	v_3	$\cdots$	$v_{\frac{n-1}{2}}$	$v_{\frac{n+1}{2}}$	$v_{\frac{n+3}{2}}$	$v_{\frac{n+5}{2}}$	$\cdots$	v_{n-1}	v_n
v_1	0	5	8	$\cdots$	$\frac{3n-5}{2}$	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$	$\frac{3n-5}{2}$	$\cdots$	8	5
v_2	5	0	5	$\cdots$	$\frac{3n-11}{2}$	$\frac{3n-5}{2}$	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$	$\cdots$	11	5
v_3	8	5	0	$\cdots$	$\frac{3n-17}{2}$	$\frac{3n-11}{2}$	$\frac{3n-5}{2}$	$\frac{3n+1}{2}$	$\cdots$	14	11
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$v_{\frac{n-1}{2}}$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\frac{3n-17}{2}$	$\cdots$	0	5	8	11	$\cdots$	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$
$v_{\frac{n+1}{2}}$	$\frac{3n+1}{2}$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\cdots$	5	0	5	8	$\cdots$	$\frac{3n-5}{2}$	$\frac{3n+1}{2}$
$v_{\frac{n+3}{2}}$	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$	$\frac{3n-5}{2}$	$\cdots$	8	5	0	5	$\cdots$	$\frac{3n-11}{2}$	$\frac{3n-5}{2}$
$v_{\frac{n+5}{2}}$	$\frac{3n-5}{2}$	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$	$\cdots$	11	8	5	0	$\cdots$	$\frac{3n-17}{2}$	$\frac{3n-11}{2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
v_{n-1}	8	11	14	$\cdots$	$\frac{3n+1}{2}$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\frac{3n-17}{2}$	$\cdots$	0	5
v_n	5	8	11	$\cdots$	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\cdots$	5	0

From the above table, we can see that the D-eccentricities of each vertex is $\frac{3n+1}{2}$, if n is odd. Hence D-radius and D-diameter is same. Thus $r^D(C_n) = d^D(C_n) = \begin{cases} \frac{3n+4}{2} & \text{if } n \text{ is even} \\ \frac{3n+1}{2} & \text{if } n \text{ is odd} \end{cases}$.

Next, we calculate D-radius and D-diameter of some families of graphs.

Theorem 3.3: For complete biaparted graph $K_{m,n},(m\geq n)$ we have $r^D(K_{m,n})=m+2(n+1)$ and $d^D(K_{m,n})=n+2(m+1)$.

Proof: Without loss of generality assume that $(m\geq n)$. N complete biparted graph $K_{m,n}$, degree of the vertices $v_1,v_2,v_3,\cdots v_m$ is n and degree $u_1,u_2,u_3,\cdots u_n$ of the vertices m. Thus $d^D(v_i,v_j)=m+2(n+1)$ if $i\neq j$, $d^D(u_i,u_j)=n+2(m+1)$ if $i\neq j$ and $d^D(v_i,u_j)=m+n+1$.

Therefore, $e^D(v_i)=m+2(n+1)\forall i$ and $e^D(u_j)=n+2(m+1)\ \forall\ j$. Hence the D-radius of $K_{m,n}$ is $m+2(n+1)$ and D-diameter of $K_{m,n}$ is $n+2(m+1)$.

Theorem 3.4: *For star graph,* $st_{n,1}$, *we have* $r^D(st_{n,1})=n+2$ *and* $d^D(st_{n,1})=n+4$.

Proof: In star graph, $st_{n,1}$, the degree of central vertex is n and degree of remaining vertices is 1. The D-distance from central vertex to other vertices is $n+2$ and the D-distance between all other vertices is $n+4$. Therefore the minimum D-eccentricity is $n+2$ and the maximum D-eccentricity is $n+4$. Hence the D-radius of $st_{n,1}$ is $n+2$ and the D-diameter of $st_{n,1}$ is $n+4$.

Theorem 3.5: *For path graph* P_n, *on* n *vertices* $(n\geq 3)$ *we have* $r^D(P_n)=\begin{cases}\dfrac{3n-1}{2} & \text{if } n \text{ is odd}\\ \dfrac{3n+2}{2} & \text{if } n \text{ is even}\end{cases}$ *and*

$d^D(P_n)=3(n-1)$ *for* $n\geq 3$.

Further $r^D(P_2)=d^D(P_2)=3$.

Proof: In path graph P_n, degree of end vertices is 1 and remaining vertices of degree is 2.

