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Center of mass acceleration of an isolated system of two particles with time variable masses interacting with each other via Newton's third law internal forces: Mach effect thrust 1

Abstract

Utilizing Newton's second law of motion, it is shown that an isolated system consisting of two particles with time variable masses interacting with each other via Newton's third law forces and no net external force can produce a DC (unidirectional) acceleration of the center of mass of the system, without any net loss or gain of mass in a cyclic process. There is no rocket type thrust in the usual sense of ejecting propellant, since it is supposed that there is no relative velocity along the direction of motion associated with the mass changes. A surprising result is that it is necessary to rederive the expression for the acceleration of the center of mass of a system when the masses are time variable, the usual expression producing zero acceleration of the center of mass under very general conditions of time variable masses and any Newton's third law forces of interaction between them. There is no violation of momentum conservation, since the total mechanical momentum of the two particle system is not conserved, a result which is independent of the exact mechanism for producing the time variable masses. Explicit expressions are obtained for the acceleration of the center of mass and time rate of change of the total momentum for a simple model of forces and mass fluctuations with harmonic time variation. Implications of these results are discussed, including their application to propellantless Mach Effect Thruster's (MET's).

Keywords

Mechanics of variable mass systems; Spacecraft propulsion; Space drive; Mach effect thruster.

INTRODUCTION

Space drive is a general term used to encompass the ambition of propulsion without propellant[1-5]. As defined by Millis (1997), it is an idealized form of propulsion where the fundamental properties of matter and spacetime are used to create propulsive forces anywhere in space without having to carry and expel a reaction mass. Such an achievement would revolutionize space travel as it would eliminate the limiting factor of requiring an extremely large mass of propellant. Without such a discovery, human interstellar exploration may not be possible. The two largest issues facing this ambition according to Millis are: first, to find a way for a vehicle to induce external net forces on itself (or the equivalent thereof); and second, to satisfy conservation of momentum in the process[4].

Numerous different proposals for space drives have appeared. Millis[5] introduced seven hypothetical proposals, three of which were collision sails of various types, and the other four involved hypothetical field drives. Four hypothetical field drives, the diametric drive, pitch drive, bias drive, and disjunction drive, were presented and discussed. These involved such concepts as negative mass[6,7] for the diametric drive. A pitch drive where somehow an unspecified mechanism could produce an asymmetric gravitational potential along the direction of motion of the vehicle. The bias drive, which would also produce an asymmetric potential or force using modifications to space, such as the Alcubierre proposal[8], which uses large quantities of positive and negative energy density to do much the same as the positive mass, negative mass proposals.

The disjunction drive assumes that it is somehow possible to separate the so called source mass from the reactant mass. The source mass is defined to have the property that it only causes a field, but does not react to one. The reactant mass is defined to react to the presence of a field, but not to cause one. Existing evidence strongly indicates that the source mass, reactant mass, and inertial mass properties are not separable. None of the four Millis field drives have been implemented and is not known if the conditions necessary for them are even possible.

In addition to the above, several individuals have made claims of propellantless propulsion methods. Millis[1] lists and briefly reviews 24 of them. Of the 20 proposals that are not misinterpretations, only 4 have experiments associated with them.

One proposed method of propellantless propulsion not reviewed by Millis is the so called Em drive developed by British engineer Roger Shawyer[9]. Thrust is claimed to be produced by the amplification of differential radiation pressure force on the flat ends of a high Q factor tapered microwave cavity. Despite reports of experimental results showing thrust, these appear in a series of unreviewed conference papers published on Shawyer's website. In response to a controversial 2006 article on the Em drive in New Scientist[10], a reviewer of Shawyer's work stated that "both theory and experiment were fatally flawed"[11]. In addition, John Costella wrote a paper[12] in which he explains why Shawyer's theory is wrong. Namely, Shawyer's analysis neglected the normal component of momentum transferred by reflections from the tapered walls, which result in additional axial forces which preserve Newton's second law (conservation of momentum) and results in zero net thrust. In addition, Shawyer neglected to explicitly account for the reduced axial components of momentum incident on the small end cap due to the increased angle of incidence caused by reflections from the tapered section. Finally, the claimed thrust levels exceed the maximum radiation pressure force available for a given electromagnetic power by several orders of magnitude.

Despite its highly questionable status, Chinese researchers recently reported[13-15] that they have performed theoretical calculations and experiments on microwave thruster devices which have validated the Em drive technology and shown it to work. In contrast to the elementary calculations of Shawyer, Yang *el al* have done finite element electromagnetic theory calculations using the Maxwell stress tensor integrated over the waveguide surface (including the side walls) to find the net force, and published their theoretical and experimental results in a peer reviewed journal[14,15]. It remains to be seen if the calculations are correct and if these results will stand up to careful scientific scrutiny. In approaching the topic of propellantless propulsion, one must be very skeptical, since many of the schemes, such as the Em drive, violate (or appear to violate) momentum conservation.

