Journal of Physics & Astronomy

Research | Vol 5 Iss 1

D Meson Decays and New Physics

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Received: April 17, 2017; **Accepted:** April 27, 2017; **Published:** May 2, 2017

Abstract

On one hand, we investigated the decay rates of meson decays from the basic computations of the Feynman diagrams. Under this approach, we derived analytically expressions for the square amplitudes of leptonic and semileptonic decays and finally used them to determine the analytical and numerical results for the decay rates. With decay rates results at our disposal, we determined the branching ratios of leptonic and semileptonic decays separately within the standard model (SM) compared our results to the latest theoretical and experimental results.

On the other hand, we focused on the effective Lagrangian (EL) under weak interaction from a general approach and used it in our calculation of differential decay rates of mesons. Thereafter, we calculated the total decay rates via integration with respect to. In addition, using the total decay rates results, the branching ratios and the contributions from the new physics (NP) operators were investigated resulting in some phenomenologies of physics beyond the SM.

Keywords: ***D meson; Leptonic decay; Semileptonic decay; New physics***

Introduction

From theory and experiments, there are several arguments to believe that the SM is just the low energy limit of a more fundamental theory. This is not necessarily true because the SM has been successfully tested at an impressive level of accuracy and provides at present our best fundamental understanding of the phenomenology of particle physics. During the 20th century, physicists made tremendous progress in observing smaller and smaller objects and today's accelerators allow us to study matter on length scales as short as 10 m to 18 m [1].

The basic questions of particle physics are:

1. What is the world made of?
2. What is the smallest indivisible building block of matter?
3. Is there such a thing?

A major goal of physics is to find a common ground that would give an integrated approach and understanding on how to solve these questions surrounding nature [2].

Despite the high level of consistence and accuracy within the SM, it does leave some phenomena unexplained and it falls short of being a complete theory of fundamental interactions [3]. Therefore, the answer to these challenges lie in probing more of physics beyond the SM [4].

D Meson Decays

D mesons contain quarks and are one of the lightest particles in this family [5].

Leptonic Decays $D \to l^+ v_l$

The purely leptonic charged D meson decays are the easiest to analyze among its decays. The factorization of its hadronic dynamics is given by

$$\langle 0 | \bar{d}\gamma^{\mu}(1-\gamma_5)c | D^+(p)\rangle = -if_{D^+} p^{\mu}_{D^+} \, . \qquad (1)$$

FIG. 1 shows the process of a D^+ meson decaying into a lepton and a neutron pair. Starting with the computation of the amplitude from the Feynman diagram, we arrive at the structure of the decay width to lowest order as

$$\Gamma\left(D^+ \to l^+ v_l\right) = \frac{G_F^2 m_{D^+} m_l^2 f_{D^+}^2}{8\pi} |V_{cd}|^2 \left(1 - \frac{m_l^2}{m_{D^+}^2}\right)^2 \qquad (2)$$

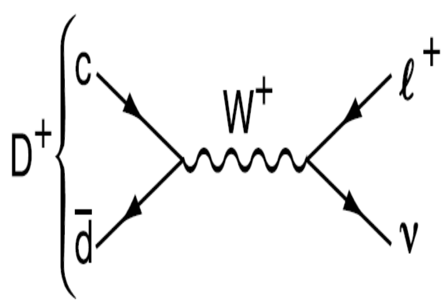

FIG. 1. **The Feynman diagram for purely leptonic D^+ decays in the Standard Model.**

From equation *(2)* the Fermi constant is given by G_F, the D^+ meson mass by m_{D^+}, and the lepton mass by m_l. V_{cd} is the CKM matrix element and f_{D^+} is the decay constant.

Analyzing equation *(1)*, it is evident that there are no leftover variables in our decay rate calculation under the leptonic decay in question. Thus, the branching ratio is determined straight forwardly without any integration by multiplying the decay rate to the mean life, τ_{D^+}

Semileptonic Decays

Due to the fact that leptons do not involve strong interaction, the lepton pair is free from the strong binding effects in the semileptonic decays. Consequently, we can factor them out and arrive at

$$A = \frac{G_F}{\sqrt{2}} V_{cq}^* \bar{v}\gamma_\mu (1-\gamma_5) l \langle X | \bar{q}\gamma^\mu (1-\gamma_5) c | D \rangle \tag{3}$$

In the above expression, $\langle X | \bar{q}\gamma^\mu (1-\gamma_5) c | D \rangle$ include all strong interactions. Leptonic and semileptonic D meson decays are ideal laboratories to study non-perturbative QCD, and to determine important quark mixing parameters. In addition, they may provide additional constraints on physics beyond the SM.

$D \rightarrow Pl^+ v_l$ **Decay:** Within the Standard Model, the D meson semileptonic decay amplitude is given by [6]

$$M\left(D \rightarrow Pl^+ v_l\right) = -i \frac{G_F}{\sqrt{2}} V_{cq}^* L^{\mu\dagger} H_\mu \tag{4}$$

with L^μ being the leptonic current and H_μ the hadronic current describing weak and strong dynamics of the interaction, respectively. Here the leptonic current is defined as

$$L^\mu = \bar{u}_l \gamma^\mu (1-\gamma_5) v_v \tag{5}$$

Where u_l and v_v are the lepton and neutrino Dirac spinors, respectively. On the other hand, the hadronic current can be written as

$$H_\mu = \langle P | \bar{q}\gamma_\mu (1-\gamma_5) c | D \rangle \tag{6}$$

It is vivid that *D* semileptonic decays involve the non-perturbative effects of quantum chromodynamics (QCD). As a result, this matrix element cannot be solved analytically. However, it can be parameterized by expanding the current in terms of all possible independent 4-vectors that can describe the decay, with each of these multiplied by a Lorentz-invariant form factor.

