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Coherent Resonances of Saturated Absorption on the Transition with Level Momenta J=1/2 in Unidirectional Wave Spectroscopy

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Abstract

The physical processes that form the saturated absorption resonance spectra on the atomic transition with level momenta J=1/2 in the field of unidirectional waves of arbitrary intensities with a change in the direction of their polarization are studied both analytically and numerically. It is shown that the anomalies of the nonlinear resonance spectra and the processes forming them are determined by the direction of light wave polarizations, openness degree of the atomic transition, and the saturating wave intensity. The conditions are found under which the nonlinear resonance is exclusively coherent, due to the magnetic coherence of transition levels.

***Keywords:* Saturated absorption resonance; Unidirectional waves; Closed and open transitions; Wave polarization; Magnetic coherence of levels.**

Introduction

Studies of the nonlinear spectroscopic effects under the resonant interaction of several light fields with degenerate atomic transitions have been conducted for a long time. This interest is due to the variety of physical processes occurring in these systems, including the interference of atomic states, which manifests itself as narrow structures in the studied spectra. Note that the occurrence of atomic state coherence in two-photon processes has been known since pre-laser times. With the ad-vent of lasers, the studies of such coherent phenomena have significantly expanded (see, for example, [1]).

Subsequently, the resonances caused by the atomic state coherences in the presence of laser radiation were called Electromagnetically Induced Transparency (EIT) and Electromagnetically Induced Absorption (EIA) resonances. Having emerged in the research field of nonlinear optical phenomena under the interaction of laser radiation with gas media, the application scope of these effects has expanded over many other systems involving practical applications. However, the need for obtaining accurate analytical solutions and interpreting detected phenomena preserved their connection with nonlinear spectroscopy of gas media and

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stimulated the research in this area, including numerical modeling of experiments. Many of the previously discovered phenomena in those years were "rediscovered" and renamed in papers on EIT and EIA. These, as well as other misconceptions in the interpretation of a number of results at that time, were shown by us in the Introduction [2].

An important example of coherent phenomena at transitions from the ground state of alkali metal atoms is the EIT resonances, which are based on the phenomenon of coherent level population trap-ping (CPT), as well as resonances of the opposite sign are the EIA resonances, first found in.. [3-5]. The occurrence of these EIA resonances was explained in by the process of spontaneous transfer of the magnetic coherence (MC) of the excited state atomic levels to the ground state, which manifestation regularities in saturated absorption spectroscopy were first considered in [6,7]. Subsequently, the observation of EIA resonances was reported in [8, 9].

However, the anomalies of EIA resonance on degenerate atom transitions recorded later in experiments could not be explained due to the level MC transfer mechanism [6], (see, in particular, [8, 9]). Therefore, to explain these effects, other processes, such as optical pumping and CPT, and collisions, were also considered and not always justified, but gave externally similar resonance structures to the experiment [8,10]. Nevertheless, in the resent paper, developing the concept, the authors stated that the main mechanism for the formation of EIA resonances at closed transitions with any level moment values is precisely the spontaneous transfer of the MC of the upper state levels to the lower state [11,6]. At the same time, the authors did not analyze other processes that can form the narrow nonlinear resonance structures.

However, later our studies showed that in the formation of structures of nonlinear resonances at degenerate transitions, including EIA resonances, the other processes are more important than the process of spontaneous MC transfer of the upper state levels. In this case, the nature of the process-ses depends on the values of the total level momenta, on the openness degree of the atomic transition, on the directions of polarizations and the intensities of optical fields.

Thus, it was shown in that in a simple system of two levels, the narrow structure of nonlinear resonance in the field of two unidirectional waves manifested itself as an EIT resonance at the open transition, and as an EIA resonance at the closed transition [12]. The reason for the appearance of these structures is the coherent beats of the transition level populations in the field of two frequencies [13, 14].

When studying the formation processes of the saturated absorption and magnetic scanning resonances at transitions with level momenta J = 1, it was shown that the narrow resonance structures were formed in Λ-schemes of transitions, and were determined by the coherent beats of level populations (for parallel polarizations of fields), as in [15,12], and by the nonlinear interference effect (for orthogonal polarizations of fields) [13]. The effect of MC of levels forms the EIT and EIA resonances in the magnetic scanning spectra. In this case, the main contribution is made by the MC of the lower state levels, and the contribution of the spontaneous transfer of MC from the upper state to the lower one is small and manifests itself only in the form of an additive. The results of paper are also valid for the J→J and J→J – 1 transitions, since the nonlinear resonance spectra at the-se transitions are also formed in the open Λ-schemes [15].

Another situation occurs on the J→J+1 transitions, where the nonlinear resonance spectrum is formed mainly in the V-transition

schemes formed by sublevels with the maximum magnetic number M [16, 17]. The closed two-level transitions are realized just in V-schemes, in which the form of narrow resonance structures radically depends on the degree of openness of the atomic transition [12]. The effect of spontaneous transfer of the level MC at these transitions also does not affect qualitatively on appearance of the narrow resonance structures. At the same time, it was found that a change of the probe field intensity could change the type of narrow resonance (from EIT to EIA and vice versa) [16].

A review of the performed studies on the nonlinear and coherent effects in two-photon processes at various types of atomic transitions shows that the transition between levels with the total momenta J = 1/2 fell out from consideration when studying the effects. The present paper fills this gap. The simple structure of this transition levels makes it possible to carry out an analytical analysis and establish quantitative relations between the processes that form the saturated absorption resonance spectrum in the probe field method and to determine the contributions of such processes as the saturation effect of level populations, coherent beats of level populations, induction of magnetic coherence of levels by optical fields and its transfer between levels of different states. The obtained relations are important for identifying the contributions of these processes in the more complex atomic systems, including for clarifying the mechanism of the EIA resonance formation [5].

The Probe Field Absorption Spectrum in a System of two levels with Momenta J=1/2

Let us consider the problem of the probe field absorption spectrum in a gas of two-level atoms with total level momenta J = 1/2 in the presence of a strong wave field. The scheme of transition levels is shown in **FIG.1**. The strong wave is assumed to be plane, monochromatic, linearly polarized (frequency ω, wave vector **k**, electric field strength **E**) and resonant to the atomic transition *m-n* (transition frequency ω_{mn}). The probe wave is also monochromatic (frequency ω_{μ}, wave vector $\mathbf{k}_{\mu}$, electric field strength $\mathbf{E}_{\mu}$) with the linear polarization directed parallel or orthogonal to the strong-field polarization. The gas is assumed to be sufficiently rarefied to neglect collisions.

We will consider the problem in a coordinate system with the quantization axis directed along the strength vector E of the strong wave (**E** along the Z-axis). In this coordinate system, a strong field induces transitions between magnetic sublevels with a change in the magnetic quantum number ΔM=0, and the probe field causes transitions with a change of ΔM=0 in case of parallel field polarizations, or with ΔM = ±1 in case of orthogonal field polarizations (see **FIG.1**).

