ISSN : 0974 - 7435 Volume 10 Issue 13

2014

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 10(13), 2014 [7196-7206]

Construction of three kinds of QR algorithm based on optimal power evaluation model for solving the matrix eigenvalue

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ABSTRACT

As the most basic, the most important knowledge system In linear algebra, matrix solution is the basis of learning and application of linear algebra. Therefore, the eigenvalues of the matrix becomes one of the focus of the study naturally.

In this paper, their are a series of studies for the eigenvalues of the matrix algorithm. It starts with the concept and nature of the matrix, the eigenvalues and algorithm. The final purpose of research is the three constraint conditions. They are simple in calculation,easy understanding, accurate results. Analysis of Schmidt Orthogonalization, elementary transformation of matrix and Givens transformation of the three algorithms, thus draw the conclusion: Computing characteristic value of matrix is not only the simple problems of mathematical evaluation and calculation, but also it relates to many areas of life such as engineering and technology. In the solution process, the algorithm is more common, the most widely one. And for Schmidt Orthogonalization, elementary transformation of matrix and Givens transformation is studied, Through the construction of optimal power evaluation model, found the elementary transformation of matrix is the most suitable for matrix eigenvalue algorithm finally.

KEYWORDS

Matrix; Characteristic value; Algorithm; Optimal power.

INTRODUCTION

The eigenvalue of the matrix is an important part of the learning process in linear algebra, So the discussion on its algorithm has become the focus of learning. There are a lot of algorithms of matrix eigenvalue, scholars have also conducted an in-depth study to this. For example, Tao Wang has carried on the concrete study, interview investigation, questionnaire on the question of using parallel machine or cluster systems though can solve many problems of matrix characteristic value, brings great consumption of resources In our country and even the world. Through the Jacobian iteration method features of the practical issues that arise in large matrix values are solved, get rid of the time consuming by serial method, problem solving in the study of the eigenvalues of a matrix has been widely concerned, and get the approval authority.

In addition, the eigenvalues of the matrix is obtained by the mathematical theory not only, also can be obtained by means of the necessary hardware. Shengguang Yuan put the matrix has been applied to want to mechanical engineering, metallurgical engineering and pharmaceutical engineering fields as background. He consulted a lot of literature, summarized the experience of previous studies, using the logic analysis, mathematical statistics, system analysis and other methods, to solve the problem of matrix eigenvalue and the application in fact was further studied. He pointed out that: the method, power is more suitable for sparse matrix to sparse matrix characteristic values of the so-called problem. Inverse power law is preferred to the problem of in turn for matrix eigenvalue matrix vector. The authors have their own views on solving the matrix and splitting method, which provides the theoretical basis for the development of our matrix cause.

Method, system analysis method of literature by Chunming Zhang, features of the optimization method for solving large matrix value problem has carried on the concrete study. He through the study of the famous Newton's theory of subspace accelerated stage specific theory of drama and for a large number of numerical experiments and numerical simulation. And finally, he point out: In the matrix has the question of how much the extreme eigenvalues of Newton theory is improved, and puts forward the practical solution of the calculating about symmetric matrix a plurality of extremists and a simple method.

This paper expounds on QR algorithm of matrix eigenvalue. Through the discussion of the matrix, eigenvalues and the QR algorithm with the concept and nature, introduces Schmidt Orthogonalization matrix elementary transformation, the Givens transformation, three algorithms, which provides guarantee for the application of matrix theory in solving practical problems.

MODEL ESTABLISHMENT

Basic researches

The related concept of matrix

Matrix is an important concept in linear algebra in the subject, and is an important part of linear algebra in the exam also. Matrix as the basic concepts of linear algebra, it and the determinant together constitute the basic theory of linear relationship. The matrix is a table, the number of rows and columns can be different. For a matrix concept, we understand the key of linear algebra is the basic nature and the common methods of learning.

