ISSN : 0974 - 7435 Volume 10 Issue 24

2014

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 10(24), 2014 [15220-15229]

Coordination of a dual-channel closed-loop supply Chain with demand disruption under revenue-sharing contract

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ABSTRACT

A manufacturer and a retailer coexist in a dual channel supply chain, where the retailer has a traditional channel and the manufacturer owns an online channel. In the dual channel, the manufacturer is also responsible for recycling remanufactured products. When the supply chain system is in a stable state, we calculate the optimal price, the sales quantity and the recovery rate. However, when demand disruption happens, the manufacturer does not need to adjust production planning within a certain range. In addition, the recovery rate and the change of demand disruption are negatively correlated. Under this circumstance, revenue-sharing contract in the stable state is no longer appropriate to coordinate the supply chain. Instead, an improved revenue-sharing contract is designed to coordinate the dual-channel closed-loop supply chain with the consideration of demand disruption.

KEYWORDS

Improved revenue-sharing contract; Closed-loop; Dual-channel; Demand disruption.

INTRODUCTION

With the development of the Internet, online shopping has become one of the important individuals' consumption patterns. Thus, in order to expand the business and obtain a dominant position in oligarchs retailers, most manufacturers try their best to build their own electronic channels. Many firms have engaged in online channels to get a chance to extend manufacturers' own industrial chain. It is reported that about 42% of the top suppliers such as IBM, Nike, Pioneer Electronics, Estee Lauder, and Dell are selling to customers through an online channel (Chiang et al [1] ; Tsay and Agrawal [2]). Manufacturers' electronic channels and retailers' traditional retail channels form a sales model which is called dual-channel sales model. The establishment of electronic channels provides a chance to extend manufacturers' own industrial chain. Ryan et al [3] considered a dual-channel supply chain in which a manufacturer sells a single product to end-users through both traditional retail channel and manufacturer-owned direct online channel. They proposed a modified revenue-sharing contract and gain/loss sharing contract to enable supply chain coordination. Cao et al [4] studied a dual-channel supply chain under wholesale contract and investigated the impact of asymmetric cost information. In recent years, closed-loop supply chain can generate profits by taking back products from consumers and recovering the remaining added value (Ferguson and Toktay [5] ; Geyer et al [6]). Ma et al [7] begun to focus on how consumption-subsidy influences dual-channel closed-loop supply chain. Their result indicated that the manufacturer and the retailer were beneficiaries of the government consumption-subsidy. The dual-channel closed-loop supply chain structure has been taken into the supply chain research.

Disruptions have a significant impact on the operation of supply chain. Natural disasters such as earthquakes, tsunamis and landslides, public health emergencies which include avian flu, and man-made emergencies such as terrorist attacks affect the operation of stable supply chain. How to cope with the supply chain's disruptions mentioned above has become a hot issue for the scholars worldwide. Qi et al [8] first used the idea of disruption management in supply chain management. In their paper, they considered deviation costs. If the demand exceeds the original production quantity, underage cost may happen. Otherwise disposal cost may happen. Based on disruption management, many researchers had extended in various scenarios in the supply chain(Yang et al. [9] ; Xu et al. [10] ; Huang et al. [11] ; Xiao and Yu [12] ; Lei et al. [13]). In the multi-retailer supply chain, Xiao et al. [14] ~ [16] studied how to coordinate a supply chain with one manufacturer and two competing retailers when demands and costs are disrupted. Huang et al [17] considered the disruption management in a dual-channel supply chain under demand disruptions. Huang et al [18] , also studied a pricing and production problem in a dual-channel supply chain when production costs are disrupted in the centralized and decentralized dual-channel supply chain.

However, in this paper we investigate the pricing, recovering rate and production quantity decisions in a dual-channel closed-loop supply chain under demand and production cost disruptions. We also design a revenue-sharing contract in a Stackelberg game, in which the manufacturer is the leader, to coordinate the decentralized suppliant chain. The contract is improved when demand changes significantly. At last, a numerical example is shown to illustrate the related results.

BASIC MODEL

In a two-stage closed-loop supply chain, there is one manufacturer and one traditional retailer. Manufacturer orders products from both traditional and electronic retail channel, and recycles the products directly (see Figure 1, dashed line indicates recycling channel).

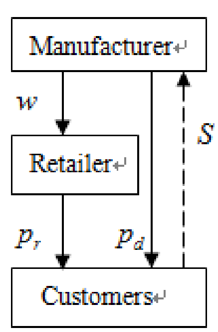

Figure 1 Dual-channel closed-loop supply chain

Notation

The notation used for the model is shown in Table1.