In our case, there are only two independent 4-vectors, which can be taken to be $p_1 + p_2$ and $p_1 - p_2$. Moreover, there is only one Lorentz invariant quantity, which is traditionally taken to be the invariant mass squared of the virtual W boson, $q^2 = (p_1 - p_2)^2$

Thus, H_μ can take a decomposed in the form

$$\langle P|\bar{q}\gamma^\mu(1-\gamma_5)c|D\rangle = (p_1+p_2)^\mu f_+(q^2) + (p_1-p_2)^\mu f_-(q^2) \tag{7}$$

with p_1 and p_2 being the initial D momenta and and pseudoscalar meson in the final state, respectively. From above, the form factors are given by $q = p_1 - p_2$, and $f_+(q^2)$ and $f_-(q^2)$. We can also express the decomposition in the form

$$\langle P|\bar{q}\gamma^\mu(1-\gamma_5)c|D\rangle = \left(p_1^\mu + p_2^\mu - \frac{m_D^2 - m_P^2}{q^2} q^\mu\right) f_+(q^2) + \frac{m_D^2 - m_P^2}{q^2} q^\mu f_0(q^2) \tag{8}$$

with $f_0(q^2)$ being the scalar and $f_+(q^2)$ the vector form factors, respectively.

Here we note that p_1^2 and p_2^2 are not variables since the initial and final particles are on-the-mass-shell; $p_1^2 = m_D^2$, $p_2^2 = m_P^2$. The form factors depend only on $p_1.p_2$, and hence for we can write an equivalently relation as

$$q^2 = p_1^2 - 2p_1.p_2 + p_2^2. \tag{9}$$

With an electron in the final state, its mass is much less with respect to parent D, therefore, only $f_+(q^2)$ contributes. As a result, taking the limit $m_l \to 0$ is an excellent approximation, and the current is further simplified to

$$H_\mu = (p_1 + p_2)_\mu f_+(q^2) \tag{10}$$

Using these expressions for the hadronic and leptonic currents, we arrive at the partial decay width:

$$\frac{d\Gamma(D \to Pe\nu_e)}{dq^2} = X \frac{G_F^2 |V_{cq}|^2}{24\pi^3} p^3 |f_+(q^2)|^2 \tag{11}$$

with p and X being the hadronic momentum and multiplicative factor, respectively. By neglecting lepton masses and plugging in all the necessary parameters we arrive at

$$\frac{d\Gamma}{dq^2}\left(D \to Pl^+\nu_l\right) = \frac{G_F^2}{192\pi^3 m_D^3}\left|V_{cq}\right|^2\left[\left(m_D^2 + m_P^2 - q^2\right) - 4m_D^2 m_P^2\right]^{3/2}\left|f_+\left(q^2\right)\right|^2 \tag{12}$$

with only $f_+\left(q^2\right)$ contributing. $f_-\left(q^2\right)$ contributions are neglected due to its proportionality relation with m_l^2. Here $0 \le q^2 \le \left(m_D - m_P\right)^2$ gives the q^2 distribution range.

Experimental studies measure $d\Gamma/dq^2$ integrated over several q^2 bins in each semileptonic mode. In order to compare these with theoretical predictions, which provide estimates of $f_+\left(q^2\right)$ at one or several points in q^2, it is convenient to fit the results using parameterizations of $f_+\left(q^2\right)$. Theoretically, a number of parameterizations of $f_+\left(q^2\right)$ have been suggested. The most theoretically motivated one is known as the "series" parameterization [7] and follows from a dispersion relation:

$$f_+\left(q^2\right) = f_+(0)\frac{1-\alpha}{1-\frac{q^2}{m_{D^+}^2}} + \frac{1}{\pi}\int_{(m_D+m_P)^2}^{\infty} dt\,\frac{\operatorname{Im} f_+(t)}{t - q^2 - i\varepsilon} \tag{13}$$

with m_D and m_P being the parent and daughter masses, respectively, and α is related to the relative meson contribution to $f_+(0)$.

Here we are not going to dwell much on this one but on another parameterization called ‘simple pole’ model which suggests that the dispersion relation given in Eq. *(13)* is described by

$$f_\pm\left(q^2\right) = \frac{f_\pm(0)}{1-\frac{q^2}{m_{pole}^2}}. \tag{14}$$

While this model can provide reasonable fits when both m_{pole} and $f_+\left(q^2\right)$ are allowed to float, experimental fits of m_{pole} are far away from the expected value of $M_{D^*_{(s)}}$, indicating the higher-order poles are not negligible [8].

The semileptonic decay branching ratio is determined by integration with respect to q^2 as

$$Br\left(D \to Pl^+\nu_l\right) = \tau_D \int_0^{(m_D-m_P)^2} dq^2\,\frac{d\Gamma\left(D \to Pe^+\nu_e\right)}{dq^2} \tag{15}$$

with τ_D being the mean life of the D meson.

