

Constant-Roll $F(R)$ Gravity Inflation Solution

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Abstract

In this letter, the issue of flatness in the standard Big Bang model is approached with the different perspective where the inflation theory is framed with the constant-roll model of modified gravity $F(R)$ by addressing the reheating phenomena.

Keywords: *Inflation; Flatness problem; Modified $F(R)$ gravity; Constant roll approximation*

Introduction

So far the visible universe which we observe in our night sky has been explained overall with its key features by the standard concept of Big Bang Cosmology on a cosmic scale. The model further showed consistency with the observed data of Cosmic Microwave Background (CMB) and Large Scale Structure (LSS) involving the nuclear as well as plasma physics of the formation of the early universe to some extent [1]. But the model has faced various shortcomings when it is observed on different levels which provoked the emergence of inflationary theory to provide necessary explanations. It is suggested that the initial conditions of the large-scale structure of the universe such as galaxies can be explained by Inflationary theory. This theory is built on the basis of the expansion of the scale factor in a very minuscule time span exponentially in the early universe in an epoch [2]. Few complications in the standard model include the horizon, flatness and monopole problem. Solving these issues required models with scalar fields either by single [3] or manifold [4] and several models of inflation with the modified gravity in its various forms [5]. $F(R)$ modified gravity theory is one such theory renders a simple theoretical framework which also proved to be compatible with the recent observations of Planck Data [6]. So, in this letter, a quick opinion of approaching one of the inconsistencies of the standard model namely, Flatness problem, with the inflation theory by incorporating $F(R)$ framework is suggested.

Let us assume the cosmological model to be Friedmann-Robertson-Walker metric (FRW) in the polar coordinates as

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$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (1)$$

where t is time, $a(t)$ is the cosmic scale factor of the universe and k is the constant representing the spatial curvature. One of the equations which govern the function $a(t)$ is

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{k}{a^2} \quad (2)$$

where Hubble parameter $H = \frac{\dot{a}}{a}$, ρ is the density of the universe and G is the gravitational constant related to Planck's mass m_{pl} by $G = \frac{1}{m_{pl}^2}$. From the Eq. (2), the density parameter Ω can be obtained as

$$\Omega = \frac{k}{a^2 H^2} + 1 \quad (3)$$

Where $\Omega = \rho_i / \rho_c$, ρ_i is the density of the various components inhabiting the universe and ρ_c is the critical density. Types of the universe depend upon the conditions of density parameter Ω and k as

Case i: if $\Omega < 1, k < 0$, then it is an open universe.

Case ii: if $\Omega = 1, k = 0$, then it is a flat universe.

Case iii: if $\Omega > 1, k > 0$, then it is a closed universe.

The standard model suggests the condition of the case (ii) (*flatness*) imparting the instability in the shape of the universe hence a minor commotion of the magnitude Ω can instigate the curvature in the geometry. Further, the entity $1/aH$ is the function of t at different early epochs of the Inflationary era:

$$\frac{d}{dt} \left(\frac{1}{aH} \right) < 0 \quad (4)$$

Hence we can manipulate the Ω value at by incorporating time and it is recommended that the observed values prefer $\Omega - 1$ to be nearly zero. This problem can be solved by the perfect fluid cosmological model with the inclusion of the term cosmological constant (Λ) where the equation of state is $p = -\rho$, where p is the pressure. At the inflation zone, the scale factor indicates the exponential expansion of the universe retaining the constant H parameter is as follows:

$$a(t) \propto \exp \left(\sqrt{\frac{\Lambda}{3}} t \right) \quad (5)$$

Thus this commences the inflation by satisfying the condition in Eq. 4 at the early phase named as de sitter stage. However, the inclusion of Λ sets up a further consequence of the reheating process. A reheating era emerges once the inflationary era ends and thus proceeds with the radiation and matter dominating epochs. When the de sitter period is reached, due to its rapid expansion, it will be cold and at when the necessity of the thermalization arises for the matter in the universe [7]. The modified gravity theory of $F(R)$ can be used to stimulate the reheating activity immediately at the exit of inflation [8].

The Essential feature includes the gravitational action of the $F(R)$.

$$S = \frac{1}{2k^2} \int d^4x \sqrt{-g} F(R) \quad (6)$$

Where g is the trace of the background metric.

Constant-roll inflation model

The recent promising analysis of reheating using the $F(R)$ gravity is with the constant-roll approximation. The interesting property of the model is the non-Gaussianities predictions, so the Non- Gaussianities in the power spectrum of the primordial curvature perturbations can be studied precipitately in the future [9]. The assumption for the constant-roll is expressed as [10]

$$\frac{\ddot{H}}{2H\dot{H}} \cong \beta \tag{7}$$

Where β is a real parameter. Holding the above condition true for the gravitational equation of motion obtained by the variation of action (Eq. 6) with the metric tensor in the case of R^2 model [11], we get

$$\ddot{H} - \frac{\dot{H}^2}{2H} + 3H_i H = -3H\dot{H}, \ddot{R} + 3HR + 6H_i R = 0 \tag{8}$$

Where R is the Ricci Scalar and H_i is a phenomenological parameter of unit mass². Substituting the Eq. (7) in the first equation of Eq. (8) and by solving the differential equation, we get

$$H(t) = H_0 - H_I(t - t_k) \tag{9}$$

where H_I is the function of H_i and β .

Now, Let the time t_h be the time at which the reheating process begins, Hubble rate can be obtained for $t < t_h$ as in equ. (9) and for $t > t_h$, it is yielded by the first part of the Eq. (9) as

$$H(t) \cong \frac{\cos^2 \omega (t - t_h)}{\frac{3}{\omega} + \frac{3}{4}(t - t_h) + \frac{3}{8\omega} \sin 2\omega (t - t_h)} \tag{10}$$

Where ω can be obtained from the condition $\frac{\dot{H}^2}{2H} = 3H\dot{H}$, combined with the Eq. (9) and (10), we yield

$$\omega = \sqrt{\frac{3H_I}{2}} \ \& \ t_h = H_I H_0 \tag{11}$$

The scale factor at the reheating era is given as

$$a(t) = a_h \left(1 + \frac{\omega(t - t_h)}{4} \right)^{2/3} \tag{12}$$

Where $a_h = a_0 e^{\left(\frac{H_0^2 - 1}{2H_I - 12}\right)}$, a_0 is the initial scale factor at the commencement of inflation.

In summary, from the Eq. (10) , we can deduce that the Hubble rate increases with the time, proving that the reheating progress with the evolution of the time indicating the early time acceleration of the time and also the scale factor, on the other hand, expands abruptly which can be easily inferred from the Eq. (12). The term R^2 has combined the inflation with the dark energy [12]. Also, the minimal curvature specifies the possible correlation with the physical observations of the universe.

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