Table1: Notation and parameters for the problems

Value	Description	Test values
a	Total market demand	100
$\mu,(0<\mu<1)$	Demand proportion of electronic channel	0.3
$\theta,(0<\theta<1)$	Substitution coefficient between the two channels	0.4
p_d	Unit sale price in online channel	Decision variable
p_r	Unit sale price in offline channel	Decision variable
c_m	Unit manufacturing cost	10
S	Unit profit from the re-manufacturing	4
$\lambda,(0\le\lambda\le1)$	Proportion of products using recycled	Decision variable
C	Recycling input coefficient, which is large enough	200
$C\lambda^2$	Manufacturer's recycling input	Variable
Δa	Demand disruption	20,10,5,0,-5,-10,-20
k_1	Marginal costs when increase production plan	4
k_2	Marginal costs when decrease production plan	4
$\tilde{p}_d$	Unit sale price in online channel after disruption	Decision variable
$\tilde{p}_r$	Unit sale price in offline channel after disruption	Decision variable
$\tilde{\lambda}$	Proportion of products using recycled after disruption	Decision variable
$\tilde{w}^{RS}$	The wholesale price of traditional channel	Decision variable
ϕ_1	The percent of the channel profit from retailer allocates to manufacturer when production quantity immovable	0.6
ϕ_2	The percent of the change cost from retailer allocates to manufacturer when production quantity change	0.15

Benchmark Model

We build a demand function considering electronic channel and traditional channel according to the literature Qi et al [8] :

$$\begin{cases} q_d = \mu a - p_d + \theta p_r \\ q_r = (1-\mu)a - p_r + \theta p_d \end{cases} .(1)$$

The profit function is derived by the following equations

$$\Pi = (p_d - c_m)(\mu a - p_d + \theta p_r) + (p_r - c_m)\left[(1-\mu)a - p_r + \theta p_d\right] + \lambda S\left[a - (1-\theta)(p_d + p_r)\right] - C\lambda^2 \quad (2)$$

$$(p_d^*, p_r^*, \lambda^*) \in \arg\max \Pi(p_d, p_r, \lambda)$$

$\Pi(p_d, p_r, \lambda)$ is strictly differential concave function of (p_d, p_r, λ) .

Proof: The Hessian matrix of $\Pi(p_d, p_r, \lambda)$, $\begin{bmatrix} -2 & 2\theta & -(1-\theta)S \\ 2\theta & -2 & -(1-\theta)S \\ -(1-\theta)S & -(1-\theta)S & -2C \end{bmatrix}$, is negative definite matrix. Its order principal minor determinant is （-2, $4(1-\theta^2)$, $-4(1-\theta^2)\left[2C-(1-\theta)S^2\right]$） , which means $\Pi(p_d, p_r, \lambda)$ is a differential concave function.

We obtain the optimal solution by solving the first-order condition, $\frac{\partial\Pi}{\partial p_d} = \frac{\partial\Pi}{\partial p_r} = \frac{\partial\Pi}{\partial\lambda} = 0$, which is

$$\begin{cases} p_d^* = \dfrac{a[\mu+(1-\mu)\theta]}{2(1-\theta^2)} + \dfrac{4c_m C - aS^2}{4[2C-(1-\theta)S^2]} \\ p_r^* = \dfrac{a[1-\mu+\mu\theta]}{2(1-\theta^2)} + \dfrac{4c_m C - aS^2}{4[2C-(1-\theta)S^2]} \end{cases}$$

$$\lambda^* = \frac{[a-2(1-\theta)c_m]S}{4C-2(1-\theta)S^2}$$

Thus, the optimal sales quantities of the dual-channel supply chain are

$$\begin{cases} q_d^* = \dfrac{\mu a-(1-\theta)c_m}{2}+\dfrac{S^2(1-\theta)[a-2(1-\theta)c_m]}{4[2C-(1-\theta)S^2]} \\ q_r^* = \dfrac{(1-\mu)a-(1-\theta)c_m}{2}+\dfrac{S^2(1-\theta)[a-2(1-\theta)c_m]}{4[2C-(1-\theta)S^2]} \end{cases}.$$

The optimal profit of the supply chain is

$$\Pi^* = \frac{a^2[4C-(1-\theta)^2S^2]}{8(1-\theta^2)[2C-(1-\theta)S^2]}-\frac{a^2\mu(1-\mu)}{2(1+\theta)}-\frac{C[a-(1-\theta)c_m]c_m}{2C-(1-\theta)S^2}.$$

CENTRALIZED DECISIONS AFTER DEMAND DISRUPTION

Emergency happens after the manufacturer formulates a production plan, which leads to some changes of market scale in the dual-channel closed-loop supply chain. In the disruption model ,the demand functions of the supply chain under disruption (if $\Delta a = 0$,which is the baseline case) are

$$\begin{cases} \tilde{q}_d = \mu(a+\Delta a)-\tilde{p}_d+\theta\tilde{p}_r \\ \tilde{q}_r = (1-\mu)(a+\Delta a)-\tilde{p}_r+\theta\tilde{p}_d \end{cases} \quad (3)$$

We assume that there is a central decision-maker who seeks to maximize the total supply chain profit after the uncertainty is resolved. The new expression for the supply chain profit function is derived by the following equations:

$$\tilde{\Pi} = (\tilde{p}_d-c_m)\tilde{q}_d+(\tilde{p}_r-c_m)\tilde{q}_r+\lambda S(\tilde{q}_d+\tilde{q}_r)-C\lambda^2-k_1(\tilde{q}_d+\tilde{q}_r-q_d^*-q_r^*)^+-k_2(q_d^*+q_r^*-\tilde{q}_d-\tilde{q}_r)^+$$
$$(x)^+ = \max(x,0) \quad (4)$$

In Eq. (4), the third part on the right side of the function is the profit from re-manufacturing, the fourth part is the manufacturer's recycling input, the fifth part is the cost associated with increase in production quantity and the sixth part is the cost associated with decrease in production quantity.