$D \to Vl^+\nu_l$ **Decay:** For the transitions to vector mesons, the structure of the hadronic matrix element of the process

$D \rightarrow V l^{+} \nu_{l}$ (FIG. 2), according to its Lorentz structure, is given by

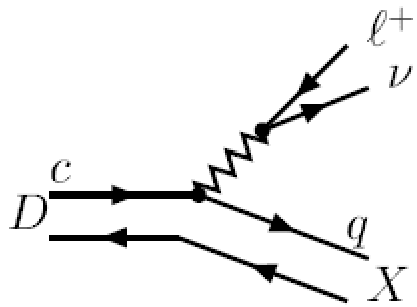

FIG. 2. **Feynman diagram of semileptonic D meson decay.**

$$\langle V(\varepsilon, p_2)|\bar{q}\gamma_{\mu}(1-\gamma_5)c|D(p_1)\rangle = \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}P_1^{\alpha}P_2^{\beta}\frac{2V(q^2)}{m_D+m_V} - i\left(\varepsilon_{\mu}^{*} - \frac{\varepsilon^{*}.q}{q^2}q_{\mu}\right)(m_D+m_V)A_1(q^2)$$

$$+i\left[(p_1+p_2)_{\mu} - \frac{m_D^2-m_V^2}{q^2}q_{\mu}\right]\varepsilon^{*}.q\frac{A_2(q^2)}{m_D+m_V} - i\frac{2m_V\varepsilon^{*}.q}{q^2}q_{\mu}A_0(q^2) \tag{16}$$

where form factors $V(q^2)$ originate from $\bar{q}\gamma_{\mu}c$, and $A_{0,1,2}(q^2)$ from $\bar{q}\gamma_{\mu}\gamma_5 c$, respectively.

By decomposing equation *(16)* we obtain the differential and total decay rate for $D \rightarrow V l^{+} \nu_{l}$ decay. Here we note that there are three polarization states namely: longitudinal and two transverse polarization states. The longitudinal polarization state differential decay rate is given by

$$\frac{d\Gamma_L}{dq^2} = \frac{G_F^2}{192\pi^3 m_D^3}\sqrt{\lambda(m_D^2, m_V^2, q^2)}\left|\frac{1}{2m_V}\left[(m_D^2-m_V^2-q^2)(m_D+m_V)A_1(q^2)\right.\right.$$

$$\left.\left. - \frac{\lambda(m_D^2, m_V^2, q^2)}{m_D+m_V}A_2(q^2)\right]\right|^2 \tag{17}$$

Where

$$\lambda(m_D^2, m_V^2, q^2) \equiv (m_D^2+m_V^2-q^2)^2 - 4m_D^2 m_V^2 . \tag{18}$$

On the other hand, we obtain the differential decay rate with respect to transverse state as,

$$\frac{d\Gamma_T^{\pm}}{dq^2} = \frac{G_F^2 |Vcq|^2}{192\pi^3 m_D^3} q^2 \lambda\left(m_D^2, m_V^2, q^2\right)^{3/2} \left| \frac{V\left(q^2\right)}{m_D + m_V} \mp \frac{\left(m_D + m_V\right) A_1\left(q^2\right)}{\sqrt{\lambda\left(m_D^2, m_V^2, q^2\right)}} \right|^2 , \tag{19}$$

with the symbols " $+$ "and " $-$ " denoting the right-handed and left-handed states, respectively. We can therefore deduce from above that the combined transverse decay rate takes the form;

$$\frac{d\Gamma_T}{dq^2} = \frac{d}{dq^2}\left(\Gamma_T^+ + \Gamma_T^-\right) . \tag{20}$$

Finally, the total differential decay rate becomes

$$\frac{d\Gamma}{dq^2} = \frac{d}{q^2}\left(\Gamma_L + \Gamma_T\right) \tag{21}$$

Where Γ_L and Γ_T are the longitudinal and the combined transverse decay rates, respectively.

Physics Beyond the Standard Model

Physics beyond the Standard Model (BSM) refers to the theoretical developments needed to explain the deficiencies of the SM. These include aspects such as the origin of mass, the strong CP problem, neutrino oscillations, matter-antimatter asymmetry, and the nature of dark matter and dark energy [9].

Another problem lies in the mathematical framework of the SM itself due to the fact that the SM is inconsistent with that of general relativity, to the point that one or both theories break down under certain conditions, for example, within known space-time singularities like the Big Bang and black hole event horizons.

The SM is inherently an incomplete theory. There are fundamental physical phenomena in nature that the SM does not adequately explain. The first area involves phenomena not explained which include gravity, dark matter and dark energy, neutrino masses, and matter-antimatter asymmetry. Secondly, most theoretical predictions have not been observed up to date.

The Effective Lagrangian and Decay Amplitude

The most general effective Lagrangian for $c \to q' e\nu$ in presence of New Physics (NP), where $q' = d, s$ can be written as [10,11]

$$L_{eff} = -\frac{4G_F}{\sqrt{2}} V_{cq'}^* \{ (1+V_L) \overline{l}_L \gamma_\mu \nu_L \overline{q'}_R \gamma^\mu c_L + V_R \overline{l}_L \gamma_\mu \nu_L \overline{q'}_L \gamma^\mu c_R$$

$$+ \tilde{V}_L \overline{l}_R \gamma_\mu \nu_R \overline{q'}_L \gamma^\mu c_L + \tilde{V}_R \overline{l}_R \gamma_\mu \nu_R \overline{q'}_R \gamma^\mu c_R$$

$$+ S_L \overline{l}_L \nu_R \overline{q'} c_L + S_R \overline{l}_R \nu_L \overline{q'} c_R + \tilde{S}_L \overline{l}_L \nu_R \overline{q'}_R c_L + \tilde{S}_R \overline{l}_L \nu_R \overline{q'}_L c_R$$

$$+ T_L \overline{l}_R \sigma_{\mu\nu} \nu_L \overline{q'}_R \sigma^{\mu\nu} c_L + \tilde{T}_L \overline{l}_L \sigma_{\mu\nu} \nu_R \overline{q'}_L \sigma^{\mu\nu} c_R \} + h.c. \tag{22}$$