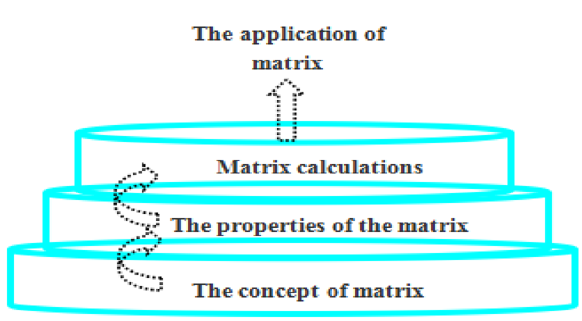

Figure 1 : The study of the matrix

In the learning process on the matrix, we must first understand the basic form of matrix and form. The matrix is composed of $m \times n$ rows of m rows and n columns of the table. Table for $a_{ij}(i=1,2,\cdots,m; j=,2,\cdots,n)$ elements. Table in the cross is called a row, vertical is called the column, the main form is shown as follows:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{nm} \end{bmatrix}$$

Secondly, focus on the basic properties of matrices is also learning. The basic properties of matrices including center special form of matrix, linear matrix operations, matrix multiplication, exponentiation and square matrix transpose content. In the matrix in the process of understanding, the contents of learning matrix contribute to the more skilled, deeper level of matrix operations, so as to master the concept and application of matrix.

Matrix operation is based on learning the basic concepts and properties of matrix on the learning process more deep discussion of its. Usually, matrix operations including the elementary transformation of matrix, computing characteristic value of matrix, characteristic and Vector similarity diagonalization of matrix, etc… In this process, to explore the matrix algorithm becomes the key of matrix operation. In this paper, the eigenvalues of the matrix operation of this important and its operation algorithm is discussed.

Finally, the study of any problem is to be applied to real life to solve practical problems, the study of matrix is no exception. Based on the matrix operations as a springboard, application of matrix is the study of the highest level, also will be the application of theoretical knowledge to practical life model.

The eigenvalue of the matrix

The eigenvalue of the matrix is not simple is the problem of evaluation in mathematics and computation problem in mathematical statistics, it relates to many problems in engineering and technology. For example, quantitative vibration, stability problems in engineering and technology, the relationship between the numbers of features can be applied to solve the matrix values.

In engineering design, application of eigenvalue is very extensive, too. We combined with the engineering design concept, engineering design knowledge, solve problem in engineering design to solve problem in engineering design by he study on characteristic value of matrix diagonalization, differential equations, and a continuous dynamical system.

First of all, introduce the concept of feature worth. The definition of A is n matrix, if the number of non-zero λ and n-dimensional column vector x, Satisfy the equation:

$$|\lambda E - A| = 0$$

To meet all the λ of the above equation are the eigenvalues of matrix A.

To solve the eigenvalue in general have a certain steps, the main steps as shown in the following flow chart:

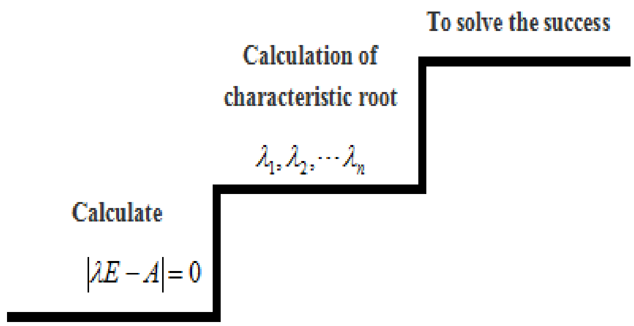

Figure 2 : Eigenvalue solution process

As shown above, solving eigenvalue main energy hungry two steps:

(1)Calculate the characteristic polynomial of the matrix A, and calculation of $|\lambda E - A| = 0$ with the same solution

(2)Obtained $\lambda_1, \lambda_2, \cdots \lambda_n$ by solving $|\lambda E - A| = 0$, $\lambda_1, \lambda_2, \cdots \lambda_n$ is square A matrix eigenvalues.

This process looks simple, but if you couldn't find the right method, you solve the problem will be complicated. Usually, there are many algorithms to calculate of characteristic value, QR algorithms are one of them. the algorithm of QR matrix include three methods, respectively is Schmidt Orthogonalization, elementary transformation of matrix and Givens transformation. In the subordinate research, this paper will introduce and analyze the specific for the 3 QR algorithms.

QR algorithm of matrix eigenvalue

In the process of calculating eigenvalues, the matrix QR decomposition, that is to say orthogonal triangular decomposition. It is a special triangular decomposition methods, feature is usually applied to solve the matrix. In addition, it plays a very important role in the method of least square. The classification below is the QR algorithm of matrix eigenvalues.