If $\tilde{q}_d+\tilde{q}_r \geq q_d^*+q_r^*$, the total profit of the supply chain can be written as:

$$\tilde{\Pi}_1 = (\tilde{p}_d-c_m)\tilde{q}_d+(\tilde{p}_r-c_m)\tilde{q}_r+\lambda S(\tilde{q}_d+\tilde{q}_r)-C\lambda^2-k_1(\tilde{q}_d+\tilde{q}_r-q_d^*-q_r^*). \quad (5)$$

If $\tilde{q}_d+\tilde{q}_r \leq q_d^*+q_r^*$, the total profit of the supply chain can be written as:

$$\tilde{\Pi}_2 = (\tilde{p}_d-c_m)\tilde{q}_d+(\tilde{p}_r-c_m)\tilde{q}_r+\lambda S(\tilde{q}_d+\tilde{q}_r)-C\lambda^2-k_2(q_d^*+q_r^*-\tilde{q}_d-\tilde{q}_r). \quad (6)$$

We can obtain the optimal retail price because the profit functions of the supply chain, $\tilde{\Pi}_1 \, and \, \tilde{\Pi}_2$,are the strictly concave function of the retail price, $(\tilde{p}_d, \tilde{p}_r, \lambda)$, which exists three cases.

Case 1: $\Delta a \geq \dfrac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$.

Solve the function

$$\tilde{\Pi}_1 = (\tilde{p}_d-c_m)\tilde{q}_d+(\tilde{p}_r-c_m)\tilde{q}_r+\lambda S(\tilde{q}_d+\tilde{q}_r)-C\lambda^2-k_1(\tilde{q}_d+\tilde{q}_r-q_d^*-q_r^*)$$
$$s.t. \; \tilde{q}_d+\tilde{q}_r \geq q_d^*+q_r^*$$

We get the solution that when $\Delta a \geq \dfrac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$, the retail price of the electronic channel and the retail price of the traditional channel are

$$\begin{cases} \tilde{p}_{d1}^* = \dfrac{(a+\Delta a)[\mu+(1-\mu)\theta]}{2(1-\theta^2)}+\dfrac{4(c_m+k_1)C-(a-\Delta a)S^2}{4[2C-(1-\theta)S^2]} \\ \tilde{p}_{r1}^* = \dfrac{(a+\Delta a)[1-\mu+\mu\theta]}{2(1-\theta^2)}+\dfrac{4(c_m+k_1)C-(a-\Delta a)S^2}{4[2C-(1-\theta)S^2]} \end{cases} \quad (7)$$

The recycling rate of product is

$$\lambda_1^* = \frac{[a-\Delta a-2(1-\theta)(c_m+k_1)]S}{4C-2(1-\theta)S^2} \quad (8)$$

The sale quantity of the electronic channel and the sale quantity of the traditional channel are

$$\begin{cases} \tilde{q}_{d1}^* = \dfrac{\mu(a+\Delta a)-(1-\theta)(c_m+k_1)}{2}+\dfrac{(1-\theta)S^2[a-\Delta a-2(1-\theta)(c_m+k_1)]}{4[2C-(1-\theta)S^2]} \\ \tilde{q}_{r1}^* = \dfrac{(1-\mu)(a+\Delta a)-(1-\theta)(c_m+k_1)}{2}+\dfrac{(1-\theta)S^2[a-\Delta a-2(1-\theta)(c_m+k_1)]}{4[2C-(1-\theta)S^2]} \end{cases} \quad (9)$$

Case 2: $\Delta a \le -\dfrac{2(1-\theta)Ck_2}{C-(1-\theta)S^2}$

Solve the function

$$\tilde{\Pi}_2 = (\tilde{p}_d - c_m)\tilde{q}_d + (\tilde{p}_r - c_m)\tilde{q}_r + \lambda S(\tilde{q}_d + \tilde{q}_r) - C\lambda^2 - k_2(q_d^* + q_r^* - \tilde{q}_d - \tilde{q}_r)$$

$$st. \tilde{q}_d + \tilde{q}_r \le q_d^* + q_r^*$$

We get the solution that when $\Delta a \le -\dfrac{2(1-\theta)Ck_2}{C-(1-\theta)S^2}$, the retail price of the electronic channel and the retail price of the traditional channel are

$$\begin{cases} \tilde{p}_{d2}^* = \dfrac{(a+\Delta a)[\mu+(1-\mu)\theta]}{2(1-\theta^2)} + \dfrac{4(c_m-k_2)C-(a-\Delta a)S^2}{4[2C-(1-\theta)S^2]} \\ \tilde{p}_{r2}^* = \dfrac{(a+\Delta a)[1-\mu+\mu\theta]}{2(1-\theta^2)} + \dfrac{4(c_m-k_2)C-(a-\Delta a)S^2}{4[2C-(1-\theta)S^2]} \end{cases} \quad (10)$$