For $n=2$, $d^D(v_1,v_2)=3$ and $d^D(v_1,v_1)=d^D(v_2,v_2)=0$. Thus the D-eccentricity of each vertex is 3. Hence the D-radius and D-diameter of P_2 are same and equal to 3.

Next consider path graph with $n\geq 3$. we can consider even and odd cases separately, in this case the D-distance between vertices are as given below.

Case 1: When n is odd, the D-distance in P_n are as below

TABLE 3. **D-distances of path graphs if n is odd.**

	v_1	v_2	v_3	$\cdots$	$v_{\frac{n-1}{2}}$	$v_{\frac{n+1}{2}}$	$v_{\frac{n+3}{2}}$	$\cdots$	v_{n-1}	v_n
v_1	0	4	7	$\cdots$	$\frac{3n-7}{2}$	$\frac{3n-1}{2}$	$\frac{3n+5}{2}$	$\cdots$	3n-5	3n-3
v_2	4	0	5	$\cdots$	$\frac{3n-11}{2}$	$\frac{3n-5}{2}$	$\frac{3n+1}{2}$	$\cdots$	3n-7	3n-5
v_3	7	5	0	$\cdots$	$\frac{3n-17}{2}$	$\frac{3n-11}{2}$	$\frac{3n-5}{2}$	$\cdots$	3n-10	3n-7
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$v_{\frac{n-1}{2}}$	$\frac{3n-7}{2}$	$\frac{3n-11}{2}$	$\frac{3n-17}{2}$	$\cdots$	0	5	8	$\cdots$	$\frac{3n+1}{2}$	$\frac{3n+5}{2}$
$v_{\frac{n+1}{2}}$	$\frac{3n-1}{2}$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\cdots$	5	0	5	$\cdots$	$\frac{3n-5}{2}$	$\frac{3n-1}{2}$
$v_{\frac{n+3}{2}}$	$\frac{3n+5}{2}$	$\frac{3n+1}{2}$	$\frac{3n-5}{2}$	$\cdots$	8	5	0	$\cdots$	$\frac{3n-11}{2}$	$\frac{3n-7}{2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
v_{n-1}	3n-5	3n-7	3n-10	$\cdots$	$\frac{3n+1}{2}$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\cdots$	0	4
v_n	3n-3	3n-5	3n-7	$\cdots$	$\frac{3n+5}{2}$	$\frac{3n-1}{2}$	$\frac{3n-7}{2}$	$\cdots$	4	0

From the above table, we can see that the D-eccentricities are $\left\{3n-3, 3n-5, 3n-7, \cdots, \frac{3n+5}{2}, \frac{3n-1}{2}, \frac{3n+5}{2}, \cdots, 3n-5, 3n-3\right\}$. Thus the maximum D-eccentricity of P_n is $3n-3$ and minimum D-eccentricity is $\frac{3n-1}{2}$ Hence the D-radius is $\frac{3n-1}{2}$ and the D-diameter is $3n-3$ if n is odd.

Case 2: When n is even, the D-distance in P_n are as below

TABLE 4. **D-distances of cyclic graphs if n is even.**

	v_1	v_2	v_3	$\cdots$	$v_{\frac{n}{2}-1}$	$v_{\frac{n}{2}}$	$v_{\frac{n}{2}+1}$	$v_{\frac{n}{2}+2}$	$\cdots$	v_{n-1}	v_n
v_1	0	4	7	$\cdots$	$\frac{3n-10}{2}$	$\frac{3n-4}{2}$	$\frac{3n+2}{2}$	$\frac{3n+8}{2}$	$\cdots$	3n-5	3n-3
v_2	4	0	5	$\cdots$	$\frac{3n-14}{2}$	$\frac{3n-8}{2}$	$\frac{3n-2}{2}$	$\frac{3n+4}{2}$	$\cdots$	3n-7	3n-5
v_3	7	5	0	$\cdots$	$\frac{3n-20}{2}$	$\frac{3n-14}{2}$	$\frac{3n-8}{2}$	$\frac{3n-2}{2}$	$\cdots$	3n-10	3n-7
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$v_{\frac{n}{2}-1}$	$\frac{3n-10}{2}$	$\frac{3n-14}{2}$	$\frac{3n-20}{2}$	$\cdots$	0	5	8	11	$\cdots$	$\frac{3n+4}{2}$	$\frac{3n+8}{2}$
$v_{\frac{n}{2}}$	$\frac{3n-4}{2}$	$\frac{3n-8}{2}$	$\frac{3n-14}{2}$	$\cdots$	5	0	5	8	$\cdots$	$\frac{3n-2}{2}$	$\frac{3n+2}{2}$
$v_{\frac{n}{2}+1}$	$\frac{3n+2}{2}$	$\frac{3n-2}{2}$	$\frac{3n-8}{2}$	$\cdots$	8	5	0	5	$\cdots$	$\frac{3n-8}{2}$	$\frac{3n-4}{2}$
$v_{\frac{n}{2}+2}$	$\frac{3n+8}{2}$	$\frac{3n+4}{2}$	$\frac{3n-2}{2}$	$\cdots$	11	8	5	0	$\cdots$	$\frac{3n-14}{2}$	$\frac{3n-10}{2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
v_{n-1}	3n-5	3n-7	3n-10	$\cdots$	$\frac{3n+4}{2}$	$\frac{3n-2}{2}$	$\frac{3n-8}{2}$	$\frac{3n-14}{2}$	$\cdots$	0	4
v_n	3n-3	3n-5	3n-7	$\cdots$	$\frac{3n+8}{2}$	$\frac{3n+2}{2}$	$\frac{3n-4}{2}$	$\frac{3n-10}{2}$	$\cdots$	4	0