When solving the problem, we will use the kinetic equations for the density matrix of the atomic system [13]. In the case of a four-level quantum system (**FIG.1**) interacting with a dichromatic (strong and probe) field, the kinetics of diagonal elements ρ_{mi}, ρ_{nk} and off-diagonal elements ρ_{ik} of the density matrix in the relaxation constant model is described by the system of equations:

$$(\frac{d}{dt}+\Gamma_m)\rho_{mi} = Q_{mi} - 2\,\mathrm{Re}(i\sum_k \rho_{ik}V_{ki}) \tag{1}$$

$$(\frac{d}{dt}+\Gamma_n)\rho_{nk} = Q_{nk} + \sum_i A_{ik}\rho_{mi} + 2\,\mathrm{Re}(i\sum_i \rho_{ki}V_{ik}) \tag{2}$$

$$(\frac{d}{dt}+\Gamma_{ik})\rho_{ik} = -i\sum_j (V_{ij}\rho_{jk} - \rho_{ij}V_{jk}) + \delta\rho_{ik}^{s} \tag{3}$$

$$\rho_{nm} = \rho_{mn}^{*},$$

Here, indices *i* and *k* denote the magnetic sublevels of the upper and lower states, Γ_m , Γ_n are the level relaxation constants; Γ_{ik} are the relaxation constants; of polarizations on allowed ($\Gamma_{ik} = \Gamma$) and forbidden ($\Gamma_{ik} = \Gamma_m, \Gamma_n$) transitions between magnetic sublevels of *m* and *n* states; A_{ik} are the rates of spontaneous decay for the magnetic sublevels of the upper *m* state for each of the channels; Q_{mi}, Q_{nk} are the excitation rates of these sublevels, assumed to be given; $V = G\exp(-i\Omega t) + G^{\mu}\exp(-i\Omega_{\mu} t)$ is the interaction operator of an atom with the strong and probe fields, where $G = \mathrm{d}_{mn}\mathrm{E}/2\hbar$, $G^{\mu} = \mathrm{d}_{mn}\mathrm{E}_{\mu}/2\hbar$, and d_{mn} is the reduced matrix element of the transition dipole moment. For unmoved atoms the frequencies are: $\Omega=\omega-\omega_{mn}$, $\Omega_{\mu}=\omega_{\mu}-\omega_{mn}$. Taking the motion of atoms into account that is reduced to the replacement $\Omega \rightarrow \Omega - \mathbf{kv}$, $\Omega_{\mu} \rightarrow \Omega_{\mu} - \mathbf{k}_{\mu}\mathbf{v}$ in the equations**,** where **v** is the atomic velocity vector. In the case of the ground state, Γ_n is replaced by the average transit width determined by the transfer size of light beams and the most probable particle velocity $\mathrm{v_T}$.

For generality, the equation for the nondiagonal elements of the density matrix (3) includes a term $\delta\rho_{ik}^{s}$ that determines the spontaneous magnetic coherence transfer of the *m*-state sublevels to the *n*-state with decay rate A_c. This term is present in the equation for orthogonal polarizations and is absent for parallel polarizations of optical fields.

According to the standard procedure of the probe field method, the solutions of equations (1) - (3) up to terms linear in G^{μ}, are found in the form [13]:

$$\rho_{mi} = \rho_{mi}^{0} + r_{mi}\exp(-i\varepsilon t) + r_{mi}^{*}\exp(i\varepsilon t), \tag{4a}$$

$$\rho_{nk} = \rho_{nk}^{0} + r_{nk}\exp(-i\varepsilon t) + r_{nk}^{*}\exp(i\varepsilon t)), \tag{4b}$$

$$\rho_{ik} = \rho_{ik}^{0} exp(-i\Omega t) + r_{ik} exp[-i(\Omega + e)t] + r_{rk}^{1} exp[-i(\Omega - \varepsilon)t], \tag{4c}$$

Or

$$\rho_{ik} = r_{jik}\exp(-i\varepsilon t] + r_{jik}^{1}\exp(-i\varepsilon t]\left(here\ j = m,\ n\right), \tag{4d}$$

$$e = \Omega_m - \Omega.$$

Solutions (4a, 4b) describe the sublevel populations of the *m*-and *n*-states. Solution (4c) describes the polarization at the allowed transitions between the sublevels of *m*- and *n*-states (optical coherence), and solution (4d) describes the polarization at the forbidden transitions between sublevels of one state (MC of levels).

In the considered level system, we have the following relations between rates of the spontaneous decay of the magnetic sublevels A_1 and A_2 and the magnetic coherence A_c: $A_1=A_{mn}/3$, $A_2=2A_{mn}/3$, $A_1+A_2=A_{mn}$, $A_c= -A_{mn}/3$, where A_{mn} is the first Einstein transition coefficient [18]. The matrix elements of the interaction operator G at the transition between the magnetic sublevels $m\mathrm{M}_m \rightarrow n\mathrm{M}_n$ are $G_{m\mathrm{M}_m n\mathrm{M}_n} = G_{m\pm 1/2\, n\pm 1/2}$, which we denote as: $\mathrm{G_{m+1/2n+1/2}}=\mathrm{G_{+}}$, $G_{m-1/2\, n-1/2} = G_{-}$, $G_{m+1/2\, n-1/2} = G_{+-}$, $\mathrm{G_{m+1/2n-1/2}}=\mathrm{G_{+-}}$. For these elements the following relations are valid: $G_{+}= -G_{-}$, $G_{+-}= -G_{-+}$.

Based on this, we specify the indices of the density matrix elements in solution (4) and denote the population coefficients of the magnetic sublevels of *m*- and *n*- states as: $\rho_{\mathrm{ik}}=\rho_{\pm \mathrm{m}}$, $r_{mi} = r_{mi}$, $\rho_{ni}^{0} = \rho_{n\pm}^{0}$, $r_{ni} = r_{n\pm}$; the polarization coefficients at the allowed transitions between the sublevels of *m*- and *n*-states as: $\rho_{ik}^{0} = \rho_{mn\pm}^{0}$, $r_{ik} = r_{\pm}$, $r_{ik}^{1} = r_{\pm}^{1}$ (for parallel polarizations of fields), or $r_{ik} = r_{\pm\mp}$, $r_{ik}^{1} = r_{\pm m}^{1}$ (for orthogonal polarizations of fields); and the coefficients at the coherence for transitions

According to the solution procedure of equations (1-3) by the probe field method for $G >> G^{\mu}$, the system of equations in the zero-

order approximation with respect to G^{μ} for the transition between magnetic sublevels with M = 1/2 has the form:

$$\begin{aligned}
\Gamma_m\rho^0_{m+} + 2\,\mathrm{Re}(iG^*_+\rho^0_{mn+}) &= Q_{m+},\\
\Gamma_n\rho^0_{n+} - A_1\rho^0_{m+} - A_2\,\rho^0_{m-} - 2\,\mathrm{Re}(iG^*_+\rho^0_{mn+}) &= Q_{n+},\\
\Gamma - i\Omega\rho^0_{mn+} + iG_+(\rho^0_{m+} - \rho^0_{n+}) &= 0.
\end{aligned} \tag{5}$$

The system of equations for respective quantities for the transition between magnetic sublevels with M = - 1/2 has a similar form.