where G_F is the Fermi constant, $V_{cq'}$ is the relevant CKM Matrix element, and $(q', c, l, v)_{R,L} = \left(\frac{1 \pm \gamma_5}{2}\right)(q', c, l, v)$. The NP couplings denoted by $V_{L,R}$, $S_{L,R}$, and T_L involve left-handed neutrinos, whereas, those denoted by $\tilde{V}_{L,R}$, $\tilde{S}_{L,R}$, and $\tilde{T}_L$ involve right-handed neutrinos. We assume the NP couplings to be real in our analysis. The projection operators are given by $P_L = (1-\gamma_5)/2$ and $P_R = (1-\gamma_5)/2$. Furthermore, we neglect the NP effects coming from the tensor couplings T_L and $\tilde{T}_L$ in our analysis. With this simplification, we obtain

$$L_{eff} = -\frac{G_F}{\sqrt{2}} V_{cq'}^* \{ G_V \bar{l} \gamma_\mu (1-\gamma_5) \nu_l \overline{q'} \gamma^\mu c - G_A \bar{l} \gamma_\mu (1-\gamma_5) \nu_l \overline{q'} \gamma^\mu \gamma_5 c$$

$$+ G_S \bar{l} (1-\gamma_5) \nu_l \overline{q'} c - G_P \bar{l} (1-\gamma_5) \nu_l \overline{q'} \gamma_5 c$$

$$+ \tilde{G}_V \bar{l} \gamma_\mu (1+\gamma_5) \nu_l \overline{q'} \gamma^\mu c - \tilde{G}_A \bar{l} \gamma_\mu (1+\gamma_5) \nu_l \overline{q'} \gamma^\mu \gamma_5 c$$

$$+ \tilde{G}_S \bar{l} (1+\gamma_5) \nu_l \overline{q'} c - \tilde{G}_P \bar{l} (1+\gamma_5) \nu_l \overline{q'} \gamma_5 c \} + h.c. \tag{23}$$

where

$$G_V = 1 + V_L + V_R, \qquad \tilde{G}_V = \tilde{V}_L + \tilde{V}_R,$$

$$G_A = 1 + V_L - V_R, \qquad \tilde{G}_A = \tilde{V}_L - \tilde{V}_R,$$

$$G_S = S_L + S_R, \qquad \tilde{G}_S = \tilde{S}_L + \tilde{S}_R,$$

$$G_P = S_L - S_R, \qquad \tilde{G}_P = \tilde{S}_L - \tilde{S}_R. \tag{24}$$

The SM contribution can be obtained once we set $V_{L,R} = S_{L,R} = 0$ in Eq. *(22)* This implies that $G_V = G_A = 1$ and all other NP couplings are zero. In order to compute the branching fractions and other observables for $D \to Pl\nu$ and $D \to Vl\nu$ decay modes, we need to find various hadronic form factors that parametrizes the hadronic matrix elements of vector (axial vector) and scalar (pseudoscalar) currents between the initial D and final state mesons.

Therefore, from above, we see that expressions for $D \to l\nu$ and $D \to (P,V)l\nu$ decay amplitude depends on non-perturbative hadronic matrix elements that can be expressed in terms of D meson decay constants and $D \to (P,V)$ transition form factors, where P denotes a pseudoscalar meson and V a vector meson, respectively [12-15].

From the Lorentz and parity invariance we obtain

$$\langle 0|\overline{q'}\gamma_\mu c|D(p)\rangle = 0,$$

$$\langle P(p')|\overline{q'}\gamma_\mu\gamma_5 c|D(p)\rangle = 0,$$

$$\langle V(p',\varepsilon^*)|\overline{q'}c|D(p)\rangle = 0. \quad (25)$$

To find the scalar and pseudoscalar matrix elements, we use the equation of motion and thus we have

$$\langle 0|\overline{q'}\gamma_5 c|D(p)\rangle = i\frac{m_D^2}{m_c(\mu)+m_{q'}(\mu)} f_{D_{q'}},$$

$$\langle P(p')|\overline{q'}c|D(p)\rangle = \frac{M_D^2 - m_P^2}{m_c(\mu)-m_{q'}(\mu)} F_0(q^2),$$

$$\langle V(p',\varepsilon^*)|\overline{q'}\gamma_5 c|D(p)\rangle = -\frac{2m_V A_0(q^2)}{m_c(\mu)+m_{q'}(q^2)}\varepsilon^*.q. \quad (26)$$

Decay Widths

Finally, using the effective Lagrangian of Eq. (23) in the presence of NP, we are capable of computing the partial decay widths of $D \to l\nu$ and $D \to (P,V)l\nu$ beyond the SM.

$D \to l\nu$

In the case of leptonic decay in the presence of NP, starting with Eq. *(23)* we arrive at

$$\Gamma(D \to l\nu) = \frac{G_F^2 m_D m_l^2 f_D^2}{8\pi} |V_{cd}|^2 \left(1 - \frac{m_l^2}{m_D^2}\right)^2 \left\{ \left[G_A - \frac{m_D^2}{m_l(m_c(\mu) + m_d(\mu))} G_P \right]^2 \right.$$

$$\left. + \left[\tilde{G}_A - \frac{m_D^2}{m_l(m_c(\mu) + m_d(\mu))} \tilde{G}_P \right]^2 \right\} \qquad (27)$$

where, in the SM, we have $G_A = 1$ and $G_P = \tilde{G}_A = \tilde{G}_P = 0$, so that we have

$$\Gamma(D \to l\nu)_{SM} = \frac{G_F^2 m_l^2 m_D f_D^2}{8\pi} |V_{cd}|^2 \left(1 - \frac{m_l^2}{m_D^2}\right)^2 \qquad (28)$$

At this point, it is important to note that the right-handed neutrino couplings denoted by $\tilde{V}_{L,R}$ and $\tilde{S}_{L,R}$ appear in the decay width quadratically, whereas, the left-handed neutrino couplings denoted by $V_{L,R}$ and $S_{L,R}$ appear linearly. The linear dependence, arising due to the interference between the SM couplings and the NP couplings, is suppressed for the right-handed neutrino couplings as it is proportional to a small factor m_ν and hence is neglected.