The longest surviving and most experimentally tested propellantless propulsion method is that of Woodward[16], who has reported on the development of a Mach Effect Thruster (MET) that produces thrust without propellant by inducing small cyclic fluctuations in mass in a system oscillating back and forth. He has published several peer reviewed articles on the experimental and theoretical aspects of this over a period of more than two decades[17], and recently written a book on the subject[18]. According to Woodward, the mass fluctuations are induced by time changing energy density in a body undergoing acceleration. He has derived formulas for the mass fluctuations using a theory based on a version of Mach's principle. Several investigators trying to measure these effects have reported positive or inconclusive experimental results. The experiments are difficult to replicate due to a number of challenging experimental problems associated with the properties of the materials used and the extreme demands put on them, as well as the experimental sensitivity required to reliably detect and characterize the small unidirectional force and rule out spurious signals from other effects.

Mach's principle has not been incorporated into mainstream physics, most often it has been thought of as a philosophical principle with little quantitative content. As it has not often been used for calculations, one is naturally skeptical and cautious of accepting quantitative claims based on it. In fact there are several versions of Mach's principle, one article lists ten different versions[19], books devoted to it have been published[20-22]. One conference book lists twenty one different versions[22], and there are several recent calculations which incorporate various versions of Mach's principle[23].

In this light, the present work was motivated by a desire to better understand and either prove or disprove the unidirectional thrust reported by Woodward. In the process of so doing, the author encountered some simple surprises in re-examining commonly held notions of Newton's second law and its application to variable mass systems, a subject that is not covered in detail in standard courses on classical mechanics, other than some simple examples of rocket thrust or transferring mass to and from moving objects[24]. To the best of our knowledge, we have not found any works on time variable masses that treat cases relevant to Woodward's proposals, in particular, cyclic variation of the masses and the required reevaluation of the center of mass acceleration and resulting notions of an isolated system. Woodward has derived an expression for the unidirectional thrust force that an accelerated mass undergoing time dependent, cyclic mass fluctuations exerts on a much heavier (effectively infinite) reaction mass[25]. However, he has not shown from Newton's 2nd and 3rd laws that the center of mass of an isolated system subject to zero net external force can have a unidirectional acceleration without any net loss or gain of mass in a cyclic process, regardless of the exact mechanism for the origin of the variable masses.

MODEL FOR ISOLATED VARIABLE MASS SYSTEM

The purpose of the present work is to explicitly demonstrate a mechanism for propellantless propulsion by showing that an isolated system using time variable masses in a cyclic variation, with cyclic internal forces having no net external force, and that explicitly satisfies Newton's 2nd and 3rd laws of motion, can produce a unidirectional acceleration. To that end, consider a system of two masses coupled by a spring and also a driving force acting on each mass, as shown in Figure 1. It is assumed that there are no external forces acting on either particle since we are considering an isolated system. Although our model is extremely simple, it is easily extended and applied to more complicated systems of interacting particles, but the essence of the effects is captured with just two variable mass particles interacting with a time dependent force obeying Newton's third law of motion. In fact a single variable mass particle interacting with a fixed mass particle is sufficient to demonstrate the primary Mach Effect thrust, but this is obtained as a limiting case of the two variable mass particles problem.

In order to study the effects on the center of mass motion and a possible acceleration of the center of mass, we take the motion to be in one dimension along the *x* axis. Then for each of the oscillator masses we write (including for the case of time dependent masses with no relative velocity of the convective momentum entering or leaving the masses)

$$m_1 \ddot{x}_1 = -k(x_1 - x_2 - l_0) + F_{12} \tag{1}$$

$$m_2 \ddot{x}_2 = -k(x_2 - x_1 + l_0) + F_{21} \tag{2}$$

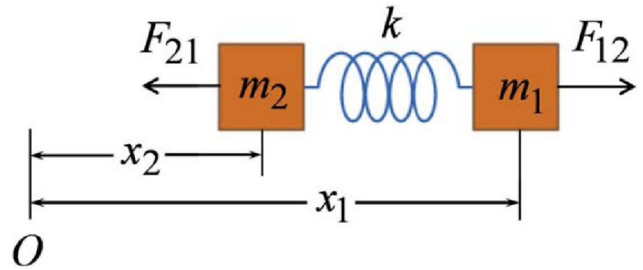

Figure 1 : Two mass system under consideration. Time variable masses m_1 and m_2 interact via a Hooke's law spring with force constant k and forces F_{12} and F_{21} obeying Newton's third law, F_{12} = -F_{21}. x_1 and x_2 are the coordinates of the masses relative to the origin *O*. There are no external forces acting on the system.