Due to the symmetry with respect to the signs of the magnetic state numbers and the relations $\rho^0_{m+}=\rho^0_{m-}$, $A_1+A_2=A_{mn}$ the solutions of the system of equations (5) are the same as for the two-level system [18]:

$$\begin{aligned}
\rho^0_{m+} &= N_m + \left[\frac{N_{nm}}{\Gamma_m T_{mn}}\frac{\kappa\Gamma^2}{\Gamma_s^2+\Omega^2}\right],\\
\rho^0_{n+} - \rho^0_{m+} &= N_{nm}\left[1 - \frac{\kappa\Gamma^2}{\Gamma_s^2+\Omega^2}\right],\\
\rho^0_{mn\pm} &= iG_\pm(\rho^0_{n+} - \rho^0_{m+})/(\Gamma - i\Omega),\ \rho^0_{m+} = \rho^0_{m-}, \rho^0_{n+} = \rho^0_{n-}, N_{nm} \equiv N_n - N_m
\end{aligned} \tag{6}$$

Where, $\Gamma_s = \Gamma\sqrt{1+\kappa}$, $\kappa = 2|G|^2\,\gamma_{mn}/\Gamma\Gamma_n\Gamma_m$, $\gamma_{mn} = \Gamma_m + \Gamma_n - A_{mn}$,

and N_n, N_m are the magnetic sublevel populations of *n*- and *m*-states in the absence of a strong field.

The systems of equations of the first-order approximation with respect to G^{μ} from (4), determining the probe field absorption spectrum, depend on the mutual direction of strong and probe field polarizations.

In the case of parallel field polarizations and the *m*M=1/2→*n*M=1/2 transition, the equations for the coefficients of level populations r_{m+}, r_{n+} and for polarization coefficients r_+, r^1_+ have the following form:

$$\begin{aligned}
(\Gamma_m - i\varepsilon)r_{m+} + iG_+^*r_+ - iG_+r^{1*}_+ &= iG^{\mu}_+\rho^0_{nm+},\\
(\Gamma_n - i\varepsilon)r_{n+} - A_1r_{m+} - A_2r_{m-} - iG_+^*r_+ + iG_+r^{1*}_+ &= -iG^{\mu}_+\rho^0_{nm+},\\
(p - i\varepsilon)r_+ + iG_+(r_{m+} - r_{n+}) &= -G^{\mu}_+i\,(\rho^0_{m+} - \rho^0_{n+}),\\
(p^* - i\varepsilon)r^{1*}_+ + iG_+^*(r_{m+} - r_{n+}) &= 0,
\end{aligned} \tag{8}$$

Where $p = G - iW.$

The system of equations for the $\mathrm{mM} = -1/2 \to \mathrm{nM} = -1/2$ transition is similar to system (8), but with the replacement of the signs + ↔ - in the indices. Both systems of equations are closed and uniquely solvable relative to the coefficients [18]. The solution for the r+ coefficient is as follows:

$$r_+ = (\mathrm{p} - i\varepsilon)^{-1}[iG^{\mu}_+\,(\rho^0_{n+} - \rho^0_{m+}) + i\mathrm{G}_+(r_{n+} - r_{m+})] = iG^{\mu}_+\,\frac{\mathrm{N}_{nm}}{\Gamma - i\Omega_{\mu}}\left(1 - \frac{\kappa\Gamma^2}{\Gamma_s^2+\Omega^2}\right)(1 - \mathrm{J}_{\|}(\varepsilon)), \tag{9}$$

where: $$J_{\|}(\varepsilon) = \left(\left|G_+\right|^2/\Delta\varepsilon\right)(2\Gamma - i\varepsilon)(\Gamma_m + \Gamma_n - A_{mn} - 2i\varepsilon)[\Gamma - i(\varepsilon - \Omega)]/(\Gamma + i\Omega),$$

$$\Delta_{\varepsilon} = [\Gamma - i(\varepsilon + \Omega)][\Gamma - i(\varepsilon - \Omega)](\Gamma_m - i\varepsilon)(\Gamma_n - i\varepsilon) + 2(\Gamma - i\varepsilon)(\Gamma_m + \Gamma_n - A_{mn} - 2i\varepsilon)\left|G_+\right|^2. \tag{10}$$

In the case of orthogonal field polarizations, the equations that determine the values linear in G^{μ} are formed from (4) by the coefficients r+−, r^1_{+-} , r−+, r^1_{-+} at the allowed transitions and by the coefficients r_j +-, r^1_{j+-}, r_j−+, r^1_{j-+} (j=m,n) at the transitions between the sublevels within each of the states (MC of levels). In this case, at the transition mM=1/2→n M= −1/2, the system of equations has the form:

$$
\begin{aligned}
(\Gamma_m - i\varepsilon) r_{m+-} + iG_-^{*} r_{+-} - iG_+ r^1_{+-} &= iG^{\mu}_{+-}\rho^0_{nm-},\\
(\Gamma_n - i\varepsilon) r_{n+-} - A_c r_{m+-} - iG_+^{*} r_{+-} + iG_- r^1_{+-} &= -iG^{\mu}_{+-}\rho^0_{nm+},\\
(p - i\varepsilon) r_{+-} - iG_+ r_{n+-} + iG_- r_{m+-} &= -iG^{\mu}_{+-}\ (\rho^0_{m+} - \rho^0_{n-}),\\
(p^{*} - i\varepsilon)\ r^1_{+-} + iG_-^{*} r_{n+-} - iG_+^{*} r_{m+-} &= 0
\end{aligned}
\qquad (11)
$$

The term $A_c r_{m+-}$ in the second equation of (11) describes the effect of transferring the MC of the upper state levels to the lower state. The equations for the coefficients at the *m* M = − 1/2→ *n* M =1/2 transition are obtained from equations (11) by replacing the signs + ↔ – in the indices. Both systems of equations are also closed and are solvable uniquely with respect to the coefficients. The solution of the equation system (11) gives the following expression for the coefficient r_{+-}:

$$
r_{+-} = iG^{\mu}_{+-}\frac{N_{nm}}{\Gamma - i\Omega_{\mu}}\ (1 - \frac{\kappa\Gamma^2}{\Gamma_s^2 + \Omega^2})(1 - J_{\perp}(\varepsilon)), \qquad (12)
$$

$$
r\partial\varepsilon:\ J_{\perp} = (|\,G_+\,|^2\ /\Delta_{\perp})(2\Gamma - i\varepsilon)(\Gamma_m + \Gamma_n + A_c - 2i\varepsilon)[\Gamma - i(\varepsilon - \Omega)]\,/\,(\Gamma + i\Omega), \qquad (13)
$$

$$
\Delta_{\perp} = [\Gamma - i(\varepsilon + \Omega)][\Gamma - i(\varepsilon - \Omega)](\Gamma_m - i\varepsilon)(\Gamma_n - i\varepsilon)\ +\ 2(\Gamma - i\varepsilon)(\Gamma_m + \Gamma_n + A_c - 2i\varepsilon)\left|G_+\right|^2.
$$