$D \to (P,V) l\nu$

Let us now proceed to discuss the $D \to Pl\nu$ and $D \to Vl\nu$ decays. In this section, we follow the helicity method of Refs. [16,17] for the semileptonic B meson decays and apply it to our case of the D meson decays. Therefore, the differential decay distribution can be written as

$$\frac{d\Gamma}{dq^2 d\cos\theta_l} = \frac{G_F^2 |V_{cq'}|^2 |\vec{p}_{(P,V)}|}{2^9 \pi^3 m_D^2} \left(1 - \frac{m_l^2}{q^2}\right) L_{\mu\nu} H^{\mu\nu} \qquad (29)$$

where $L_{\mu\nu}$ and $H^{\mu\nu}$ are the usual leptonic and hadronic tensors, respectively. θ_l is the angle between $P(V)$ meson and the lepton three momentum vector in the q^2 rest frame. The three-momentum vector $|\vec{p}_{(P,V)}|$ is defined as

$$|\vec{p}_{(P,V)}| = \frac{\sqrt{\lambda(m_D^2, m_{P(V)}^2, q^2)}}{2m_D} \qquad (30)$$

where

$$\lambda(a,b,c)=a^2+b^2+c^2-2(ab+bc+ca)\;. \quad (31)$$

Furthermore, the differential decay distribution for $D \to Pl\nu$ in terms of helicity amplitudes H_0, H_t, and H_S is given by

$$\frac{d\Gamma}{dq^2 d\cos\theta_l}=2N\left|\vec{p}_P\right|\left\{H_0^2\sin^2\theta_l\left(G_V^2+\tilde{G}_V^2\right)+\frac{m_l^2}{q^2}\left[H_0 G_V\cos\theta_l-\left(H_t G_V+\frac{\sqrt{q^2}}{m_l}H_S G_S\right)\right]^2\right.$$
$$\left.+\frac{m_l^2}{q^2}\left[H_0\tilde{G}_V\cos\theta_l-\left(H_t\tilde{G}_V+\frac{\sqrt{q^2}}{m_l}H_S\tilde{G}_S\right)\right]^2\right\} \quad (32)$$

where

$$N=\frac{G_F^2\left|V_{q'c}\right|^2 q^2}{256\pi^3 m_D^2}\left(1-\frac{m_l^2}{q^2}\right)^2, \qquad H_0=\frac{2m_D\left|\vec{p}_P\right|}{\sqrt{q^2}}F_+\left(q^2\right)$$

$$H_t=\frac{m_D^2-m_P^2}{\sqrt{q^2}}F_0\left(q^2\right), \qquad H_S=\frac{m_D^2-m_P^2}{m_c(\mu)-m_{q'}(\mu)}F_0\left(q^2\right). \quad (33)$$

Performing the integration over $\cos\theta_l$, we can determine the differential decay $d\Gamma^P/dq^2$ rate and arrive at

$$\frac{d\Gamma^P}{dq^2}=\frac{8N\left|\vec{p}_P\right|}{3}\left\{H_0^2\left(G_V^2+\tilde{G}_V^2\right)\left(1+\frac{m_l^2}{2q^2}\right)\right.$$
$$\left.+\frac{3m_l^2}{2q^2}\left[\left(H_t G_V+\frac{\sqrt{q^2}}{m_l}H_S G_S\right)^2+\left(H_t\tilde{G}_V+\frac{\sqrt{q^2}}{m_l}H_S\tilde{G}_S\right)^2\right]\right\} \quad (34)$$

In the SM, we have $G_V=1$ and the rest of the NP couplings are equal to zero and henceforth we

Obtain

$$\left(\frac{d\Gamma^P}{dq^2}\right)_{SM} = \frac{8N\left|\vec{p}_P\right|}{3}\left\{H_0^2\left(1+\frac{m_l^2}{2q^2}\right)+\frac{3m_l^2}{2q^2}H_t^2\right\}. \quad (35)$$

On the other hand, let us now draw our attention to the differential decay distribution for $D \to Vl\nu$ in terms of the helicity amplitudes ξ_0, ξ_1, ξ_2, ξ_P, and ξ_t. After a series of derivations, we arrive at a more simplified expression

$$-4\xi_1\xi_2\cos\theta_l\left(G_A G_V - \tilde{G}_A\tilde{G}_V\right)+\frac{m_l^2}{q^2}\sin^2\theta_l\left[\xi_1^2\left(G_A^2+\tilde{G}_A^2\right)+\xi_2^2\left(G_V^2+\tilde{G}_V^2\right)\right]$$

$$+\frac{2m_l^2}{q^2}\left[\left\{\xi_0 G_A\cos\theta_l-\left(\xi_t G_A+\frac{\sqrt{q^2}}{m_l}\xi_P G_P\right)\right\}^2\right.$$

$$\left.+\left\{\xi_0\tilde{G}_A\cos\theta_l-\left(\xi_t\tilde{G}_A+\frac{\sqrt{q^2}}{m_l}\xi_P\tilde{G}_P\right)\right\}^2\right]\Bigg\}, \quad (36)$$

where from Eq. *(36)* we have

$$\xi_0 = \frac{1}{2m_V\sqrt{q^2}}\left[\left(m_D^2-m_V^2-q^2\right)\left(m_D+m_V\right)A_1\left(q^2\right)-\frac{4m_D^2\left|\vec{p}_V\right|^2}{m_D+m_V}\right],$$

$$\xi_1 = \frac{2\left(m_D+m_V\right)A_1\left(q^2\right)}{\sqrt{2}},$$

$$\xi_2 = -\frac{4m_D V\left(q^2\right)\left|\vec{p}_V\right|}{\sqrt{2}\left(m_D+m_V\right)},$$

$$\xi_t = \frac{2m_D\left|\vec{p}_V\right|A_0\left(q^2\right)}{\sqrt{q^2}},$$

$$\xi_P = -\frac{2m_D\left|\vec{p}_P\right|A_0\left(q^2\right)}{m_c(\mu)+m_d(\mu)}. \quad (37)$$