From the above table, we can see that the D-eccentricities are $\left\{3n-3, 3n-5, 3n-7, \cdots, \frac{3n+8}{2}, \frac{3n+2}{2}, \frac{3n+2}{2}, \frac{3n+8}{2} \cdots, 3n-5, 3n-3\right\}$. Thus the maximum D-eccentricity of P_n is $3n-3$ and minimum D-eccentricity is $\frac{3n+2}{2}$ Hence the D-radius is $\frac{3n+2}{2}$ and the D-diameter is $3n-3$ if n is even.

Thus $r^D(P_n) = \begin{cases} \dfrac{3n-1}{2} & \text{if } n \text{ is odd} \\ \dfrac{3\text{n}+2}{2} & \text{if } n \text{ is even} \end{cases}$

and $d^D(P_n) = 3(n-1)$ for $n \geq 3$.

Theorem 3.6: For wheel graph, $W_{n,1}$, *with* $n+1$ vertices we have $r^D(W_{n,1}) = n+4$ and $d^D(W_{n,1}) = n+8$ for $n \geq 6$. Further we have $r^D(W_{3,1}) = d^D(W_{3,1}) = 7, r^D(W_{4,1}) = 8, d^D(W_{4,1}) = 11, r^D(W_{5,1}) = 9$ *and* $d^D(W_{5,1}) = 11$.

Proof: For $n = 3, d^D(v_i, v_j) = 7$ if $v_i \neq v_j$. Thus the D-eccentricity of every vertex is 7. Hence the D-radius and D-diameter of $W_{3,1}$ is 7 (and hence $W_{3,1}$ is self-centered).

For $n = 4, d^D(v_i, v_j) = 8$ if v_i is central vertex and $d^D(v_i, v_j) = 7$ or 11, if v_i is not central vertex. Thus the D-eccentricity central vertex is 8 and the D-eccentricity of other vertices is 11. hence the D-radius of $W_{4,1}$ is 8 and D-diameter of $W_{4,1}$ is 11.

For $n = 5, d^D(v_i, v_j) = 9$ if v_i is central vertex and $d^D(v_i, v_j) = 7$ or 11, if v_i is not central vertex. Thus the D-eccentricity central vertex is 9 and the D-eccentricity of other vertices is 11. hence the D-radius of $W_{5,1}$ is 9 and D-diameter of $W_{5,1}$ is 11.

In wheel graph $W_{n,1}$ degree of central vertex is n and degree of remaining vertices is 3. Thus D-eccentricity of central vertex is $n+4$ and D-eccentricity of remaining vertices is $n+8$ for $n \geq 6$. Thus the D-radius of $W_{n,1}$ is $n+4$ and D-diameter of $W_{n,1}$ is $n+8$ for $n \geq 6$.

REFERENCES

1. Buckley F, Harary F. Distance in Graphs, Addison-Wesley. Longman. 1990.
2. Reddy Babu D, Varma PLN. D-distances in graphs, Golden Research thoughts,2013; vol 2(9): 53-58
3. Reddy Babu D, Varma PLN. A note on radius and diameter of a graph w.r.t, D-distance. Int J chem sci. 2016;14(2):1725-29.