According to [13], the absorption spectrum of the probe wave is determined through the work of the field as: $P_{\mu} = -4\hbar\omega_{\mu} Re\left\langle i\sum r_+ G_+^{\mu^*}\right\rangle$ (for parallel polarizations), or $P_{\mu} = -4\hbar\omega_{\mu} Re\left\langle i\sum r_{+-} G_{+-}^{\mu^*}\right\rangle$ (for orthogonal polarizations of fields). Using solutions (9, 12), we obtain the following expressions for the work of the probe field:

$$
P_{\mu\perp} = 4\hbar\omega_{\mu}\left|G^{\mu}_{+-}\right|^2 \mathrm{Re}[\frac{N_{nm}}{\Gamma - i\Omega_{\mu}}(1 - \frac{\kappa\Gamma^2}{\Gamma_s^2 + \Omega^2})(1 - J_{\parallel}(\varepsilon))], \qquad (14)
$$

(for parallel polarizations of fields);

$$
P_{\mu\parallel} = 4\hbar\omega_{\mu}\left|G^{\mu}_{+}\right|^2 \mathrm{Re}[\frac{N_{nm}}{\Gamma - i\Omega_{\mu}}(1 - \frac{\kappa\Gamma^2}{\Gamma_s^2 + \Omega^2})(1 - J_{\perp}(\varepsilon))], \qquad (15)
$$

(for orthogonal polarizations of fields).

Expressions (14, 15) describe the absorption spectra of the probe field in a system with the total level momenta J=1/2, that are formed by the effect of saturation of the level populations (incoherent process) and by the coherent processes, including the level splitting effect by a strong field and nonlinear interference effects (NIEF) [13]. The differences between expressions (14) and (15) appear only in the coherent terms $J_{\parallel}(\varepsilon)$ and $J_{\perp}(\varepsilon)$. These differences are significant at transitions with the relaxation constant ratio of $\Gamma_m >> \Gamma_n$ in the case of unmoved atoms, and when taking into account the atomic motion, in the case of unidirectional light waves, when the field frequency detuning **ε** does not depend on the atom velocity, and the resonance parameters are determined by the transition relaxation constants appearing in terms $J_{\parallel}(\varepsilon)$ and $J_{\perp}(\varepsilon)$.

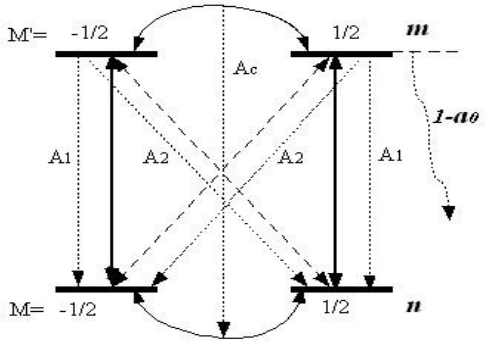

FIG 1. **Schematic diagram of kinetic processes for the transition with level momenta J=1/2. Solid and dashed arrows denote transitions under the action of strong and probe field, respectively; dotted arrows denote spontaneous transitions (rates A1, A2 and Ac); solid arc arrows denote magnetic coherence of levels.**

The case of unmoved atoms:

In the case of unmoved atoms, for parallel polarizations of fields and level relaxation constant ratios $\Gamma_m >> \Gamma_n$, the work of the probe field in a weak saturation field ($\kappa << 1$) near the center of the line ($\varepsilon << \Gamma, \Gamma_m$; $\Omega=0$) is determined from (14), as in the two-level system, by the expression [12]:

$$P_{\mu\|} = 4\hbar\omega_\mu \left|G_+^\mu\right|^2 \mathrm{Re}\{\frac{\delta N_{nm}}{\Gamma - i\varepsilon}\left[1 - \frac{2\left|G_+\right|^2}{\Gamma\Gamma_m}\left(\frac{\Gamma_m + A_{mn}}{\Gamma_m} + \frac{\Gamma_m - A_{mn} - \Gamma_n}{\Gamma_n - i\varepsilon}\right)\right. \tag{16}$$

Where $$\delta \mathrm{N}_{nm} = \mathrm{N}_{nm}(1 - \frac{\kappa\Gamma^2}{\Gamma_s^2 + \Omega^2}) = \frac{N_{nm}}{1 + k_0}, \kappa_0 = \kappa\Gamma^2/(\Gamma^2 + \Omega^2).$$

It follow from (16), that the probe field absorption spectrum has a Lorentzian shape with a half-width of Γ and a narrow structure with a half-width Γ_n near the frequency detunings $\varepsilon=0$. The structure amplitude is determined by the factor $S_\|=\Gamma_m-\Gamma_n-A_{mn}$, depending on the relations between the transition relaxation constants and the first Einstein coefficient A_{mn}, that can change its sign. When the ratio of constants is $\Gamma_m - A_{mn} > \Gamma_n$ the $S_\|$factor is positive (>0), and the structure manifests itself as a narrow dip, and when the ratio is $\Gamma_m - A_{mn} < \Gamma_n$ the $S_\|$ value is negative (<0), and the structure manifests itself as a peak. When $A_{mn}=\Gamma_m-\Gamma_n$ there is no narrow structure in the probe field spectrum, and the nonlinear resonance shape is determined by the population term in (16).

In the case of orthogonal field polarizations, the work of the probe field under the same conditions will be determined from (15) as:

$$P_{\mu\perp} = 4\hbar\omega_\mu \mathrm{Re}\{\frac{\delta \mathrm{N}_{nm}}{\Gamma - i\varepsilon}\left[1 - \frac{2\left|\mathrm{G}_+\right|^2}{\Gamma\Gamma_m}\left(\frac{\Gamma_m - A_c}{\Gamma_m} + \frac{\Gamma_m + A_c - \Gamma_n}{\Gamma_n - i\varepsilon}\right)\right]\} \tag{17}$$

It follows from expression (17) that a narrow Lorentz type structure with a half-width of Γ_n and amplitude determined by the factor $S_\perp = \Gamma_m - \Gamma_n + A_c$ is also formed in the absorption line shape. Since at considered transition $A_c = -A_{mn}/3$ (see above), then at the ratio $\Gamma_m >> \Gamma_n$, the factor $S_\perp \approx \Gamma_m - A_{mn}/3 > 0$ is always positive, and the structure at any transitions appears as a narrow dip.