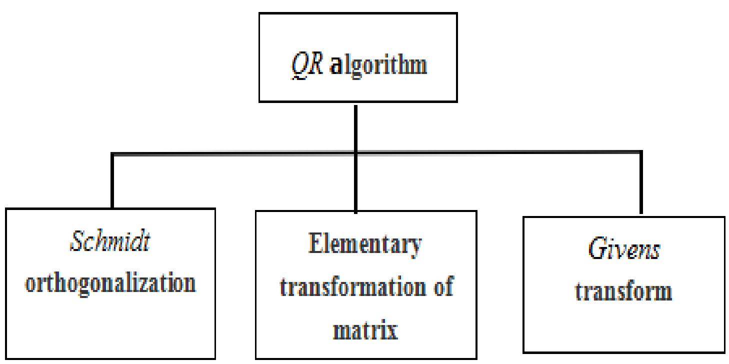

Figure 3 : Matrix eigenvalue QR algorithm

The above can be obtained, the QR algorithm includes three methods, respectively is Schmidt Orthogonalization, elementary transformation of matrix and Givens transformation. To further understand the QR algorithm, analyzes on the first.

Definition $A \in C^{n \times n}$, if there are n order orthogonal matrix of Q order and n order upper triangular matrix of R, making $A = QR$, it is called the A QR decomposition or orthogonal triangular decomposition. When $A \in R^{n \times n}$, then known as the decomposition of triangle A. The QR decomposition is: any one of full rank matrix of A real (complex), can be uniquely decomposed $A = QR$, of which Q orthogonal (unitary) matrix, R is the upper triangular matrix with diagonal elements.

Schmidt orthogonalization

Schmidt Orthogonalization is the process of linear independent vectors of norm $\alpha_1, \alpha_2, \cdots, \alpha_r$ for orthogonal vectors equivalent to $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_r$, the basic steps are as follows:

(1) Orthogonal

Take:

$$\beta_1 = \alpha_1$$

$$\beta_2 = \alpha_2 - \frac{[\beta_1, \alpha_2]}{[\beta_1, \beta_1]}\beta_1$$

$$\beta_3 = \alpha_3 - \frac{[\beta_1, \alpha_3]}{[\beta_1, \beta_1]}\beta_1 - \frac{[\beta_2, \alpha_3]}{[\beta_2, \beta_2]}\beta_2$$

$$\vdots$$

$$\beta_r = \alpha_r - \frac{[\beta_1, \alpha_r]}{[\beta_1, \beta_1]}\beta_1 - \frac{[\beta_2, \alpha_3]}{[\beta_2, \beta_2]}\beta_2 - \cdots - \frac{[\beta_{r-1}, \alpha_r]}{[\beta_{r-1}, \beta_{r-1}]}\beta_{r-1}$$

(2) Unitization

Take:

$$\varepsilon_1 = \frac{1}{\|\beta_1\|}\beta_1, \varepsilon_2 = \frac{1}{\|\beta_2\|}\beta_2, \cdots, \varepsilon_r = \frac{1}{\|\beta_r\|}\beta_r$$

To obtain orthonormal vector group $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_r$, and group $\alpha_1, \alpha_2, \cdots, \alpha_r$ equivalent vector.

For further analysis of the above process, we cited specific examples to explain:

Example: using Schmidt Orthogonalization method for matrix QR decomposition, the matrix is

$$A = \begin{pmatrix} 0 & 3 & 1 \\ 0 & 4 & -2 \\ 2 & 1 & 2 \end{pmatrix}$$

Solution:

Hypothesis $x_1 = (0,0,2)^T, x_2 = (3,4,1)^T, x_3 = (1,-2,2)^T$, then x_1, x_2, x_3 linearly independent, first orthogonalization:

$$y_1 = x_1 = (0,0,2)^T$$

$$y_2 = x_2 - \frac{(x_2, y_1)}{(y_1, y_1)}y_1 = x_2 - \frac{1}{2}y_1 = (3,4,0)^T$$

$$y_3 = x_3 - \frac{(x_3, y_1)}{(y_1, y_1)}y_1 - \frac{(x_3, y_2)}{(y_2, y_2)}y_2 = x_3 - y_1 + \frac{1}{5}y_2 = (\frac{8}{5}, -\frac{6}{5}, 0)^T$$