The recycling rate of product is

$$\tilde{\lambda}_2^* = \frac{[a-\Delta a-2(1-\theta)(c_m-k_2)]S}{4C-2(1-\theta)S^2}. \quad (11)$$

The sale quantity of the electronic channel and the sale quantity of the traditional channel are

$$\begin{cases} \tilde{q}_{d2}^* = \dfrac{\mu(a+\Delta a)-(1-\theta)(c_m-k_2)}{2} + \dfrac{(1-\theta)S^2[a-\Delta a-2(1-\theta)(c_m-k_2)]}{4[2C-(1-\theta)S^2]} \\ \tilde{q}_{r2}^* = \dfrac{(1-\mu)(a+\Delta a)-(1-\theta)(c_m-k_2)}{2} + \dfrac{(1-\theta)S^2[a-\Delta a-2(1-\theta)(c_m-k_2)]}{4[2C-(1-\theta)S^2]} \end{cases} \quad (12)$$

Case 3: $-\dfrac{2(1-\theta)Ck_2}{C-(1-\theta)S^2} < \Delta a < \dfrac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$

Solve the function

$$\tilde{\Pi}_3 = (\tilde{p}_d - c_m)\tilde{q}_d + (\tilde{p}_r - c_m)\tilde{q}_r + \lambda S(\tilde{q}_d + \tilde{q}_r) - C\lambda^2$$

$$st. \tilde{q}_d + \tilde{q}_r = q_d^* + q_r^*$$

When $-\dfrac{2(1-\theta)Ck_2}{C-(1-\theta)S^2} < \Delta a < \dfrac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$, we get $\tilde{p}_{d3} = \dfrac{a+\Delta a-(q_d^*+q_r^*)}{(1-\theta)} - \tilde{p}_{r3}$ under the condition $\tilde{q}_d + \tilde{q}_r = q_d^* + q_r^*$. Then, the retail price of the electronic channel and the retail price of the traditional channel are

$$\begin{cases} \tilde{p}_{d3}^* = \dfrac{(a+\Delta a)[\mu+(1-\mu)\theta]}{2(1-\theta^2)} + \dfrac{4c_mC-(a+\Delta a)S^2}{4[2C-(1-\theta)S^2]} + \dfrac{\Delta aC}{2(1-\theta)[2C-(1-\theta)S^2]} \\ \tilde{p}_{r3}^* = \dfrac{(a+\Delta a)(1-\mu+\mu\theta)}{2(1-\theta^2)} + \dfrac{4c_mC-(a+\Delta a)S^2}{4[2C-(1-\theta)S^2]} + \dfrac{\Delta aC}{2(1-\theta)[2C-(1-\theta)S^2]} \end{cases} \quad (13)$$

The recycling rate of product is

$$\tilde{\lambda}_3^* = \frac{[a-2(1-\theta)c_m]S}{4C-2(1-\theta)S^2} - \frac{\Delta aS}{2C}. \quad (14)$$

The sales quantity of the electronic channel and the sales quantity of the traditional channel are

$$\begin{cases} \tilde{q}_{d3}^* = \dfrac{\mu(a+\Delta a)-(1-\theta)c_m}{2} + \dfrac{S^2(1-\theta)[a-2(1-\theta)c_m]}{4[2C-(1-\theta)S^2]} - \dfrac{\Delta a}{4} \\ \tilde{q}_{r3}^* = \dfrac{(1-\mu)(a+\Delta a)-(1-\theta)c_m}{2} + \dfrac{(1-\theta)S^2[a-2(1-\theta)c_m]}{4[2C-(1-\theta)S^2]} - \dfrac{\Delta a}{4} \end{cases} \quad (15)$$

Theorem 1. *If a dual-channel closed-loop supply chain faces a demand function shown in Eq. (1), the revenue function with demand disruption is shown in Eq. (4). If* $\Delta a \ge \dfrac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$*, the optimal decision is given by Eq.(7)-(9). If* $\Delta a \le -\dfrac{2(1-\theta)Ck_2}{C-(1-\theta)S^2}$*, the optimal decision is given by Eq.(10)-(12). If* $-\dfrac{2(1-\theta)Ck_2}{C-(1-\theta)S^2} < \Delta a < \dfrac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$*, the optimal decision is given by Eq.(13)-(15).*

It can be seen in Theorem 1:

(1)The relationship between demand disruption and recovery rate is negative. When demand increases, the optimal recovery rate decreases; when demand decreases, the optimal recovery rate increases.

(2)If market scale increases greatly, the manufacturer increases production plan and channel sales quantity. Then, the recovery rate will decrease greatly.

(3)If market scale decreases greatly, the manufacturer decreases production plan and channel sales quantity. Then, the recovery rate will increase greatly.

(4)If the change of market scale is relatively small, the manufacturer does not need to adjust production plan. The allocated sales quantities between the manufacturer and the retailer are related to the market share but the total sales quantities do not change.

(5)If the change of market scale is zero, the supply chain in a stable stage, the optimal prices, recovery rate and production quantities are eagle to the benchmark model.