Thus, the appearance of a narrow structure in the form of the saturated absorption resonance is due to the coherent beats of

magnetic sublevel populations of the transition (in the case of parallel field polarizations), as in a two-level system [12], or due to the MC induced by the optical fields between magnetic sublevels of the each state, as well as the level MC transfer from the upper state to the lower one with the rate constant $A_c = -A_{mn}/3$ (in the case of orthogonal field polarizations).

The ratio of the narrow structure amplitudes is determined from relations (16) and (17) by the factor $\delta S = (\Gamma_m - \Gamma_n - A_{mn})/(\Gamma_m - \Gamma_n - A_c)$. For the constant ratios $\Gamma_m >> \Gamma_n$ and $\Gamma_m - A_{mn} > \Gamma_n$ (open transitions), this factor is equal to $\delta S \approx (\Gamma_{mn} - A_{mn})/(\Gamma_m + A_{mn}/3) < 1$, i.e., the difference in the amplitudes of the coherent dips is small, in particular, $\delta S \approx 0.4$ for the branching parameter $a_0 = 0.5$. In the case of $\Gamma_m = A_{mn}$ (closed transitions), the amplitude ratio is as follows: $\delta S \approx 3\Gamma_n / 4\Gamma_m >> 1$, i.e. the amplitude of the coherent dip in orthogonal polarizations significantly exceeds amplitude of the coherent peak in parallel polarizations of fields.

In the case of orthogonal field polarizations, the relative contribution of the MC transfer process from the upper to the lower state is determined from (17) by the quantity $A_c/(\Gamma_m - \Gamma_n) \approx -A_{mn}/3\Gamma_m$. Since $A_c < 0$, the MC transfer effect leads to an increase in absorption at the line center. The maximum change in the absorption line occurs on the closed transitions and amounts to 30% of the resonance amplitude due to the MC of levels. In the case of open transitions the change in absorption value is significantly smaller (<10%).

The case of moving atoms:

In the case of moving atoms, the contribution of coherent processes to the work of the probe field in the counter propagating light waves is known to be insignificant (suppressed in the ratio of Γ/kv_T) and appears only at a high saturating field ($\kappa >> 1$) [13, 19]. In this case, the expressions obtained from relations (14, 15) for the work of probe field do not depend on the field polarizations and are represented as [18]:

$$P_{\mu\|} / |G_{\mu+}|^2 = P_{\mu\perp} / |G_{\mu+-}|^2 = 4\hbar\omega_\mu \langle N_{nm} \rangle \frac{\sqrt{\pi}}{kv_T} exp[-(\Omega / kv_T)^2] Re\left(F(x,y)\right), \quad (18)$$

Where $F(x,y) = 1 - \frac{1}{1-ix} + \frac{(y^2/2)(3-ix)}{(1-ix)[(1-ix)(3-ix)+y^2]}$, $x = \frac{2\Omega}{\Gamma_s}$, $y^2 = \frac{4|G_+|^2}{\Gamma_s^2}$.

In this case, the resonance is represented on the Doppler line by a dip at $\Omega = 0$ with a half-width of $\Gamma_s = \Gamma\sqrt{1+\kappa}$. The third term in F(x,y) is generated by the interference terms in $J_{\|}(\varepsilon)$ and $J_{\perp}(\varepsilon)$. It determines the resonance amplitude, its value at x=0 is positive and is equal to $J_o = \frac{6|G_+|^2}{3\Gamma_s^2 + 4|G_+|^2} = \frac{3}{3\Gamma T_{mn} + 2}$. In the value intervals $1 \geq \Gamma T_{mn} \geq ½$ and $\Gamma T_{mn} \geq 1$, the corresponding inequalities are fulfilled for the value J_0: $3/5 \leq J_o \leq 6/7$ and $3/5 > J_o > 0$.

In the case of unidirectional waves, when averaging expressions (14, 15) over the Maxwell velocity distribution of atoms at the large Doppler broadening ($kv_T >> \Gamma_s$) and the work of the probe field in the approximation of the first nonlinear corrections on the saturating field ($\kappa << 1$) for the field frequency detunings ($\Omega_\mu << kv_T$, $\Omega << kv_T$) is determined by the following expressions:

$$\left\langle P_{\mu\|,\perp} \right\rangle = 4\hbar\omega_\mu \left| G^{\mu}_{+,+-} \right| \left\langle N_{nm} \right\rangle \frac{\sqrt{\pi}}{kv_T} exp[-(\Omega_\mu / kv_T)^2] Re(F_{\|,\perp}(\varepsilon)), \quad (19)$$

$$\text{Where, } F_{\parallel,\perp}(\varepsilon) = 1 - \frac{\kappa\Gamma}{2\Gamma - i\varepsilon} - \frac{2|G|^2 \eta_{\parallel,\perp}(\varepsilon)}{2\Gamma - i\varepsilon}, \tag{20}$$

$$\eta_{\parallel} = \frac{\Gamma_m + \Gamma_n - A_{mn} - 2i\varepsilon}{(\Gamma_m - i\varepsilon)(\Gamma_n - i\varepsilon)} = \frac{\Gamma_m - \Gamma_n + A_{mn}}{\Gamma_m - \Gamma_n} \cdot \frac{1}{\Gamma_m - i\varepsilon} + \frac{\Gamma_m - \Gamma_n - A_{mn}}{\Gamma_m - \Gamma_n} \cdot \frac{1}{\Gamma_n - i\varepsilon} \tag{21}$$

$$\eta_{\perp} = \frac{\Gamma_m + \Gamma_n + A_c - 2i\varepsilon}{(\Gamma_m - i\varepsilon)(\Gamma_n - i\varepsilon)} = \frac{\Gamma_m - \Gamma_n - A_c}{\Gamma_m - \Gamma_n} \cdot \frac{1}{\Gamma_m - i\varepsilon} + \frac{\Gamma_m - \Gamma_n + A_c}{\Gamma_m - \Gamma_n} \cdot \frac{1}{\Gamma_n - i\varepsilon} \tag{22}$$

It follows from expressions (19-22) that, in a weak saturating field, a resonance is formed on the Doppler absorption line of the probe wave in the form of a hole with a half-width of 2Γ centered at the frequency difference ε=0. Near the line center, as in the case of unmoved atoms, the structures of an interference nature are formed, and their shapes are determined by the factors $\eta_{\parallel,\perp}(\varepsilon)$.

The factor values depend on the transition relaxation constants and the directions of field polarizations. The shapes of the resonance structures differ significantly at the transitions with a significant difference in the level relaxation constants (at $\Gamma_m >> \Gamma_n$). In this case, the factors $\eta_{\parallel,\perp}(\varepsilon)$ are representable near the line center (at $\varepsilon/\Gamma_m << 1$) in the forms:

$$\eta_{\parallel}(\varepsilon) \approx \frac{1}{\Gamma_m}\left(\frac{\Gamma_m + A_{mn}}{\Gamma_m} + \frac{\Gamma_m - \Gamma_n - A_{mn}}{\Gamma_n - i\varepsilon}\right) \tag{23}$$

$$\eta_{\perp}(\varepsilon) \approx \frac{1}{\Gamma_m}\left(\frac{\Gamma_m + A_c}{\Gamma_m} + \frac{\Gamma_m - \Gamma_n + A_c}{\Gamma_n - i\varepsilon}\right) \tag{24}$$

It follows from relations (23, 24) that, in the center of a broad dip (20) the narrow Lorentz type structures with half-width Γ_n are formed. The amplitudes of the structures and their signs are determined, as in the case of unmoved atoms, by the level relaxation constants, the first Einstein coefficient A_{mn} and the upper state MC relaxation constant A_c.