Then unit the matrix, so as to get:

$$e_1 = \frac{1}{2}y_1 = (0,0,1)^T, e_2 = \frac{1}{5}y_2 = (\frac{3}{5}, \frac{4}{5}, 0)^T$$

$$e_3 = \frac{1}{2} y_3 = (\frac{4}{5}, -\frac{3}{5}, 0)^T$$

Thus:

$$x_1 = y_1 = 2e_1$$

$$x_2 = \frac{1}{2} y_1 + y_2 = e_1 + 5e_2$$

$$x_3 = y_1 - \frac{1}{5} y_2 + y_3 = 2e_1 - e_2 + 2e_3$$

Therefore, the final results can be:

$$A = QR = \begin{pmatrix} 0 & \frac{3}{5} & \frac{4}{5} \\ 0 & \frac{4}{5} & -\frac{3}{5} \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 2 \\ 0 & 5 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

Elementary transformation of matrix

Elementary transformation of matrix in three similar origin of solutions of linear equations solution, not only for solving linear equations, but also for the rank in the matrix, the eigenvalue of the matrix and the matrix inversion etc.. Elementary transformation of matrix has many forms, one of the most employees for meals are the following forms:

TABLE 1 : The form of matrix elementary transformation

	content
First kind	The corresponding element of the exchange matrix of arbitrary two rows. (exchange the corresponding elements i and j ,denoted as $r_i \leftrightarrow r_j$);
Second kind	The non zero k multiplied by all the elements of a row of the matrix. (line i is multiplied by the number of k ,denoted as $r_i \times k$);
Third kind	The k times in a row from the matrix added to the corresponding elements which belong to another line. (k times in line j added to the line i , denoted as $r_i + kr_j$);

Elementary transformation of matrix not only includes the elementary row transformation, also includes a column elementary transformation. Three kinds of elementary transformation of matrix are reversible, and their inverse transform are the same type of their transformation, that is

TABLE 2 : The inverses transform of the row elementary transformation of a matrix

	the inverses transform
$r_i \leftrightarrow r_j$	$r_i \leftrightarrow r_j$
$r_i \times k$	$r_i \times (\frac{1}{k})$
$r_i + kr_j$	$r_i + (-k)r_j$

TABLE 3 : The inverses transform of the column elementary transformation of a matrix

	the inverses transform
$c_i \leftrightarrow c_j$	$c_i \leftrightarrow c_j$
$c_i \times k$	$c_i \times (\frac{1}{k})$
$c_i + kc_j$	$c_i + (-k)c_j$

Through the row and column elementary transformation of matrix, bring matrix A into simple shape matrix B ,. Even if the matrix elements are changed, but the basic properties and properties between the two matrix does not change, is still inseparable relationship. Among them, the most special and important property is that, when a row or column of elementary changes, the rank of a matrix is not changed.

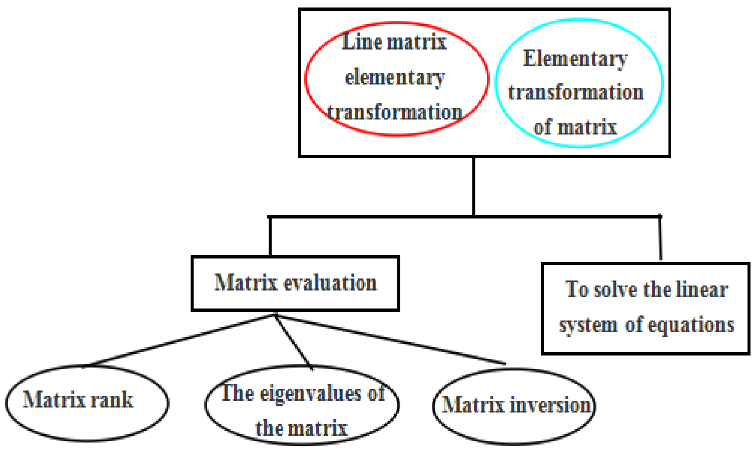

Figure 4 : Elementary transformation of matrix

Elementary transformation of matrix can achieve a linear equation group, matrix rank, the characteristic value of the matrix, matrix inversion and other content. It's one of the widely algorithm (QR) to seek the characteristic value of matrix.