(6)The turning point that the manufacturer adjusts his production plan is closely related to channel substitution coefficient, recovery-input coefficient, unit variable cost and recovery-saving coefficient, which means that the robustness of production range is determined by the four elements mentioned above.

DECENTRALIZED COORDINATION OF THE SUPPLY CHAIN WITH IMPROVED REVENUE SHARING CONTRACT

Decentralized coordination strategy in the supply chain

There exists a manufacturer-led Stackelberg game in the dual-channel supply chain, which means that the manufacturer and the retailer negotiate ex ante and the retailer allocates $\phi_1 (0 \le \phi_1 \le 1)$ percent of the channel profit to the manufacturer. Then, market scale changes due to some disruptions. When the change is big enough to adjust the production quantity, the retailer bears $\phi_2 (0 \le \phi_2 \le 1)$ percent of the cost related to the production change and the manufacturer offers the retail price of the electronic channel and the retail price of the traditional channel, which are $\hat{p}_d^{RS}$ and $\hat{w}^{RS}$ respectively.

However, when demand change happens in a small stage, $-\frac{2(1-\theta)Ck_2}{C-(1-\theta)S^2} < \Delta a < \frac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$, and the manufacturer does not need to adjust the production quantity, the retailer's profit function is:

$$\hat{\pi}_r = (1-\phi_1)(\hat{p}_r - \hat{w})\left[(1-\mu)(a+\Delta a) - \hat{p}_r + \theta\hat{p}_d\right] \quad (16)$$

We calculate the retailer's reply function about the wholesale price and the price of the electronic channel, which is

$$\hat{p}_r = \frac{\hat{w} + \theta\hat{p}_d + (1-\mu)(a+\Delta a)}{2}. \quad (17)$$

Because the total profit either in centralized or decentralized decision remains equal as a result of the double marginal effect, the dual-channel supply chain coordinates at this time. In order to coordinate the supply chain, we need:

$$\hat{p}_d^{RS} = \hat{p}_d^*, \quad \hat{p}_r^{RS} = \hat{p}_r^*, \quad \hat{\lambda}^{RS} = \hat{\lambda}^*$$

$$\hat{w}^{RS} = 2\hat{p}_r^* - \theta\hat{p}_d^* - (1-\mu)(a+\Delta a). \quad (18)$$

Therefore, when $-\frac{2(1-\theta)Ck_2}{C-(1-\theta)S^2} < \Delta a < \frac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$,

$$\hat{\lambda}_3^{RS} = \hat{\lambda}_3^* = \frac{[a-2(1-\theta)c_m]S}{4C-2(1-\theta)S^2} - \frac{\Delta a S}{2C}$$

$$\hat{p}_{d3}^{RS} = \hat{p}_{d3}^* = \frac{(a+\Delta a)[\mu+(1-\mu)\theta]}{2(1-\theta^2)} + \frac{4c_m C-(a+\Delta a)S^2}{4[2C-(1-\theta)S^2]} + \frac{\Delta a C}{2(1-\theta)[2C-(1-\theta)S^2]}$$

$$\hat{w}_3^{RS} = \frac{2(a+\Delta a)(1-\mu+\theta\mu)+(1+\theta)\Delta a}{4(1-\theta^2)} + \frac{(S^2+2C)[4Cc_m-(a-\Delta a)S^2]}{4S^2[2C-(1-\theta)S^2]} - \frac{4Cc_m+(a+3\Delta a)S^2-2\mu(a+\Delta a)S^2)}{4S^2}.$$

When the change of market scale is so big that the manufacturer needs to adjust the production quantity, we consider two cases according to the method in part 3.

Case 1 : If $\Delta a \ge \frac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$,

the retailer's profit function is $\hat{\pi}_r = (1-\phi_1)(\hat{p}_r - \hat{w})\left[(1-\mu)(a+\Delta a) - \hat{p}_r + \theta\hat{p}_d\right] - \phi_2 k_1(\tilde{q}_d + \tilde{q}_r - q_d^* - q_r^*)$.(19)

The retailer's reply function about the wholesale price and the price of the electronic channel is

$$\hat{p}_r = \frac{\hat{w} + \theta\hat{p}_d + (1-\mu)(a+\Delta a)}{2} + \frac{k_1\phi_2(1-\theta)}{2(1-\phi_1)} \quad .(20)$$

We obtain: $\hat{\lambda}_1^{RS} = \hat{\lambda}_1^* = \frac{[a-\Delta a-2(1-\theta)(c_m+k_1)]S}{4C-2(1-\theta)S^2}$

$$\hat{p}_{d1}^{RS} = \hat{p}_{d1}^* = \frac{(a+\Delta a)[\mu+(1-\mu)\theta]}{2(1-\theta^2)} + \frac{4(c_m+k_1)C-(a-\Delta a)S^2}{4[2C-(1-\theta)S^2]}$$

$$\tilde{w}_1^{RS} = \frac{(a+\Delta a)(1-\mu+\theta\mu)}{2(1-\theta^2)} + \frac{(S^2+2C)[4C(c_m+k_1)-(a-\Delta a)S^2]}{4S^2[2C-(1-\theta)S^2]} - \frac{4C(c_m+k_1)+(a+3\Delta a)S^2-2\mu(a+\Delta a)S^2)}{4S^2} - \frac{k_1\phi_2(1-\theta)}{1-\phi_1}$$