At this stage, we now perform the integration with respect to $\cos\theta_l$ and arrive at the differential decay rate $d\Gamma^V/dq^2$ in its simplicity form given by

$$\frac{d\Gamma^V}{dq^2}=\frac{8N\left|\vec{p}_V\right|}{3}\left\{\xi_{AV}^2+\frac{m_l^2}{2q^2}\left[\xi_{AV}^2+3\xi_{tP}^2\right]+\tilde{\xi}_{AV}^2+\frac{m_l^2}{2q^2}\left[\tilde{\xi}_{AV}^2+3\tilde{\xi}_{tP}^2\right]\right\}, \tag{38}$$

Where

$$\xi_{AV}^2=\xi_0^2G_A^2+\xi_1^2G_A^2+\xi_2^2G_V^2,$$

$$\tilde{\xi}_{AV}^2=\xi_0^2\tilde{G}_A^2+\xi_1^2\tilde{G}_A^2+\xi_2^2\tilde{G}_V^2,$$

$$\xi_{tP}=\xi_tG_A+\frac{\sqrt{q^2}}{m_l}\xi_PG_P,$$

$$\tilde{\xi}_{tP}=\xi_t\tilde{G}_A+\frac{\sqrt{q^2}}{m_l}\xi_P\tilde{G}_P. \tag{39}$$

It is wise to remind ourselves at this stage that in the SM, $G_V=G_A=1$ and all the other NP couplings are zero. As a result Eq. *(3.17)* simplifies to

$$\left(\frac{d\Gamma^V}{dq^2}\right)_{SM}=\frac{8N\left|\vec{p}_P\right|}{3}\left\{\left(\xi_0^2+\xi_1^2+\xi_2^2\right)\left(1+\frac{m_l^2}{2q^2}\right)+\frac{3m_l^2}{2q^2}\xi_t^2\right\}. \tag{40}$$

Having obtained this gigantic task of deriving the analytical results for the decay rates, we now define a very important physical observable called the differential branching ratio (DBR). This is given by

$$DBR\left(q^2\right)=\frac{\left(d\Gamma/dq^2\right)}{\Gamma_{total}}=\tau_D\frac{d\Gamma}{dq^2}, \tag{41}$$

where $\Gamma_{total}=\frac{1}{\tau_D}$ is the total decay width of D meson.

Results and Discussion

For definiteness, we first present all the inputs that are pertinent for our calculation from Ref [4]. For the quark and lepton masses we use $m_c(m_c)=1.275\pm0.025\,\mathrm{GeV}$, $m_d(2\mathrm{GeV})=4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$, $m_s(2\mathrm{GeV})=95\pm5\,\mathrm{MeV}$, $m_e=0.511$

MeV, $m_\mu = 105.7$ MeV, and $m_\tau = 1776.82$ MeV. In addition, for the meson masses and lifetime we use $m_{D^\pm} = 1869.61 \pm 0.09$ MeV, $m_{K^\pm} = 493.677 \pm 0.016$ MeV, $m_{K^0} = 497.611 \pm 0.013$ MeV, $m_{K^*} = 895.5 \pm 0.8$ MeV, $m_{\pi^0} = 134.9766 \pm 0.0006$ MeV, $m_\eta = 547.862 \pm 0.017$ MeV, $m_\rho = 775.26 \pm 0.25$ MeV, and $\tau_{D^\pm} = (1040 \pm 7) \times 10^{-5}$ s.

From Ref [14] we obtain the input values for decay constant and the Fermi constant as $f_{D^+} = (209.2 \pm 3.3)$ MeV, $f_{D_s} = (248.6 \pm 2.7)$ MeV and $G_F = 1.16637876 \times 10^{-5}$ Ge/V^2. On the aspect of the CKM matrix elements, we used numerical values given in TABLE. 1.

TABLE 1. **Numerical values of $|V_{cd}|$ and $|V_{cs}|$.**

$\lvert V_{cd}\rvert$	$\lvert V_{cs}\rvert$	**References**
0.222 ± 0.008	0.986 ± 0.016	[14]

TABLE 2. shows some of the decay modes considered in this work and their numerical values obtained with a comparison from theory. Our results are in the same range with those obtained by other scholars who has worked in the same line of research. The differences in the numerical values may be due to variation in our input values. Secondly, differences may also be due to the process of calculating resulting from rounding off figures.

TABLE 2. **Leptonic decay width.**

Mode	$\Gamma(D^+ \to l^+ \nu_l)$ **(s^{-1})**		
	This work	**Decay width**	**Reference**
$D^+ \to \mu^+ \nu$	6.754×10^{-13}	4.72×10^{-13}	[17]
$D^+ \to e^+ \nu$	3.122×10^{-13}	1.79×10^{-13}	[17]
$D^+ \to \tau^+ \nu$	5.706×10^{-15}	-	-

From Eq. *(15)* we obtained the branching ratios shown in TABLE. 3.

