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Cauchy integral theorem and complex integral several equivalence relations

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ABSTRACT

By document method, we get that Cauchy integral theorem actually is residue theorem that integrand functions as analytic function in integration region; Cauchy integral formula actually is the residue theorem that integrand has first order pole in integration region; derivatives of high order formula actually is the residue theorem that integrand has order pole in integration region. Make comparison summary on it, and effective combine with other documents, utilize Cauchy integral theorem to prove Cauchy integral formula, derivatives of high order formula, residue theorem, and find out their inner equivalence relations.

KEYWORDS

Cauchy integral theorem; Cauchy integral formula; Residue theorem; Derivatives of high order formula.

INTRODUCTION

From later half of 18th Century to the beginning of 19th century, it started exploring complex function and partial derivative as well as integration property. Complex analysis really as one research field in modern analysis was established in 19th century, the main founders were Cauchy, Riemann and Weierstrass. Cauchy established complex variables functions differential and integral theories. The paper "Report regarding integration limit as imaginary number's definite integral" in 1814, 1825, it established Cauchy integral theorem; in 1826, it proposed residue concept; in 1831, it obtained Cauchy integral formula; in 1846, it found integral and path irrelevance theorem.

"Complex function theory" is normal university mathematics and applied mathematics major required course, meanwhile it is also comprehensive university science and engineering foundation course, is real variable functions calculus promotion and development. Among them, Cauchy integral theorem is the basis of complex function theory, is the key to research on complex function theory, is also most unique creation in 19th century, is one of most harmonious theories in abstract science, complex variables functions' lots of important properties and theorems are directly or indirectly deduced from it. The paper mainly by documents: it gets Cauchy integral theorem actually is the residue theorem that integrand has first order pole in integration region; derivatives of high order formula actually is the residue theorem that integrand has order pole in integration region. Make comparative summary on them, and effective combine with documents, it concludes Cauchy integral theorem and Cauchy integral formula, derivatives of high order formula, residue theorem their deduction relations.

Since Cauchy, complex function theory has already more than 170 years' history. It becomes one of important compositions in mathematics with its perfect theory and exquisite technique. It has ever promoted some disciplines development, and tends to be applied into practical problems as a powerful tool, its basic contents have already become science and engineering many majors required courses. Now, complex function theory still has some subjects that to be researched, so it will continue to move forward and get more applications.

PRE-KNOWLEDGE

The realization of this work supposes the availability of a Cauchy integral theorem Given $f(z)$ analyze in complex plane simply connected region D, C is D anyone contour, then $\int_c f(z)dz = 0$.

Complex contour Cauchy integral theorem given D to be complex contour $C = C_0 + C_1^- + C_2^- + \cdots + C_n^-$ surrounding bounded multiple connected regions, $f(z)$ analyzes in D, and continues in $\overline{D} = D + C$, then $\int_c f(z)dz = 0$.

Cauchy integral formula Given region D boundary is contour (or complex contour) C, function $f(z)$ analyzes in D, and continues in $\overline{D} = D + C$, then it has $f(z) = \frac{1}{2\pi i}\int_C \frac{f(\xi)}{\xi - z} d\xi$ ($z \in D$).

Residue theorem $f(z)$ is in the contour (or complex contour) C ranged region D, analyzes except for $a_1, a_2, \cdots, a_n$, it continues in closed domain $\overline{D} = D + C$ except for $a_1, a_2, \cdots, a_n$, then $\int_c f(z)dz = 2\pi i \sum_{k=1}^{n} \operatorname{Res}_{z=a_k} f(z)$.

Derivatives of high order formula given region D boundary is contour (or complex contour) C, function $f(z)$ analyzes in D, and continues in $\overline{D} = D + C$, function $f(z)$ in region D has each order derivative, and has $f^{(n)}(z) = \frac{n!}{2\pi i}\int_c \frac{f(\xi)}{(\xi - z)^{n+1}}d\xi$ ($z \in D$).

CAUCHY INTEGRAL THEOREM AND COMPLEX INTEGRAL RELATIONSHIP EVIDENCE

Cauchy integral theorem and Cauchy integral formula relationship

Complex contour Cauchy integral theorem can also be recorded as

$$\int_{c_0} f(z)dz = \int_{c_1} f(z)dz + \int_{c_2} f(z)dz + \cdots + \int_{c_n} f(z)dz.$$

Any fixed $z \in D$, $F(\xi) = \frac{f(\xi)}{\xi - z}$ as ξ function in D all analyzes except for point z. Center on point z, with enough small $\rho > 0$ as radius to make periphery γ_ρ, let γ_ρ and its interiors all included in D, for complex contour $\Gamma = C + \gamma_\rho^-$ and function $F(\xi)$, apply complex contour Cauchy integral theorem, it gets: $\int_c \frac{f(\xi)}{\xi - z}d\xi = \int_{\gamma_\rho} \frac{f(\xi)}{\xi - z}d\xi$.