Elementary transformation is one of the most simple and direct algorithm, not only easy to understand, but also easy to use. Now take the simplest example: By using elementary transformation take the matrix $A = \begin{pmatrix} 0 & 3 & -6 & 2 \\ 1 & -7 & 8 & -1 \\ 1 & -9 & 12 & 1 \end{pmatrix}$ becomes the row simplest form.

Solving:

$$A = \begin{pmatrix} 0 & 3 & -6 & 2 \\ 1 & -7 & 8 & -1 \\ 1 & -9 & 12 & 1 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -9 & 12 & 1 \\ 1 & -7 & 8 & -1 \\ 0 & -9 & -6 & 2 \end{pmatrix}$$

$$\xrightarrow{r_2 - r_1} \begin{pmatrix} 1 & -9 & 12 & 1 \\ 0 & 2 & -4 & -2 \\ 0 & -9 & -6 & 2 \end{pmatrix} = B$$

$$B=\begin{pmatrix}1&-9&12&1\\0&2&-4&-2\\0&-9&-6&2\end{pmatrix}\xrightarrow{r_2\times 1/2}\begin{pmatrix}1&-9&12&1\\0&1&-2&-1\\0&-9&-6&2\end{pmatrix}$$

$$\xrightarrow{r_3-3r_2}\begin{pmatrix}1&-9&12&1\\0&1&-2&-1\\0&0&0&5\end{pmatrix}\xrightarrow{r_3\times 1/5}\begin{pmatrix}1&-9&12&1\\0&1&-2&-1\\0&0&0&1\end{pmatrix}=C$$

$$C=\begin{pmatrix}1&-9&12&1\\0&1&-2&-1\\0&0&0&1\end{pmatrix}\xrightarrow{r_1+9r_2}\begin{pmatrix}1&0&-6&-8\\0&1&-2&-1\\0&0&0&1\end{pmatrix}\xrightarrow[r_1+8r]{r_2+r_{33}}\begin{pmatrix}1&0&-6&0\\0&1&-2&0\\0&0&0&1\end{pmatrix}$$

The request for the process by using elementary transformation take the matrix A becomes the row simplest form.

Givens transform

Givens transform is a kind of algorithm in QR algorithm. Similar to some of the characteristics of elementary transformation, it can be interpreted as a primary rotating transform. By the following Figure can explain the meaning of the *Givens* transform:

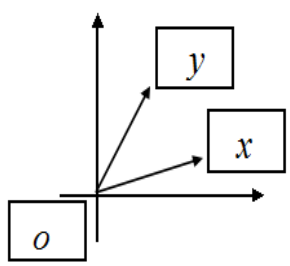

Figure 5 : *Givens* **conversion**

For the plane coordinate system in R^2, the rotation angle of the θ transform can be expressed as:

$$\begin{pmatrix}y_1\\y_2\end{pmatrix}=T\begin{pmatrix}x_1\\x_2\end{pmatrix},T=\geq\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}$$

Among them, T is an orthogonal matrix, also known as the plane of rotation matrix. In the orthogonal n-dimensional space in general, you can get a primary rotating transform, is the *Givens* transform.

For the *Givens* matrix test basic form can be obtained by the definition of n order matrix. Now suppose

$$c,s\in C^n;|c|^2+|s|^2=1$$

Can be obtained that

$$T_{kl}=\begin{array}{cc}
\left(\begin{array}{ccccccccccc}
1 &&&&&&&&&& \\
& \ddots &&&&&&&&& \\
&& 1 &&&&&&&& \\
&&& \bar{c} &&&& \bar{s} &&& \\
&&&& 1 &&&&&& \\
&&&&& \ddots &&&&& \\
&&&&&& 1 &&&& \\
&&& -s &&&& c &&& \\
&&&&&&&& 1 && \\
&&&&&&&&& \ddots & \\
&&&&&&&&&& 1
\end{array}\right)
&
\begin{array}{c}
\\ \\ \\ (k) \\ \\ \\ \\ (l) \\ \\ \\ \\
\end{array}
\\
\begin{array}{ccccccccccc}
&&& (k) &&&& (l) &&&
\end{array}
&
\end{array}$$

Among them, the matrix T_{kl} is an orthogonal matrix, and $\det T_{kl}=1$. And identified by the matrix T_{kl} transform is called the *Givens* transform.