TABLE 3. **Leptonic branching ratios.**

Mode	**This work**	**Branching ratio**	**Experiment**	**References**
$D^\pm \to \mu^\pm \nu$	4.12×10^{-4}	(3.82 ± 0.32 ± 0.09) $\times 10^{-4}$	CLEO	[18]
		(3.71 ± 0.19 ± 0.06) $\times 10^{-4}$	BES III	[19]
		2.2 $\times 10^{-4}$	-	[20]

		2.87×10^{-4}	-	[17]
		4.3×10^{-3}	-	[21]
		3.82×10^{-4}	Exp [2]	[16]
$D^{\pm}\to e^{\pm}\nu$	9.42×10^{-8}	$<8.8\times10^{-6}$	CLEO	[18]
		1.0×10^{-8}	-	[21]
		0.5×10^{-8}	-	[20]
		$<8.8\times10^{-6}$	Exp [2]	[16]
$D^{\pm}\to\tau^{\pm}\nu$	9.73×10^{-4}	$<1.2\times10^{-3}$	CLEO	[18]
		1.5×10^{-3}	-	[20]
		7.54×10^{-4}	-	[17]
		1.05×10^{-3}	-	[21]
		$<1.2\times10^{-3}$	[2]	[16]

The pseudoscalar semileptonic $D\to Pl\nu$ decay width for the K channel via the mode $D^{+}\to Ke^{+}\nu_{e}$ was found to be 5.51×10^{-15} s^{-1}. Furthermore, some comparison of the branching ratios was looked at and the numerical values obtained are under Table 4. The numerical values obtained in the SM appear to be different from experimental values as a result of computation of the decay rates as this process encompassed a good number of constants determined theoretically and experimentally.

TABLE 4. **Branching ratios for** $D^{+}\to\pi^{0}e^{+}\nu_{e}$, $D^{+}\to\overline{K}^{0}e^{+}\nu_{e}$ **and** $D^{+}\to\overline{K}^{0}\mu^{+}\nu_{\mu}$.

Mode	**This work**	**Experiment**	**References**
$D^{+}\to\pi^{0}e^{+}\nu_{e}$	3.91×10^{-2}	(0.44 ± 0.06 ± 0.03)%	[15]
$D^{+}\to\overline{K}^{0}e^{+}\nu_{e}$	9.98×10^{-2}	(8.71 ± 0.38 ± 0.37)%	[15]
$D^{+}\to\overline{K}^{0}\mu^{+}\nu_{\mu}$	10.45×10^{-2}	(9.3 ± 0.7)%	[4]

There are other $D(D_{s})$ to pseudoscalar semileptonic decay channels which deserve further studies. Our calculations and experimental measurements are listed in TABLE 5. The calculations under these channels were taken with different trial of different decay mode and then compared to theoretical and experimental numerical values. As clearly seen our values seem to be in the acceptable range of values numerically despite the difference being so much especially with respect to the theoretical values cited under the same table. Without much clarification, it is clear that our values are a bit nearer to experimental than theoretical values. It is vital to mention that this work did not deal much on the computation of form factors under pseudoscalar decays as well as other forms of D meson decays.

TABLE 5. **Branching ratios for D^+ and D_s^+ to η.**

Mode	BR	BR(%)		References
	This work	Theoretical	Experiment	
$D^+ \to \eta e^+ \nu_e$	2.01×10^{-2}	0.10	<0.5	[15]
$D_s^+ \to \eta e^+ \nu_e$	3.15×10^{-2}	1.7	2.5 ±0.7	[15]

For the vector $D \to Vl\,\nu$ decays, we took into consideration are $D^+ \to \rho e^+ \nu_e$ and $D^+ \to K^* e^+ \nu_e$. TABLE 6 shows a comparison of the values obtained with other theoretical values. In addition to the partial decay rates under TABLE 6, this work also computed total decay width for $D \to K_0^* e\nu$ and arrived at $\Gamma\left(D^+ \to K_0^* e^+ \nu_e\right) = 2.9\times10^{-16}$ s^{-1}. On the other hand, the branching ratio was found to be $Br\left(D^+ \to K_0^* e^+ \nu_e\right) = 4.6\times10^{-4}$.

TABLE 6. **Branching ratios for $D^+ \to \rho e^+ \nu_e$ and $D^+ \to K^* e^+ \nu_e$.**

Mode	Branching ratios		References
	This work	Theoretical	
$D^+ \to \rho e^+ \nu_e$	3.12×10^{-3}	$2.18\,^{+0.17}_{-0.25}\times10^{-3}$	[4]
$D^+ \to K^* e^+ \nu_e$	4.98×10^{-2}	(3.68 ± 0.10)%	[4]

We have seen the transition from Eq. *(27)* to Eq. *(28)* via fixing of our NP couplings by assigning $G_A = 1$ and the rest of the couplings equal zero. The same approach applies BSM.

It is vital to note that experiments on a general view do not follow any theory hence making us check our results both from the theoretical aspect as well as experimental results. With this in mind, we now set the standard value of our NP coupling constants as per theory. The standard is usually set at 1.25 ± 0.04 where $\sigma = \pm$ 0.04.

Now for our work, we needed to set the NP coupling constants in such a way as to arrive at the constraint contribution of each one of them. We began by focusing on the pure leptonic decay $D^\pm \to e^\pm \nu$. We used Eq. (27) and by setting the NP coupling constants appropriately, we checked the contribution of each. The first step was to vary G_A and set the rest of the couplings to zero.