Here *k* is the spring constant for the harmonic interaction between the particles, l_0 the equilibrium or relaxed length of the spring, F_{12} is the (possibly time dependent) force acting on particle 1 due to particle 2 and F_{21} the force on particle 2 due to particle 1. Adding equations 1 and 2 yields

$$m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = -k(x_1 - x_2 - l_0) - k(x_2 - x_1 + l_0) + F_{12} + F_{21} \tag{3}$$

Simplifying slightly

$$m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = F_{12} + F_{21} \tag{4}$$

Now F_{12} and F_{21} are internal forces and form an action – reaction pair which obeys Newton's third law so that

$$F_{21} = -F_{12} \tag{5}$$

Substituting eqn. 5 into eqn. 4 we obtain

$$m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = 0 \tag{6}$$

Using the usual result that the acceleration of the center of mass is given by[24]

$$a_{cm} = \frac{m_1 \ddot{x}_1 + m_2 \ddot{x}_2}{m_1 + m_2} \tag{7}$$

and substituting eqn. 6 into eqn. 7 shows that

$$a_{cm} = 0 \tag{8}$$

Equation 8 was derived under very general circumstances, and seems to show that the acceleration of the center of mass of an isolated system with no net external force must be zero, even with time dependent masses, and time dependent interactions within the system, as long as they obey Newton's third law, which is not being questioned. Frictional damping can be easily added to the equations of motion, eqns. 3 and 4, but it is an unnecessary complication at this point.

Such considerations bode very poorly for a Mach Effect thruster (MET) producing any center of mass acceleration for an isolated system, at least using the ordinary Newton's 2nd law with time dependent masses, and interaction forces between the particles which obey Newton's third law. How then can it be possible to get a net thrust, as consistently reported by Woodward[16-18], since the center of mass of the isolated system does not accelerate?

FORM OF NEWTON'S SECOND LAW FOR VARIABLE MASS SYSTEMS

There are two major possible answers to the question posed above. The first possibility that immediately comes to mind is that one should modify the form used for Newton's second law, eqns. 1 and 2, to read something like *dp/dt* on the left hand side. One must be very careful when trying to use Newton's 2nd law in this form for time dependent masses, as it can often produce wrong results if not properly understood and applied. As noted by

Sommerfeld[26] and others, $dp/dt = d(mv)/dt = F$ only applies to the total system and not a single body with time variable mass due to mass entering or leaving the body. There is a significant amount of literature[26-39] on variable mass dynamics which discusses this point and the fallacies that result from using $d(mv)/dt = F$ in naïve application.

Despite the fact that variable mass dynamics has been an active research field for many years[28-30], several misapplications of Newton's second law for variable mass systems continue to appear in the literature[27,40-43]. The fact that experienced and well known physicists make such elementary errors shows how widespread the misunderstanding of Newton's second law applied to variable mass systems is. Because of its relevance to the evaluation of the work of Woodward and the present work, we take this opportunity to point out these errors in the work of Cramer[42] and Whealton[43]. Both authors have misapplied Newton's second law in the form $d(mv)/dt = F$ for variable mass systems, which does not properly take account of the momentum flux entering or leaving the body. Both authors attempt to analyze Mach effects, and as a result of their same error, wrongly conclude that there is no time average force associated with an oscillating time variable mass. Woodward has written a response[44] to the work of Whealton wherein he performed a simple mechanics experiment on a time varying mass having zero net relative velocity between the ejected mass flux and the body undergoing acceleration, and explicitly showed the form of the second law appropriate to such situations, refuting the erroneous claims of Whealton[42].

There are also other investigators who have undertaken similar, variable mass experiments[45-48] who get the form of Newton's second law right, especially when the ejected mass flux has zero relative velocity to the body undergoing acceleration.

Sommerfeld[26] has a careful and explicit discussion of how Newton's second law is to be applied in cases of variable masses and the form it takes in different circumstances. For a body gaining or losing mass, the second law takes the form

$$\mathbf{m\dot{v} = F + \dot{m}v_{rel}} \tag{9}$$

Where F is the net force acting on the body and v_{rel} is the relative velocity of the convective momentum being added to or lost from the body, measured with respect to the center of mass of the moving body and positive in the same sense as v. In order to use the form of Newton's second law given in eqns. 1 and 2 for time variable masses, it is necessary that the mass increase or decrease does not have any relative velocity to the body in question, thus it does not impart any acceleration or ordinary "rocket thrust" to the body. In order to have a truly propellantless propulsion method, we can not have any net ejection of mass in a cyclic process, so this condition is satisfied if, we assume that the mass fluctuations are of this type, i.e. that $v_{rel} = 0$. Thus Newton's second law for this type of time variable mass is of the usual form, but with the possibility that m is time variable.