For any vector $x \in C^n$, having *Givens* transform T_{kl}, making T_{kl} first a component is 0; the k component is non negative real numbers, the remaining components.

When given a vector $x \in C^n$, then there exists a set of *Givens* matrix $T_{12}, T_{13}, \cdots, T_{1n}$, , which makes $T_{1n}\cdots T_{13}T_{12}x=\|x\|_2 e_1$. So called *Givens* transformation vector $x \in C^n$ with the first natural basis vector e_1 collinear.

The evaluation model about QR algorithm for solving matrix characteristic value based on the optimal weight

As the above mentioned the QR algorithm for solving matrix characteristic values mainly are *Schmidt* orthogonal, the elementary matrix transformation and the *Givens* transformation. By expounding the three algorithms, can be a preliminary understanding of the basic principles and steps of various algorithms, but for solving matrix characteristic value, which algorithm I more suitable, more convenient is another important problem that at present we should discuss.

Goal programming is the system problem of the given planning through mathematic method, so as to obtain the optimal scheme accord with a set of practical objectives.

Now, this paper by using the method of goal programming, starting from the simple calculation, easy understanding, accurate results of three conditions, to evaluate the three methods.

Usually, the goal programming is to establish mathematical model.

The known $x_j\,(j=1,2,\cdots,n)$ is a decision variable for goal programming. There are m constraints, among them, have l weak target constraint. The deviation of the goal programming constraints is d^+ , $d_i^-\,(i=1,2,\cdots,l)$. Now suppose q priority, respectively $P_1, P_2, P_3, \cdots, P_q$. Because the weights are different in the same priority in P_k, respectively ω_{kj}^+ , $\omega_{kj}^-\,(j=1,2,\cdots,l)$. So the general mathematical expression for the goal programming is

$$\min z=\sum_{k=1}^{q}P_k\left(\sum_{j=1}^{l}\omega^-{}_{kj}d_j^- + \omega^+{}_{kj}d_j^+\right)$$

$$\begin{cases} \sum_{j=1}^{n} a_{ij} x_j \le (=, \ge) b_i, i = 1, \cdots, m \\ \sum_{j=1}^{n} c_{ij} x_j + d_i^- - d_i^+ = g_i, i = 1, \cdots, l \\ x_j \ge 0, j = 1, 2, \cdots, n \\ d_i^-, d_i^+ \ge 0, i = 1, 2, \cdots, l \end{cases}$$

On the objective function is calculated by MATLAB, to obtain the optimal solution. Now, it can use the ideas of goal programming list goals programming function:

$$\min z = P_1 d_1^- + P_2 (d_2^+ + d_2^-) + P_3 (3d_3^+ + 3d_3^-)$$

$$x_1, x_2, \cdots, x_i, d_i^+, d_i^- \ge 0, i = 1, 2, 3$$

Finally obtains the optimal solution about goal programming, namely in the calculation is simple, easy to understand, the exact result of three constraint conditions, an QR algorithm is most suitable for the characteristic values of a matrix,is also the most widely used as the elementary transformation of matrix, then *Schmidt* orthogonal, *Givens* transform.

CONCLUSION

On the characteristic value of matrix QR algorithm in the process of study, in view of the matrix, characteristic value, the concept and properties of QR algorithm for the basic set. And through the goal programming ideas, under simple in calculation, easy understanding, accurate results of three constraints, carried on the optimal power algorithm evaluation. Thus draw the conclusion.

First of all, computing characteristic value of matrix is not only simple mathematical evaluation problem, also not only is the problem of computing in mathematical statistics, it involves many problems in the field of engineering technology and life. And in the solving process for the characteristic value of matrix, the QR algorithm is a more common, is also one of the most widely used.

Secondly, in the *Schmidt* orthogonal, elementary transformation of matrix, *Givens* transform, three algorithms, the elementary transformation of matrix is simple in calculation, easy understanding, more accurate results algorithm, is also the most suitable method ofr solving matrix characteristic value, the most widely used.

ACKNOWLEDGEMENTS

This research is supported by the National Nature Foundation of China (Grant No. 61170317) and the National Science Foundation for Hebei Province (Grant No.A2012209043), all support is gratefully acknowledged.

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