While $f(z)$ and integral variable ξ are uncorrelated, and $2\pi i = \int_{\gamma_\rho} \frac{d\xi}{\xi - z}$, then it has

$$\left|\int_{\gamma_\rho} \frac{f(\xi)}{\xi - z}d\xi - 2\pi i f(z)\right| = \left|\int_{\gamma_\rho} \frac{f(\xi)}{\xi - z}d\xi - f(z)\int_{\gamma_\rho} \frac{d\xi}{\xi - z}\right| = \left|\int_{\gamma_\rho} \frac{f(\xi) - f(z)}{\xi - z}d\xi\right|.$$

According to $f(\xi)$ continuity, for any $\varepsilon > 0$, it exists $\delta > 0$, only $|\xi - z| = \rho < \delta$, it will have $|f(\xi) - f(z)| < \frac{\varepsilon}{2\pi}, (\xi \in \gamma_\rho)$ So $\left|\int_{\gamma_\rho} \frac{f(\xi) - f(z)}{\xi - z}d\xi\right| < \left|\int_{\gamma_\rho} \frac{\varepsilon}{2\pi\rho}d\xi\right| < \frac{\varepsilon}{2\pi\rho} \cdot 2\pi\rho = \varepsilon$.

Therefore, it has $\int_{\gamma_\rho} \frac{f(\xi)}{\xi - z}d\xi = 2\pi i f(z)$.

That

$$\mathbf{f(z) = \frac{1}{2\pi i}\int_c \frac{f(\xi)}{\xi - z}d\xi, (z \notin C)} \qquad \textbf{(1)}$$

So it promotes from Cauchy integral theorem to Cauchy integral formula as formula (1).

Cauchy integral theorem and residue theorem relationship

$f(z)$ is in contour or complex contour C ranged region D, it analyzes except for $a_1, a_2, \cdots, a_n$, and it continues in closed domain $\overline{D} = D + C$ except for $a_1, a_2, \cdots, a_n$.

Center on a_k, with enough small positive number ρ_k as radius to draw periphery $\Gamma_k : |z - a_k| = \rho_k (k = 1,2,\cdots,n)$, let these peripheries and their interiors all included in D, and they isolate from each other, apply pre-knowledge complex contour Cauchy integral theorem, it gets $\int_c f(z)dz = \sum_{k=1}^{n} \int_{\Gamma_k} f(z)dz$.

By residue definition, it has $\int_{\Gamma_k} f(z)dz = 2\pi i \operatorname*{Res}_{z=a_k} f(z)$. Input into above formula that has:

$$\int_c \mathbf{f(z)dz = 2\pi i \sum_{k=1}^{n} \operatorname*{Res}_{z=a_k} f(z)} \qquad \textbf{(2)}$$

So promote Cauchy integral theorem, it gets residue theorem the formula (2).

Cauchy integral theorem and derivatives of high order formula relationship

From above deduction, it can promote from Cauchy integral theorem to residue theorem, in the following, it will promote from residue theorem to derivatives of high order formula.

If integrand in integral loop C has $n+1$ order poles, investigate integral $\int_c \frac{f(z)}{(z-a)^{n+1}}dz$, from which a is integral loop C interior point, and $z=a$ is integrand $n+1$ order poles. From residue theorem formula (2) and $n+1$ order poles residue computational formula, it has

$$\int_c \frac{f(z)}{(z-a)^{n+1}} dz = 2\pi i \operatorname{Res} F(a)$$

$$= 2\pi i \lim_{z\to a} \frac{1}{n!} \cdot \frac{d^n}{dz^n}\left[(z-a)^{n+1} \cdot \frac{f(z)}{(z-a)^{n+1}} \right]$$

$$= 2\pi i \cdot \frac{1}{n!} \cdot f^{(n)}(a).$$

So

$$\mathbf{f^{(n)}(a) = \frac{n!}{2\pi i} \int_c \frac{f(z)}{(z-a)^{n+1}} dz} \qquad \textbf{(3)}$$

There by residue theorem, it can promote and get derivatives of high order formula the formula (3).

CONCLUSIONS

This paper by collecting and sorting out correlated information, it started from Cauchy integral theorem and Cauchy integral formula, concluding their and complex variables functions internal connections, and summarized singular point contour Cauchy integral formula. From above, it could get that residue theorem and complex variables functions integral Cauchy theorem, Cauchy formula and derivatives of high order formula relationships were:

(1) Cauchy integral theorem actually is residue theorem that integrand functions as analytic function in integration region;

(2) Cauchy integral formula actually is the residue theorem that integrand has first order pole in integration region;

(3) Derivatives of high order formula actually is the residue theorem that integrand has $n+1$ order poles in integration region.

Therefore, any complex integral that could use Cauchy integral theorem, Cauchy integral formula and analytic functions' derivatives of high order formula to calculate, all could use residue theorem to calculate.

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