$$\mathbf{m\dot{v} = F} \tag{10}$$

One example of this type of variable mass corresponds to isotropic mass loss and gain, in the rest frame of the accelerating body[27]. This fact was recognized long ago by Meshcherskii[49], whose work laid the foundation for the development of variable mass dynamics as a special discipline of mechanics[28]. An example of isotropic mass loss is a droplet evaporating isotropically into a vacuum[34]. However the angular distribution of mass loss and/or gain need not be restricted to spherical symmetry only, the variable mass only need be such that the net flux of variable mass or convective momentum flux have no net relative velocity to the body in question. This can be accomplished by angular distributions of momentum fluxes that are symmetric perpendicular to the axis of motion and also symmetric in both the forward and backward directions so as to impart no relative velocity term (i.e. produce no conventional "rocket thrust" and attendant mass loss). A simple example of a non isotropic mass variation with no net relative velocity is ejection or absorption of equal and opposite momentum fluxes in directions perpendicular to the direction of motion of the object. This type of momentum flux was employed in the variable mass experiment of Woodward[44], which quantitatively showed the validity of eqn. 10 for cases with no relative velocity between the body and the momentum flux leaving or being added to the accelerating body.

In a letter to P. G. Tait, written from the Cavendish Laboratory on 15 February 1878, J. C. Maxwell makes the following comment, apparently replying to a question in a previous letter from Tait: "I don't know how to apply laws of motion to bodies of variable mass, for there are no experiments on such bodies any more than on bodies of negative mass. All such questions should be labeled "Cambridge, Mass." and sent to U.S." Thus, it is clear that Maxwell did not regard the Newtonian laws as settling the question of variable mass[33]. It is also interesting that Maxwell mentions the concept of negative mass in this letter.

CENTER OF MASS ACCELERATION

The second and more straightforward thing to investigate is to notice the fact that the usual expression for the center of mass acceleration, eqn. 7, is not simply defined by eqn. 7, but rather derived from the definition of the center of mass,

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \tag{11}$$

and taking two time derivatives, *assuming the time derivatives of the masses are zero*[24].

To get the correct result for the acceleration of the center of mass when there are variable mass effects, we begin with eqn. 11 as the proper definition of the instantaneous position of the center of mass of the two mass system, but we allow for time dependent mass effects when we take the time derivatives. Taking the first derivative of eqn. 11 we obtain the modified result for the velocity of the center of mass

$$v_{cm} = \dot{x}_{cm} = \frac{dx_{cm}}{dt} = \left[\begin{array}{l} \dfrac{m_1 \dot{x}_1 + m_2 \dot{x}_2}{m_1 + m_2} + \dfrac{\dot{m}_1 x_1 + \dot{m}_2 x_2}{m_1 + m_2} \\ -\dfrac{(m_1 x_1 + m_2 x_2)}{(m_1 + m_2)^2}(\dot{m}_1 + \dot{m}_2) \end{array} \right] \tag{12}$$

The first term is the usual result for the velocity of the center of mass. Note how there are now two additional terms associated with the time derivatives of the masses. Differentiating eqn. 12 yields the new result for the acceleration of the center of mass

$$a_{cm} = \ddot{x}_{cm} = \left[\begin{array}{l} \dfrac{m_1 \ddot{x}_1 + m_2 \ddot{x}_2}{m_1 + m_2} + \dfrac{(2\dot{m}_1 \dot{x}_1 + 2\dot{m}_2 \dot{x}_2 + \ddot{m}_1 x_1 + \ddot{m}_2 x_2)}{m_1 + m_2} \\ -\dfrac{(m_1 x_1 + m_2 x_2)}{(m_1 + m_2)^2}(\ddot{m}_1 + \ddot{m}_2) - \\ 2\dfrac{(m_1 \dot{x}_1 + m_2 \dot{x}_2 + \dot{m}_1 x_1 + \dot{m}_2 x_2)}{(m_1 + m_2)^2}(\dot{m}_1 + \dot{m}_2) \\ +2\dfrac{(m_1 x_1 + m_2 x_2)}{(m_1 + m_2)^3}(\dot{m}_1 + \dot{m}_2)^2 \end{array} \right] \tag{13}$$

The first term is the usual expression, eqn. 7, for the acceleration of the center of mass. However, note the presence of several additional terms that depend on both the first and second time derivatives of both of the masses. However, due to the exact relation eqn. 6, which holds for an isolated system with time variable masses and *any* Newton's third law forces between the particles, eqn 13 can be *exactly* simplified to

$$a_{cm} = \ddot{x}_{cm} = \left[\begin{array}{l} \dfrac{(2\dot{m}_1 \dot{x}_1 + 2\dot{m}_2 \dot{x}_2 + \ddot{m}_1 x_1 + \ddot{m}_2 x_2)}{m_1 + m_2} \\ -\dfrac{(m_1 x_1 + m_2 x_2)}{(m_1 + m_2)^2}(\ddot{m}_1 + \ddot{m}_2) - \\ 2\dfrac{(m_1 \dot{x}_1 + m_2 \dot{x}_2 + \dot{m}_1 x_1 + \dot{m}_2 x_2)}{(m_1 + m_2)^2}(\dot{m}_1 + \dot{m}_2) \\ +2\dfrac{(m_1 x_1 + m_2 x_2)}{(m_1 + m_2)^3}(\dot{m}_1 + \dot{m}_2)^2 \end{array} \right] \tag{14}$$