In the case of parallel field polarizations and the ratios of transition constants $A_{mn} < \Gamma_m - \Gamma_n$ (open transitions), the structure amplitude in (23) is positive, and the structure manifests itself in the shape of a resonance [19] as a narrow dip against the background of a wide one. At the constant ratios $\Gamma_m \geq A_{mn} > \Gamma_m - \Gamma_n$ (closed transitions), the amplitude of the structure in (23) is negative, and the structure manifests itself as a narrow peak against the background of a wide hole. At the constant ratio $A_{mn} = \Gamma_m - \Gamma_n$ there is no narrow structure shaped as a resonance [19].

In case of orthogonal field polarizations and $\Gamma_m >> \Gamma_n$ the value of multiplier $\Gamma_m - \Gamma_n + A_c = \Gamma_m - \Gamma_n - A_{mn}/3$ in (24) is always positive, and the structure at any transitions manifests itself as a narrow dip.

Let us compare the contributions of the populational and coherent terms to the resonance amplitude near the line center. For parallel polarizations of fields, the values of these contributions are as follows: $(1+2\Gamma_n/(\Gamma_m + \Gamma_n - A_{mn})):(\Gamma_m - \Gamma_n - A_{mn})/(\Gamma_m + \Gamma_n - A_{mn})$. Hence, at the constant values $\Gamma_m >> \Gamma_n$, the ratio of these contributions at the open transitions (with $a_0 = 0.5$) is equal to one, and at the closed transition ($a_0 = 1$), the ratio of amplitudes of the wide dip and the narrow peak is correlated as 3:1. These relations are observed also in the two-level system [12].

In the case of orthogonal field polarizations, the ratio of amplitudes of the populational and coherent dips is defined as: $(1+2\Gamma_n/(\Gamma_m + \Gamma_n - A_{mn})):(\Gamma_m - \Gamma_n - A_{mn}/3)/(\Gamma_m + \Gamma_n - A_{mn})$. At the open transition with $a_0 = 0.5$, these amplitudes correspond approximately as 1:2, and at the closed transition the contribution of the MC to the resonance amplitude will significantly exceed the contribution of the

incoherent term (in the proportion of $\Gamma_m / 3\Gamma_n >> 1$). The contribution of the transfer of the upper state MC to the lower state in the resonance amplitude (19) is determined by the quantity $A_c/(\Gamma_m - \Gamma_n)$. With $A_c = -A_{mn}/3$, the contribution of the MC transfer leads to an increase in the absorption at the line center (due to $A_c<0$), its maximum value is reached at the closed transition and amounting to ~30% of the amplitude of the narrow resonance structure.

Thus, the nonlinear resonance at a closed transition between levels with momenta J=1/2 for orthogonal field polarizations in a weak saturating field is exclusively coherent in its nature and is due to the MC of the transition levels(manly levels of the lower state). The resonance will also have a coherent nature in a strong saturating field (see below).

Numerical simulation of the nonlinear resonance shape in the saturating field of arbitrary intensity in unidirectional waves:

To identify the features of the saturated absorption resonance shape and the processes forming the resonance, we performed numerical simulations of the probe wave absorption spectrum based on the exact solution according to formulas (14, 15) when the saturating field intensity and the atomic transition parameters changed in a wide range of values.

When modeling the resonance shapes, the contributions of the incoherent effect of saturation of level populations and coherent effects, such as beating of the level populations (for parallel field polarizations), or effects of the level MC, induced by optical fields, and its transfer from the upper to the lower state (for orthogonal field polarizations), as well as splitting of the transition levels by the saturating field, were determined.

The calculations were carried out at the following parameters of the atomic transition: $\Gamma_m = 5.5 \cdot 10^7 c^{-1}$, $\Gamma_n = (10^{-2} \div 1)\Gamma_m$, $\Gamma_{mn} = (\Gamma_m + \Gamma_n)/2$. The ratio of the initial population levels was assumed to be $N_m/N_n \sim 10^{-2}$, and the Doppler line width was $kv_T = 5 \times 10^9 c^{-1}$. When integrating, the range of particle velocities was taken $\pm 3kv_T$ with a step of $\Delta kv_T = (10^{-3} \div 10^{-4})kv_T$, the strong field saturation parameter κ varied within 0.01÷50, and the branching parameter $a_0 = A_{mn}/\Gamma_m$ varied in the range 0÷1.

The calculations showed that the solutions in the approximation of the first nonlinear correction in the strong field intensity (16, 17) practically did not differ from the exact solution for the saturation parameters $\kappa<0.1$. For the parameters $\kappa \geq 0.2$ the differences between the solutions of both the first-order and next approximations become significant (especially at the closed transition) and it is impossible to use the analytical formulas of these approximations. The calculations revealed also significant differences between the spectra of nonlinear resonances and the resonance forming processes for open (with $a_0<1$) and closed ($a_0=1$) atomic transitions.

The characteristic shapes of the saturated absorption resonances and the contributions to the resonance of the incoherent saturation effect of the level populations are shown for open ($a_0=0.5$, **FIG. 2**) and closed ($a_0=1$, **FIG. 3**) transitions at width ratio $\Gamma_n/\Gamma_m = 2 \cdot 10^2$, $\kappa = 0.01 \div 10$, and parallel ore orthogonal polarizations of fields**.**

In the case of open transitions (**FIG. 2**), both for parallel and orthogonal polarizations of fields a resonance is formed on the Doppler absorption line contour of the probe wave shaped as a wide hole and a narrow structure (dip) near the line center. From the above dependences of the processes forming the resonance spectrum, it can be seen that the wide hole is due to the incoherent effect of saturation of the level populations by the strong field, and the coherent processes manifest themselves only near the line center (at $\varepsilon \sim 0$) in the form narrow dip structure of small amplitude. The calculations showed that, at saturation parameters $\kappa \leq 0.1$(curves 1), the narrow resonance structure is due to the coherent beats of the level populations (at parallel field polarizations) ore to the MC induced by optical fields between the magnetic sublevels of the same state (at orthogonal polarizations of fields). In this case, the widths of narrow structures are approximately the same and are determined by the width of the long-lived lower state. The amplitude of the structure at orthogonal polarizations is still greater than the amplitude of the structure at parallel polarizations of fields. The maximum ratio of these amplitudes at $\kappa \leq 0.1$ was ~2.