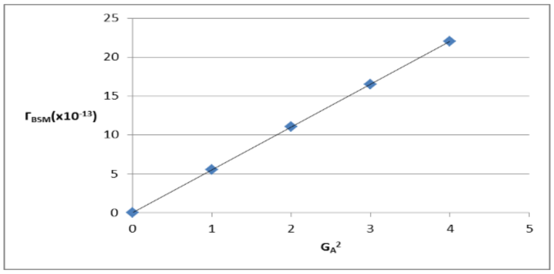

FIG. 3. **Graph of Γ_{BSM} against G_A^2 for $D^{\pm} \rightarrow e^{\pm}\nu_e$.**

From FIG. 3. we see the dependence of the decay rate BSM on the NP coupling constant G_A . For $\Gamma\left(D^{\pm} \rightarrow e^{\pm}\nu_e\right)$, the constraint for G_A is found to be $C_{G_A} = [0.2, 0.8]$. The same method was applied to the other couplings and the results were as shown in TABLE 7.

TABLE 7. **NP coupling constraints for leptonic decay $D^{\pm} \rightarrow e^{\pm}\nu_e$.**

NP coupling constant	Constraints
G_A	[0.2,0.8]
G_P	[0.9,1.4]
$\tilde{G}_A$	[0.2,0.8]
$\tilde{G}_P$	[0.9,1.4]

On the other hand, we looked at the pseudoscalar decay mode $D^{+} \rightarrow Ke^{+}\nu_e$. FIG. 4 shows the dependence of the decay width on the NP coupling constant G_V . Like-wise, this was achieving by varying G_V and setting the rest of the NP coupling constants to zero. Thereafter, varying each one of then and setting the rest to zero in order to attain the constraints for each constant.

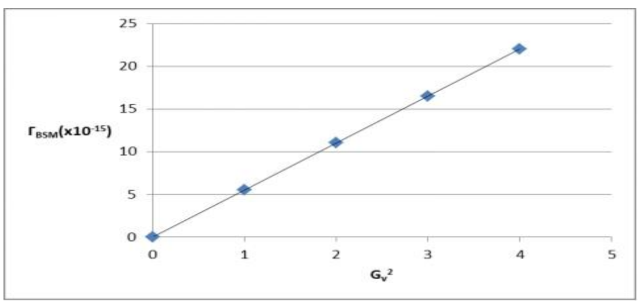

FIG. 4. **Graph of Γ_{BSM} against G_V^2 for $D^{+} \rightarrow Ke^{+}\nu$.**

It was deduced that the constraint for G_V is $C_{G_V} = [1.2, 1.6]$. The contribution of the rest of the couplings are shown in TABLE 8. Finally, we considered the decay mode $D^+ \rightarrow K_0^* e^+ \nu_e$. The dependence of the decay width on G_V is shown in FIG. 5.

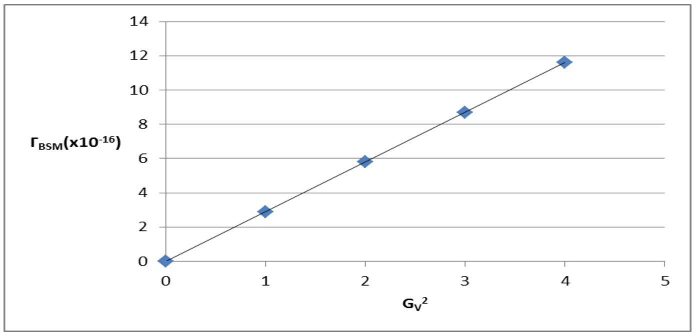

FIG. 5. **Graph of Γ_{BSM} against G_V^2 for $D^+ \rightarrow K_0^* e^+ \nu$.**

The constraint for G_V were estimated to be $C_{G_V} = [0.2, 0.9]$. The rest of the constraints for the other couplings are shown in TABLE 9.

TABLE 8. **NP coupling constraints for pseudoscalar decay $D^+ \rightarrow K e^+ \nu$.**

NP coupling constant	Constraints
G_V	[1.2,1.6]
G_S	[1.3,1.8]
$\tilde{G}_V$	[1.1,1.7]
$\tilde{G}_S$	[0.8,1.5]

TABLE 9. **NP coupling constraints for vector decay $D^+ \rightarrow K_0^* e^+ \nu$.**

NP coupling constant	Constraints
G_V	[0.2,0.9]
G_A	[0.5,1.6]
$\tilde{G}_V$	[0.6,1.2]
$\tilde{G}_A$	[0.3,1.4]

Conclusion

The leptonic, pseudoscalar and vector decays have been studied under this work with the aid of the effective Lagrangian by including the allowed direct NP couplings. Charm decays has been and still is an exciting field for both theoretical and experimental investigations. Charm quark transition amplitudes, described in this work, represent a crucial tool to understand strong interaction dynamics in the non-perturbative regime. Complementary information that constrains model building and lattice gauge calculations is coming from the rich spectroscopy of charmed mesons and baryons, which is beyond the scope of this paper.

Coming to our results, we see from our results that more needs to be done in order to understand the real phenomenology of physics BSM. Parameterization of decay constants into physical numerical values still remain a challenge among theorists. Nevertheless, more progress has been made in recent years to try and fuse in the gaps that the SM has left scholars with more questions than answers.

From our results, we can see that a search for NP under D meson decay is equally important as well as that of the B meson. The results obtained under this work do not entail pure accuracy as earlier on mentioned as they are equally prone to errors just like those from experiments too.

Acknowledgment

I also wish to thank all the Professors that taught me during the course of my study for their guidance, support, and their efforts to teach and equip me for this task. I wish to thank Professor Xin-Qiang Li for his expertise in the field and his advice there on.

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