Note that eqn. 14 now exhibits the interesting property that it does not explicitly depend on the accelerations of either of the particles, but only on products of velocities and positions with first or second order time derivatives of the masses. The second derivatives of the positions have been eliminated by Newton's third law, which is more general than just momentum conservation. Note also that the center of mass acceleration is symmetric or even under interchange of particle mass and coordinate labels 1 and 2, as it must be since neither particle is preferred over the other.

MOMENTUM CONSERVATION

Even as the ordinary expression for the center of mass acceleration was shown to be incorrect for the variable mass system under consideration, the total momentum of the system in the usual sense is not conserved for the variable mass system under consideration, despite the fact that there is no external force. This surprising conclusion follows from very simple considerations.

The normal concept of conservation of the total momentum of an isolated system follows from assuming the mass of the individual particles remain constant with time, so that the mass of the individual particles can be brought inside the m dv/dt terms and they can be written as m dv/dt = d(mv)/dt = dp/dt. The terms for the individual particles are all summed leading to an equation for the system of the form $dP_{tot}/dt = F_{net}^{external}$[24], which as noted earlier, is incorrect for the time variable mass system under consideration. Assuming $F_{net}^{external} = 0$, as in the case of an isolated system, then $dP_{tot}/dt = 0$, so that P_{tot} = constant, and thus the total momentum of the system is conserved, a conclusion that is valid when the masses of the system themselves are not time dependent.

For the variable mass two particle system under consideration the total (mechanical) momentum is given by

$$P_{tot} = m_1 v_1 + m_2 v_2 = m_1 \dot{x}_1 + m_2 \dot{x}_2 \tag{15}$$

so that the time derivative is

$$\begin{aligned} \frac{dP_{tot}}{dt} &= m_1 \dot{v}_1 + m_2 \dot{v}_2 + \dot{m}_1 v_1 + \dot{m}_2 v_2 \\ &= m_1 \ddot{x}_1 + m_2 \ddot{x}_2 + \dot{m}_1 \dot{x}_1 + \dot{m}_2 \dot{x}_2 \end{aligned} \tag{16}$$

Utilizing eqn. 6 for the variable mass system, eqn. 16 simplifies to

$$\frac{dP_{tot}}{dt} = \dot{m}_1 v_1 + \dot{m}_2 v_2 = \dot{m}_1 \dot{x}_1 + \dot{m}_2 \dot{x}_2 \tag{17}$$

Equation 17 shows that in general, the time derivative of the total momentum of the isolated two particle system is not zero as in the case of constant masses, and thus *the total momentum of the two particle system need not be conserved for the class of variable mass systems under consideration. Thus unidirectional acceleration of the center of mass of such systems does not violate momentum conservation, since the momentum of the system is not conserved! In addition, using eqns. 12 and 15, one can show that the total mechanical momentum of the system is no longer equal to the center of mass momentum for the variable mass system, as it is for systems of constant mass objects.*

Though this seems strange at first sight, a similar situation exists for the rocket equation, where the rocket, considered as a system by itself, does not conserve momentum, it accelerates without external forces and loses mass in the process. However the total momentum of the rocket plus exhaust gases is conserved. In the case of the two body variable mass system under consideration, there is no exhaust gas, but the time dependent mass implies some kind of interaction with an outside agent in order to make the mass variable so that there is no net mass change in a single cycle. The mechanism producing the acceleration is not the same as rocket thrust however, since the mass is changed with zero relative velocity to the system, thus giving no direct acceleration to the system as in the case of rocket thrust. Also, for the cases we will consider, the mass change is cyclic, meaning the time average mass of the body does not change, and thus there is no propellant necessary to produce the acceleration, as in the case of ejecting matter with a velocity relative to the body.

By focusing on the isolated system, as is often done in rocket thrust problems, we can talk about the equations governing the body of interest without discussing what happens to the momentum carried off by the rocket exhaust. Such an equation would apparently violate momentum conservation if the rocket is considered an isolated system and one does not account for the momentum of the rocket exhaust. Here too, by focusing on the two particle system, we find an equation for the system that produces an acceleration of the center of mass of the isolated system. In both cases, we are dealing with open systems, where mass or momentum can flow into or out of a control volume. If we wish to retain our concepts of momentum conservation, the time variable mass that has been hypothesized clearly transfers momentum to and from the system due to some kind of external agent. Thus the additional momentum and associated acceleration, thrust force and resulting increase in kinetic energy of the system is supplied by some kind of coupling to an outside agent which allows the mass to be time variable, despite the fact that there is no net external force applied to the isolated system. For the purposes of the present work, it is not necessary to specify the origin of the outside agent, only that its net effect is to allow production of a time variable mass without producing any explicit thrusting type force on the system.