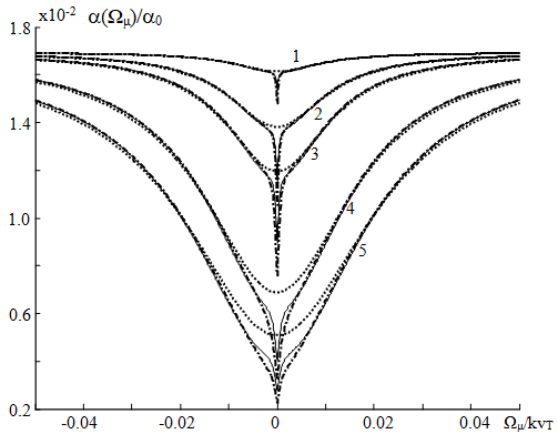

FIG. 2 **Shapes of the population part (dotted line) and total resonance for the open transition (a0 = 0.5) at Ω = 0, Γn/ Γm = 0.02, κ = 0.1 (1), 0.5 (2), 1.0 (3), 5.0 (4), 10 (5); solid lines - parallel polarization, dashed lines - orthogonal polarizations of fields.**

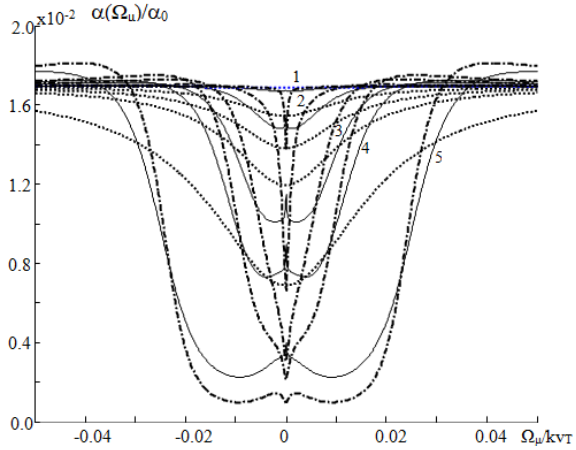

FIG. 3 **Shapes of population part (dotted line) and total resonance for the closed transition at Ω = 0, Γn/ Γm = 0.02, κ = 0.01 (1), 0.1 (2), 0.5 (3), 1.0 (4), 5 (5); solid lines - parallel polarizations, dashed lines - orthogonal polarizations of fields.**

An increase in the strong wave intensity (in the range of saturation parameters κ=0.01÷10) lead to an increase in the width and amplitude of the population part of a resonance, as in the two-level system [13, 19], and to an increase in the widths and amplitudes of narrow coherent structures, as well as to a decrease in the ratio between the amplitudes of narrow structures and their contrast with respect to the incoherent dip. At the same time, at parameters κ>1, a small amplitude lining appears in the spectrum of narrow structures (see curves 3-5). It is established from the spectra of **FIG. 2** that the half-widths of coherent dips obey the linear law as $\Gamma=\Gamma_n(1+\alpha\kappa_s)$, where α≈1 (for parallel polarizations), as in, or α≈2 (for orthogonal polarizations of fields). The estimation of the broadening coefficient from the relations (12, 13) for the transition parameters used in the calculations gives a value α≈5/3 [20].

In the case of closed transitions, the saturated absorption resonance shapes (**FIG. 3**, **FIG. 4**) turn out to be qualitatively different, depending on the directions of light wave polarizations, and are mainly due to the contributions of coherent processes. If the incoherent process, as in open transitions, forms a wide hole with the half-width of Γ in the absorption spectrum (**FIG. 3**, dotted

curves), then the coherent processes form a complex contour (**FIG. 4**) containing narrow structures near the line center (at ε=0), which appear as an additional dip, in case of orthogonal polarizations (dashed curves), and as a peak of small amplitude in case of parallel polarizations of fields (continuous curves). At the same time, at small saturation parameters κ<0.1 (curves 1, 2), the contribution of the saturation effect to the absorption resonance shape is small, and a resonance spectrum is formed exclusively by the coherent processes. For parallel polarizations of fields, these are the beats of the level populations forming a narrow peak structure, and for orthogonal polarizations, these are the MC forming a narrow dip structure. Moreover, the amplitude of a dip significantly exceeds the peak amplitude.

An increase of the saturation parameter κ in the range 0.01÷10 lead, as in the case of open transitions, to an increase in the amplitude and width (close to the square root of κ) of the incoherent part of the resonance, but it manifested itself differently in the spectra of coherent processes (**FIG. 3**, **FIG. 4**).

In the case of orthogonal field polarizations and saturation parameter κ=0.01÷0.5 (**FIG. 4**, curves 1-3), the coherent dip amplitude and its width change (according to the linear law in κ), and, at parameters κ~1 (curve 4), the additional structures appear in the line wings. The frequency interval between the maxima of these structures is $\Delta\omega\sim10^{-2}\Delta\omega_D\approx\Gamma_m\approx2\Gamma$.

With a further increase in the saturation parameter, the resonance shape is represented as three spectral components (curves 4, 5).With an increase of κ, the amplitude of the central component decreases and the amplitudes of the extreme components increase in their magnitude. The frequency distance between their maxima obeys the root dependence on parameter κ (linear dependence on the interaction parameter *G*). This fact indicates that the extreme components of spectrum are caused by the splitting of atomic levels under the strong field action.

In the case of parallel field polarizations with an increase of parameter κ, the amplitude of coherent peak and its contrast, increase also first with an increase of parameter κ, reach their maximum at values of κ~0.5÷1 and then decrease. In this case, the width of the coherent peak is narrowing slightly. At values κ>1, an increase in amplitudes and widths of the minima in the line wings is observed, as well as their shift relative to the line center. The shift magnitude obeys, as in the case of orthogonal polarizations of fields, to the root law on κ values.

Note that, for parameters κ>1, the shapes of resonance wings (for detuning frequencies ε>2Γ) are almost the same both for orthogonal and parallel polarizations of fields. In this case, the frequency shifts of the wing maxima obey to the root dependence on κ. These facts indicate that at parameters κ>1, the spectrum in the resonance wings is caused by the field splitting of transition levels (a coherent process), and, near the line center, it is caused by the beats of level populations (with parallel polarizations), or by the induced MC of levels (with orthogonal polarizations of fields).

A change in the lower level width (at $\Gamma_n\rightarrow\Gamma_m$, $\Gamma_{mn}\rightarrow\Gamma_m$) leads at any transitions to an increase in the widths of the main resonance hole and its narrow coherent structures with a decrease in their amplitudes at any polarizations of fields.