Case 2 : If $\Delta a \le -\frac{2(1-\theta)Ck_2}{C-(1-\theta)S^2}$,

the retailer's profit function is $\tilde{\pi}_r = (1-\phi_1)(\tilde{p}_r - \tilde{w})\left[(1-\mu)(a+\Delta a) - \tilde{p}_r + \tilde{\theta} p_d\right] - \phi_2 k_2(q_d^* + q_r^* - \tilde{q}_d - \tilde{q}_r)$ (21)

The retailer's reply function about the wholesale price and the price of the electronic channel is

$$\tilde{p}_r = \frac{\tilde{w} + \theta \tilde{p}_d + (1-\mu)(a+\Delta a)}{2} - \frac{k_2\phi_2(1-\theta)}{2(1-\phi_1)} \qquad (22)$$

We obtain: $\tilde{\lambda}_2^{RS} = \tilde{\lambda}_2^* = \frac{[a-\Delta a-2(1-\theta)(c_m-k_2)]S}{4C-2(1-\theta)S^2}$

$$\tilde{p}_{d2}^{RS} = \tilde{p}_{d2}^* = \frac{(a+\Delta a)[\mu+(1-\mu)\theta]}{2(1-\theta^2)} + \frac{4(c_m-k_2)C-(a-\Delta a)S^2}{4[2C-(1-\theta)S^2]}$$

$$\tilde{w}_2^{RS} = \frac{(a+\Delta a)(1-\mu+\theta\mu)}{2(1-\theta^2)} + \frac{(S^2+2C)[4C(c_m-k_2)-(a-\Delta a)S^2]}{4S^2[2C-(1-\theta)S^2]} - \frac{4C(c_m-k_2)+(a+3\Delta a)S^2-2\mu(a+\Delta a)S^2)}{4S^2} + \frac{k_2\phi_2(1-\theta)}{1-\phi_1}$$

Pareto-improvement strategy

In order to improve the situation that the manufacturer and the retailer face when there is not contract, we can calculate the range of distribution coefficient, $\phi_1(0 \le \phi_1 \le 1)$, under revenue-sharing contract. The original revenue- sharing contract is still efficient within a wider range of disruption because there exists more optimal profit that is generated by the change of production quantity and we set $\phi_2 = \tilde{\pi}_r^* / (\tilde{\pi}_r^* + \tilde{\pi}_d^*)$, which makes that the distribution percentage of cost with the change of production quantity is equal to the percentage of profit of the two parties without any disruptions i.e. $(\Delta a = 0)$. $\tilde{\pi}_i^*(i = d, r)$ is the profit of the two parties without any disruptions. Therefore, we only discuss the situation that the disruption is relatively small.

When $-\frac{2(1-\theta)Ck_2}{C-(1-\theta)S^2} < \Delta a < \frac{2(1-\theta)Ck_1}{C-(1-\theta)S^2}$, the manufacturer does not need to adjust the original production plan.

When there does not exist the contract, the manufacturer only needs to maximize its own profit, and the corresponding retail price, the wholesale price and the recovery rate are

$$\begin{cases} \tilde{p}_d^0 = \frac{8c_m C-(a+\Delta a)S^2}{2[8C-(1-\theta)(1+3\theta)S^2]} + \frac{\mu(a+\Delta a)[4C-2(1-\theta)S^2-\theta(1-\theta)S^2]}{2(1+\theta)[8C-(1-\theta)(1+3\theta)S^2]} + \frac{(a+\Delta a)\theta[4C-(1-\theta)S^2-(1-\theta^2)S^2]}{2(1-\theta^2)[8C-(1-\theta)(1+3\theta)S^2]} \\ \tilde{w}_0 = \frac{4c_m C}{[8C-(1-\theta)(1+3\theta)S^2]} - \frac{\mu(a+\Delta a)[4C-(1-\theta)S^2]}{(1+\theta)[8C-(1-\theta)(1+3\theta)S^2]} + \frac{(a+\Delta a)[8C-(4-\theta-2\theta^2-\theta^3)S^2]}{2(1-\theta^2)[8C-(1-\theta)(1+3\theta)S^2]} \\ \tilde{\lambda}_0 = \frac{[(a+\Delta a)(1+\theta)-2(1-\theta)c_m-(1-\theta^2)c_m+(1-\theta)(a+\Delta a)\mu]S}{8C-(1-\theta)(1+3\theta)S^2} \end{cases}$$

$\tilde{p}_d^0 \ne \tilde{p}_d^*$ and $\tilde{\lambda}_0 \ne \tilde{\lambda}^*$, the supply chain does not realize coordination.