UNIDIRECTIONAL ACCELERATION

We now proceed to investigate if eqn. 14 can exhibit a DC or unidirectional value of the acceleration of the center of mass for the case of two particles interacting with a Hooke's law spring, time variable masses, and a time dependent driving force. The solutions of the equations of motion are complicated somewhat by the presence of time variable masses.

Defining a new variable *u* as

$$u = (x_1 - x_2 - l_0) \tag{18}$$

substituting eqn. 18 into eqns. 1 and 2, using the result eqn. 5, dividing by the mass of each particle and subtracting the eqns. we obtain the result

$$\ddot{u} = -\frac{k}{\mu}u + \frac{F_{12}}{\mu} \tag{19}$$

Where the reduced mass μ is given by

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \tag{20}$$

In this form, eqn. 19 is recognized as that of a driven harmonic oscillator, with one important difference, the reduced mass is time dependent, making the spring constant term parametrically modulated, and the driving force also multiplied by a parametric term, or alternatively, the acceleration term can be taken to have all the parametric dependence. Note that eqns. 19 and 20 involve no approximation at this point and are valid for time variable masses of the form already discussed.

To properly account for the motion, we should at least have a solution that is complete thru the first order in the mass fluctuations, since the mass fluctuations provide phase information for the oscillator, and phasing is critical in such applications.

Nevertheless, in order to see if there are first order in the mass fluctuation, DC unidirectional effects in the acceleration of the center of mass, we first investigate the possibilities using the zero order solution to the oscillator equation for constant masses, then see what consequences that has when properly used in eqn. 14, which is already first order in the mass fluctuations. *Note that zero order effects in the solutions for the position and velocity are sufficient to get all first order effects in the acceleration of the center of mass using eqn. 14.*

In order to clearly demonstrate the effects, a simple, time harmonic model for the forces and masses is used in what follows. Since there is a bit of algebra involved, some of the mathematical details are skipped. Doing a systematic expansion in powers of the mass fluctuation amplitude over the non fluctuating mass of the solutions for $x_1(t)$ and $x_2(t)$ yields the following expression for the *DC, unidirectional acceleration of the center of mass* of an isolated two particle system interacting with each other using Newton's third law forces, using the proper, modified formula for the center of mass acceleration, eqn. 14. *To the best of our knowledge, this is a new result that has never appeared in the literature, previously.*

$$a_{cm}\Big|_{DC} = \frac{\left(\frac{\delta m_1}{m_{01}}\cos\phi_1 - \frac{\delta m_2}{m_{02}}\cos\phi_2\right)\omega^2 F_0}{(m_{01}+m_{02})(\omega_0^2-\omega^2)} \tag{21}$$

Where we have the following definitions and relationships.

The driving force for each particle (which obeys Newton's third law, eqn. 5) is taken as,

$$\mathbf{F}_{12}(t) = -\mathbf{F}_{21}(t) = \mathbf{F}_0 \cos(\omega t) \tag{22}$$

The time variable masses $m_1(t)$ and $m_2(t)$

$$m_1(t) = m_{01} + \delta m_1 \cos(\omega t + \phi_1) \tag{23a}$$

$$m_2(t) = m_{02} + \delta m_2 \cos(\omega t + \phi_2) \tag{23b}$$

where ϕ_1 and ϕ_2 are the phases of the mass fluctuations relative to the driving force F_{12} and δm_1 and δm_2 the amplitudes of the mass fluctuations. *Note that in the Mach Effect theory of Woodward[17], the mass fluctuations depend on the squares of the accelerations themselves, so they never go negative as in the model here, but the essential rectification property of the products of time variable masses with position and velocity is preserved for cases of oscillatory or cyclic motion. The fact that the mass fluctuations are themselves dependent on the state of motion, modifies the equations of motion in terms of the second derivatives of the positions*. Here we assume that we can specify the time dependence of the mass fluctuations, which take the simple form of eqns. 23 for the purpose of demonstrating an explicit formula for the center of mass acceleration of the isolated system. This is correct to lowest order in the fractional mass fluctuations, which are presumed very small compared to one in the present work.