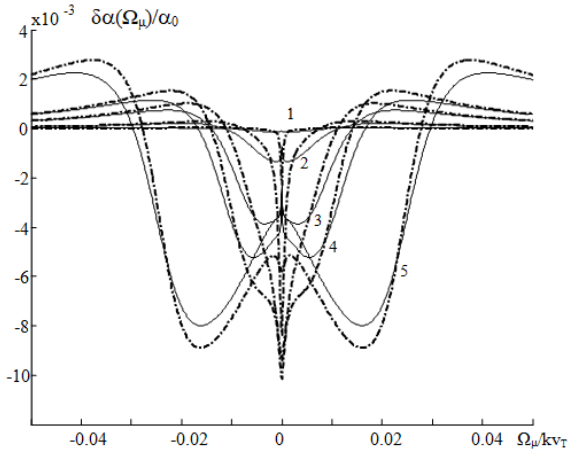

FIG. 4 **Shapes of coherent part of the resonance for the closed transition at Ω = 0, Γn/ Γm = 0.02, κ = 0.01 (1), 0.1 (2), 0.5 (3), 1.0 (4), 5.0 (5); solid lines - parallel polarizations, dash-dotted line - orthogonal polarizations of fields.**

FIG. 5 shows the shapes of contributions to the resonance from the transfer process of the MC from the upper state levels to the lower state for a closed transition at various values of the saturation parameter κ. It can be seen that the MC transfer process leads to an increase in the absorption coefficient in the line center, and its maximum value for the used relaxation constant values is realized at κ<~0.1, and amounts to ~30% of the resonance amplitude. In the case of open transition with a_0=0.5, this contribution is much smaller (less than 10% of the narrow structure amplitude). In this case, the dependences of the MC transfer shapes from the strong field intensity are specific for coherent processes.

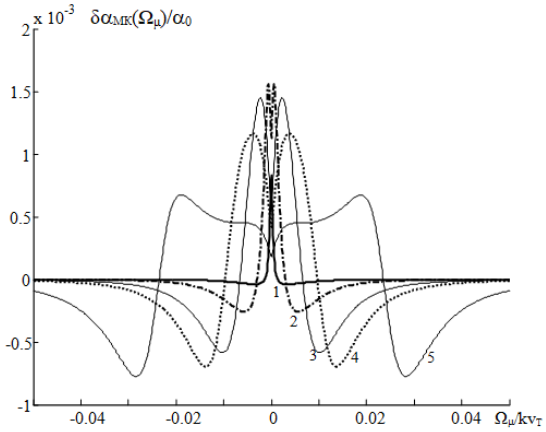

FIG. 5 **Shapes of the MC transfer from the upper state to the lower one for the closed transition with orthogonal polarizations of fields at Ω = 0; Γn/ Γm = 0.02; κ = 0.01 (1), 0.1 (2), 0.5 (3), 1.0 (4), 5.0 (5).**

According to [7], the MC transfer process forms in the saturated absorption spectrum of the probe wave the sign-changing interference structures, the integral of which is zero in the spectrum, which is typical for the NIEF [13]. In our case, the shape of the MC transfer process at parameters

k<0.05 is represented by the sign-changing interference structure (curve 1). At values κ=0.05÷0.5, the spectrum of contributions

splits near the line center into two components (curves 2, 3), and, at values $\kappa \geq 1$, the spectra acquire a complex shape (curves 4, 5). Estimation of the splitting in the case of curve 2 (the beginning of the effect, $\kappa=0.05$) gives the value $\Delta\omega \approx 6\times10^{-2}\Gamma_m \approx 3\Gamma_n$, i.e., it is determined by the half-width of the lower state levels. For curve 4 ($\kappa=1$), the splitting value is $\Delta\omega \approx 36\Gamma_n \approx 1.4\Gamma$, i.e., it is already determined by the homogeneous half-width of the transition line. This splitting manifests itself also as a total absorption resonance (**FIG. 4**, curve 4). Estimates of the interaction parameter G from relation (7) give the values $G \approx 0.1\Gamma_m \approx 5\Gamma_n$ (at $\kappa=0.05$) and $G \approx 0.5\Gamma_m \approx \Gamma$ (at $\kappa=1$).

The analysis of the dependences of the splitting value in the MC transfer spectra (Fig. 5) on the saturation parameter in the range $\kappa=0.01\div1$ shows that the splitting grows faster than the root dependence on κ, but slower than the linear one. In this case, the minimum splitting in the MC transfer spectrum, as follows from the estimates, is determined by the relaxation constant of the lower levels Γ_n and is due to the splitting of the lower state levels by the strong saturating field.

Thus, the spectrum of the MC transfer turn out to be more sensitive to the intensity of the saturating field, since here the splitting effect begins to manifest itself already at the splitting of the lower long-lived levels, while the splitting in the total absorption resonance spectrum (see **FIG. 4**) manifests itself at the level splitting larger than the homogeneous transition line width, which in the absence of collisions is determined by the relaxation constants of the both state levels [13].

It should be noted that, for closed transitions, in contrast to open ones, the action of the saturating field manifests itself more radically in the shape of resonances-both near the center and in the line wings. Since the incoherent (population) part of resonances is determined by the saturation parameter κ, while the coherent part is determined by the intensity of the saturating field $|G|^2$ (see formulas (10, 13)), then, with the same values of the saturation parameter, the intensity of the saturating field at closed transitions will exceed in $[(1-a_0)\Gamma_m/\Gamma_n+1]$ times the corresponding intensities at open (with $a_0<1$) transitions, which causes the spectral differences between contributions of coherent processes and the total resonance shapes at these transitions.

Conclusion

The presented analytical and numerical studies of the saturated absorption spectra on transition with level momenta J=½ in the probe field method with unidirectional laser waves demonstrate their dependence on the values of the level relaxation constant, on the degree of openness of the atomic transition, the intensities and the mutual orientation of polarizations of the strong and probe waves. In this case, the mutual orientation of wave polarizations is decisive in the formation of the saturated absorption spectra, since the specific features of the spectra are determined by the contributions of coherent processes, the character of which at the considered transition depends on the directions of the light wave polarizations.

In case of parallel wave polarizations, these are coherent beats of the level populations in two level transition schemes, the specific relaxation of which determines the type of narrow resonance structures: a peak at the closed transition and a dip at the open transition. In case of orthogonal wave polarizations, it is the magnetic coherence of levels induced by the optical fields and its transfer from the upper to the lower state, which create a narrow structure shaped as a dip at any transitions.

Moreover, the main contribution to the resonance amplitude is made by the levels of the lower state, the contribution of the MC transfer from the upper state to the lower one is small and manifests itself only in the form of an additive near the line center. At the same time, the shape of the MC transfer has the sign-changing interference structures inherent to coherent processes. The MC transfer spectrum turn out to be more sensitive to the saturating field intensity, since here the level splitting effect begins to manifest itself already with the splitting of the lower long-lived state levels

while, in the full resonance spectrum, this effect, at any wave polarizations, manifests itself in the wings of resonance with the

splitting of levels larger than the homogeneous transition line width.

Note that the above studies of the saturated absorption resonance at transition with level momenta J=1/2 are important for studying the formation processes of nonlinear resonances in more complex atomic systems and, in particular, for clarifying the mechanism of formation of the EIA resonance.

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