Suppose that $\tilde{\pi}_i^0$ is the total profit of the two parties when there does not exist the contract and $\tilde{\pi}_i^N$ is the total profit of the two parties when the retailer does not share the total profit. $w = \tilde{w}^{RS}$ is used by the manufacturer to coordinate the supply chain. When $w = \tilde{w}^{RS}$, the manufacturer's profit decreases $(\tilde{\pi}_d^0 > \tilde{\pi}_d^N)$ but the retailer's profit increases $(\tilde{\pi}_r^0 < \tilde{\pi}_r^N)$ .Then, the total profit increases $(\tilde{\pi}_d^0 + \tilde{\pi}_r^0 < \tilde{\pi}_d^N + \tilde{\pi}_r^N = \pi_d^* + \pi_r^*)$.

The principle to set the revenue-sharing coefficient, ϕ_1, is

$$\begin{cases} \tilde{\pi}_d^{RS} = \tilde{\pi}_d^N + \phi_1 \tilde{\pi}_r^N \ge \pi_d^0 \\ \tilde{\pi}_r^{RS} = (1-\phi_1)\tilde{\pi}_r^N \ge \pi_r^0 \end{cases},$$

and we can obtain the range of the corresponding revenue-sharing coefficients,

$\phi_1 \in [\frac{\pi_d^0 - \pi_d^N}{\pi_r^N}, 1 - \frac{\pi_r^0}{\pi_r^N}]$ and $\phi_2 = \frac{\pi_r^*}{\pi_r^* + \pi_d^*}$.

There exist the revenue-sharing coefficients, (ϕ_1, ϕ_2) with the revenue-sharing contract (w, ϕ_1, ϕ_2) and the wholesale price $(w = \tilde{w}^{RS})$, which can realize Pareto improvement for the two parties.

Theorem 2. *As to a dual-channel closed-loop supply chain, if there exists a demand function as shown in Eq. (1) and demand disruption happens, we can coordinate the supply chain by the improved revenue-sharing contract* (w,ϕ_1,ϕ_2) *when we make decentralized decision. The corresponding parameters are:*

$$\tilde{p}_d^{RS}=\tilde{p}_d^{*},\tilde{\lambda}^{RS}=\tilde{\lambda}^{*},\ \phi_1\in[\frac{\pi_d^0-\pi_d^N}{\pi_r^N},1-\frac{\pi_r^0}{\pi_r^N}],\ \phi_2=\frac{\pi_r^*}{\pi_r^*+\pi_d^*},$$

$$\tilde{w}^{RS}=\begin{cases}\dfrac{(a+\Delta a)(1-\mu+\theta\mu)}{2(1-\theta^2)}+\dfrac{(S^2+2C)[4C(c_m+k_1)-(a-\Delta a)S^2]}{4S^2[2C-(1-\theta)S^2]}-\dfrac{4C(c_m+k_1)+(a+3\Delta a)S^2-2\mu(a+\Delta a)S^2)}{4S^2}-\dfrac{k_1\phi_2(1-\theta)}{1-\phi_1}, & (\text{Case }1)\\ \dfrac{(a+\Delta a)(1-\mu+\theta\mu)}{2(1-\theta^2)}+\dfrac{(S^2+2C)[4C(c_m-k_2)-(a-\Delta a)S^2]}{4S^2[2C-(1-\theta)S^2]}-\dfrac{4C(c_m-k_2)+(a+3\Delta a)S^2-2\mu(a+\Delta a)S^2)}{4S^2}+\dfrac{k_2\phi_2(1-\theta)}{1-\phi_1}, & (\text{Case }2)\\ \dfrac{2(a+\Delta a)(1-\mu+\theta\mu)+(1+\theta)\Delta a}{4(1-\theta^2)}+\dfrac{(S^2+2C)[4Cc_m-(a-\Delta a)S^2]}{4S^2[2C-(1-\theta)S^2]}-\dfrac{4Cc_m+(a+3\Delta a)S^2-2\mu(a+\Delta a)S^2)}{4S^2}, & (\text{Case }3)\end{cases}$$

NUMERICAL EXAMPLES

Let $a=100$ (the market scale of a given product), θ=0.4(substitution coefficient between the electronic channel and the traditional channel), μ=0.3 (the rate of the online market scale), c_m=10 (unit production cost), $C=200$ (recovery input coefficient), $S=4$ (unit saving cost which derives from re-manufacturing), $k_1=k_2=4$ (unit cost which deviates from original production plan), $\phi_1=0.6$ (revenue-sharing coefficient negotiated ex ante). Several numerical examples are given to illustrate the results derived throughout the paper when different demand disruption happens.

If the manufacturer still uses the original revenue-sharing policy under stable state when demand disruption happens, i.e. Situation I, the total profits of the supply chain and the profits under centralized decision are shown in Table 2.