Finally, the oscillator resonant frequency ω_0 is given by

$$\omega_0 = \sqrt{\frac{k}{\mu_0}} \tag{24}$$

Where the reduced mass for the non-fluctuating masses μ_0 is given by (compare to eqn. 20)

$$\frac{1}{\mu_0} = \frac{1}{m_{01}} + \frac{1}{m_{02}} \tag{25}$$

A few remarks on eqn. 21 are in order. First note that the dimensions of the right hand side of eqn. 21 are manifestly those of acceleration, so it is dimensionally correct. Note that the direction of the thrust and its magnitude are highly dependent on the relative phasing of the mass fluctuations relative to the driving force. The maximum acceleration is obtained when the mass fluctuations are 180° out of phase with each other, for then the mass fluctuations add directly. One of the phases should also have zero relative phase with the driving force to maximize the magnitude of the center of mass acceleration.

Although eqn. 21 appears to be antisymmetric under interchange of particles 1 and 2, this is not the case, since when the particle labels are interchanged, one must also change sign in the forces, due to Newton's third law, and since F_{12} was taken with the positive sign of F_0, when it gets interchanged to F_{21}, we must take F_0 to $-F_0$, making the center of mass acceleration even under interchange of the particles 1 and 2, which it must be, as noted earlier, since one particle is not preferred or distinguished over the other in regards to the direction of the acceleration.

TIME RATE OF CHANGE OF TOTAL MOMENTUM

Performing similar operations on eqn. 17 as were applied to eqn. 14 to obtain eqn. 21 for the acceleration of the center of mass, we find the DC or unidirectional component of the time rate of change of the total momentum

$$\frac{dP_{tot}}{dt}\Big|_{DC} = \frac{\left(\frac{\delta m_1}{m_{01}}\cos\phi_1 - \frac{\delta m_2}{m_{02}}\cos\phi_2\right)\omega^2 F_0}{2\left(\omega_0^2 - \omega^2\right)} \tag{26}$$

Equation 26 explicitly demonstrates that the total mechanical momentum of the system is not constant in time (unless the relative mass fluctuations are of equal amplitude and 180° out of phase), despite the fact that there is no external force applied to the system. Comparing eqns. 21 and 26 for the time harmonic force and variable mass model of eqns. 22 and 23 yields,

$$\frac{dP_{tot}}{dt}\Big|_{DC} = \frac{1}{2}\left(m_{01} + m_{02}\right) a_{cm}\Big|_{DC} \tag{27}$$

thus explicitly showing that

$$\frac{dP_{tot}}{dt}\Big|_{DC} \neq \left(m_{01} + m_{02}\right) a_{cm}\Big|_{DC} \tag{28}$$

The fact that eqn. 28 is true was discussed earlier in the section on the form of Newton's second law for variable mass systems and in the references cited[26-39]. Equation 26 was calculated in a completely different manner than eqn. 21, using eqn. 17, a formula with far fewer terms than eqn. 14, so it is not too surprising that it should yield a smaller result than the right hand side eqn. 28.

DISCUSSION OF RESULTS AND RELEVANCE TO MACH EFFECT THRUSTERS

The simple and innocent looking expressions given by eqns. 21 and 26 have profound consequences and apparent contradictions that need further exploration beyond the scope of the present work. The center of mass of the two particle system has a unidirectional acceleration, which implies that the kinetic energy of the system continually increases, which increase in energy must come from somewhere if we believe in energy conservation. However, the work-energy theorem is modified for variable mass systems[36], leading to some subtleties.

If we consider a simple system of a frictionless cart constrained to move only in one dimension and equal size masses lined up on each side of it and spaced at regular intervals, we can see the essence of how the system moves forward in each cycle and yet does not violate momentum conservation. A mechanism on board the cart at rest grabs a mass from each side of the cart and draws them in perpendicularly to the direction that the cart is constrained to move. The mechanism then moves the masses to the back of the cart, stops relative to the cart, and then off loads the added masses, again perpendicular to the direction of the cart, and then the mechanism moves forward back to its original position, with the cart at rest again at the end of the cycle.

The center of mass of the cart (including the mechanism) moved forward, while the two "external" masses moved backward, keeping the overall mass of the cart constant, and the center of mass of the cart plus external mass system the same, thus conserving overall momentum, which in this example started at zero and ended at zero. However, this was at the expense of moving the mass lined up on each side of the track backwards. Note that there was no rocket thrust at any time during the cycle, only conservation of momentum at each of the four processes in the cycle. In this mechanical example, the cart is at rest at the beginning of the cycle and also at rest at the end of the cycle. Clearly there had to have been some kind of acceleration of the cart forward during the process, the average acceleration in this case related to the rate at which the mechanism took the masses, moved them to the back of the cart, then returned back to its original position in the front of the cart. The difference between this mechanical example and the two body system that we used is that in this example, the system starts at rest, speeds up, then comes back to rest at the end of the cycle, moving forward in the process without expelling any propellant. This example makes it clear that there needs to be some kind of mass exterior to the system with which one can temporarily "borrow" in order to move forward, and then give it back. In other words, our "isolated" system must really be open if the mass is to vary in the proscribed manner.