Table 2. Comparison between decentralized decision and centralized decision in the supply chain when the manufacturer does not respond to demand disruption

Δa	Case	p_d^{RS}	w^{RS}	λ^*	p_r^{RS}	π_d^{RS}	π_r^{RS}	$\prod^{RS}$	$\tilde{\Pi}^*$	ϕ_2
0	3	38.82*	53.11*	0.35*	69.318*	1268.4*	-231.6*	1036.82*	1609.92	-
20	1	38.62	20.37	0.45	59.9079	1486.9	288.733	1775.63	2389.35	-
10	1	38.62	20.37	0.45	56.4079	1290.1	263.176	1553.28	1986.88	-
5	3	38.62	20.37	0.45	54.6579	1191.8	250.397	1442.2	1799.64	-
-5	3	38.62	20.37	0.45	51.1579	994.988	224.84	1219.83	1419.61	-
-10	2	38.62	20.37	0.45	49.4079	896.603	212.061	1108.66	1240.75	-
-20	2	38.62	20.37	0.45	45.9079	699.831	186.503	886.335	911.247	-

(* is the result in the supply chain when there exists no contract.)

When demand disruption is limited, the manufacturer uses the corresponding parameters under the improved revenue-sharing contract including the wholesale price, the recovery rate, the optimal retail price and the quantities. When the range of demand disruption is wide, the results are shown in Table 3, which include ϕ_2 (the coefficient in the improved contract) and the profits under centralized and decentralized decision.

Table3. Parameters in the supply chain when manufacturer responds to demand disruption

Δa	Case	$\tilde{p}_d^{RS}$	$\tilde{w}^{RS}$	$\tilde{\lambda}^*$	$\tilde{p}_r^{RS}$	$\tilde{q}_d^{RS}$	$\tilde{q}_r^{RS}$	$\tilde{q}_d^{RS}+\tilde{q}_r^{RS}$	$\tilde{\pi}_d^{RS}$	$\tilde{\pi}_r^{RS}$	$\tilde{\Pi}^{RS}$	$\tilde{\Pi}^*$	ϕ_2
0	3	38.62	20.37	0.45	52.908	12.541	32.54	45.08	1372.3	237.62	1609.92	1609.92	-
20	1	47.78	25.84	0.32	64.924	14.189	38.19	52.38	2058.5	330.85	2389.35	2389.35	0.15
10	1	44.23	23.51	0.38	59.941	12.75	34.75	47.5	1732.7	254.18	1986.88	1986.88	0.15
5	3	42.43	24.39	0.4	57.432	12.041	33.04	45.08	1561.2	238.44	1799.64	1799.64	-
-5	3	34.81	16.34	0.5	48.384	32.041	13.04	45.08	1182.9	236.71	1419.61	1419.61	-
-10	2	33.02	16.44	0.53	45.875	12.332	30.33	42.66	1041.1	199.65	1240.75	1240.75	0.15
-20	2	29.46	14.9	0.58	40.892	10.893	26.89	37.79	758.944	152.3	911.247	911.247	0.15

As is shown in Table2, when there is no demand disruption, the profits of the supply chain without contract are less than that of the supply chain with contract. If the manufacturer optimizes his own profits, it will make that the retailer's profits are less than zero, which makes the retailer to leave the supply chain. If the wholesale price that the manufacturer offers is larger than the retail price of the electronic channel, it will make the retailer give up purchasing from the manufacturer. The case that the retailer directly purchases product in electric channel and sells it in traditional channel reflects what is called as fleeing goods in reality. The supply chain can be coordinated under the revenue-sharing contract in the stable state. If the manufacturer still uses the contract in the stable contract when demand disruption happens, the profits of the supply chain are less than those in centralized decision, which means that the supply chain cannot be coordinated.

As is shown in Table3, when there is no demand disruption, the profits of the supply chain with no contract are less than that of the supply chain with contract. If the manufacturer optimizes his own profits, it will make that the retailer's profits are less than zero, which makes the retailer to leave the supply chain. If the wholesale price that the manufacturer offers is larger than the retail price of the electronic channel, it will make the retailer give up purchasing from the manufacturer. The case that the retailer directly purchases product in electric channel and sells it in traditional channel reflects what is called as fleeing goods in reality. The supply chain can be coordinated under the revenue-sharing contract in the stable state. If the manufacturer still uses the contract in the stable contract when demand disruption happens, the profits of the supply chain are less than those in centralized decision, which means that the supply chain cannot be coordinated.

As is shown in Table3, when demand increases, the manufacturer's recovery rate decreases; and when demand decreases, the manufacturer's recovery rate increases. The total profits of the supply chain are equal to those in centralized decision, which means that the supply chain is coordinated.

CONCLUSIONS

This paper studies a dual-channel closed-loop supply chain in which the manufacturer is the leader and is responsible for recycling product. There The competition exists between the two channels. We calculate the optimal retail price, the sale quantities in the channels and recovery rate when the supply chain is in stable state. When demand disruption happens in the supply chain, we obtain an optimal centralized decision. The manufacturer needs not to adjust the original production plan when the disruption is under a certain range; and there is negative correlation between the recovery rate and the change of demand disruption. We cannot coordinate the supply chain under the revenue-sharing contract in the stable state when making decentralized decision，and we coordinate the supply chain under the improved revenue-sharing contract when demand disruption happens.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

ACKNOWLEDGEMENT

This work was supported by National Natural Science Funds of China (No. 71472026), Humanities and Social Sciences Youth Foundation of Chinese Ministry of Education (No. 12YJA630174), Research Fund for the Doctoral Program of Higher Education of China (No.. 20110185110022).

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