In the case of an MET thruster, this reaction mass is the coupling of the local system to the rest of the matter in the universe that allows this back and forth variation in the mass of the system. In essence, one pushes off a mass when it is heavy, and pulls on it when it is light, leading to an overall increase in the local systems momentum. This momentum must be made up by the rest of the universe allowing the isotropically ejected mass (in the rest frame of the accelerating body) to return isotropically to the body in its rest frame, that has in the mean time accelerated forward. to keep pace with the MET device, thus requiring the "spherical shell" of ejected mass to move forward. Quantitative expressions for the mass variations have been given by Woodward[17] when they are a result of Machian effects.

As expected from a similar calculation applied to a single fixed oscillator with variable mass and damping, the "thrust" or center of mass acceleration reverses sign above and below the resonance frequency, where the thrust is maximized. Similarly, if one can adjust the relative phases of the mass fluctuations compared to the driving force, one should be able to maximize the thrust. In piezoelectric and electrostrictive MET devices[17] temperature dependent material effects can shift the resonance frequency, and drastically alter the thrust, causing it to vary wildly when driving near resonance at high powers causing heating. One should compare the off resonance temperature dependence of the thrust with the on or near resonance cases to characterize how large these effects are for a given device. It is probably best to operate on the side of the resonance that will not cause a sign change in the acceleration when the device heats up. For example, if the heating causes the resonance frequency to shift downward, then one should drive above the resonance frequency so as not to pass thru the resonance peak due to heating and subsequent cooling effects, causing temperature dependent sign changes in the thrust, one on heating, and another on cooling!

One important modification to our main result eqn. 21 must be made for the current generation of MET thrusters. In order to illustrate our point, we used the simplest possible waveform for both the mass fluctuations and the driving force, a single trigonometric function. However, according to Woodward, the expression for the mass variations due to acceleration of a body in a time varying energy density field is proportional to the time derivative of the product of the force acting on the particle times the velocity. If the force is only a pure sine wave, then the resulting periodic component of velocity in a linear system is also first harmonic, so the time derivative of the product and thus the mass fluctuations are proportional to the second harmonic, or even harmonic in time. This then gets multiplied by another first harmonic term from a coordinate or velocity and possibly other even harmonic terms from other products of mass or mass time derivatives when computing the center of mass acceleration from eqn. 14. The result is to make the acceleration of the center of mass time variation consist of only odd harmonics in time, resulting in no unidirectional, DC terms. Thus the lowest order pure force signal that can be used for Mach effects is quadratic in voltage. Using a cosine squared forcing function so that one obtains the mass fluctuations as even harmonics, which when multiplied by the second harmonic velocity and displacement response when computing the center of mass acceleration, will yield a DC response term. Thus one needs a quadratic response force or cosine squared driving function for the type of mass fluctuations proposed by Woodward. This can be supplied by a pure electrostrictive material driven by a single sine wave, which ideally produces a strain or stress that is proportional to the square of the driving voltage. More detailed calculations of the center of mass acceleration and thrust force of realistic MET devices will be presented elsewhere.

CONCLUSION

It is emphasized that the main results of this paper, demonstrating the possibility of a mechanism for propellantless propulsion and obtaining an explicit expression for the unidirectional acceleration of the center of mass of an isolated system having time variable masses, Newton's third law internal forces and no net external force, is not a violation of Newton's second law of motion, but rather a consequence of it. The combination of Newton's second and third law, together with the required revisions of long held ideas about the velocity and acceleration of the center of mass based on experience with constant mass systems, results in new concepts and the possibility of new devices based on them. The unidirectional acceleration found here is a consequence of our primary assumption, the possibility of changing the mass of an isolated system with zero relative velocity associated with the net convective momentum flux which produces the changing mass.

Although such a process can be demonstrated with mechanical examples, for a useful thruster for space exploration applications, a field mechanism is required to provide some kind of coupling to an agent external to the system which provides work to accelerate the time variable part of the mass to keep up with the acceleration of the center of mass. At present, Woodward's proposed Mach effect is the only mechanism known to be able to provide the coupling to distant matter in the universe. As was mentioned in the text, isotropic coupling is not the only possibility. However, in order for the time varying coupling to be isotropic, one should have monopole gravitational coupling from the bodies undergoing acceleration, a controversial notion in itself, since gravitational radiation is normally thought to be quadrupolar in lowest order. However, time varying masses allow the possibility of monopole gravitational radiation.

The present work demonstrated the possibility of unidirectional center of mass acceleration with a two particle system of time varying masses, Newton's third law internal forces, and no net external force. Similar considerations apply to extended bodies consisting of arbitrary number of particles with time varying masses and third law internal forces, having no net external force.

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