Bounded error segmentation algorithm and focusing delay generation

Wang Ping¹*, Wang Siqi¹, Wang Linhong², Cheng Na¹, Gong Zhihui¹, Pan Zhen¹
1Department of Electrical and Electronics Engineering, Chongqing University, Chongqing, 400044, (CHINA)
2Department of Applied Electronics, Chongqing College of Electronic Engineering, Chongqing 401331, (CHINA)
E-mail: cqu_dqwp@163.com

ABSTRACT

The highly efficient and real-time generation of focusing delays in dynamic focusing systems is always difficult to implement. This paper proposes a segmentation method for dynamic focusing on the basis of bounded error, as well as a scheme for compressing and generating focusing delays. The segmentation boundaries satisfying the restricting error and the corresponding focusing delays in the segmentation depth can be calculated according to the number of focusing channels, the sampling frequency, and the given bounded error. Moreover, the quantization units are dynamically changed to reduce further the errors in the focusing delays of each channel. Finally, schemes for compressing and generating focusing delays are proposed. The numerical analysis and experimental results indicate that the proposed approach can obtain the least number of segmentations under the bounded error in the detecting range and significantly reduce the memory requirement of focusing delays. The SSIM of generated images by using the proposed approach and ideal dynamic focus is 0.948.

KEYWORDS

Ultrasound imaging; Dynamic focusing; Bounded error; Focusing delays.
INTRODUCTION

In dynamic focusing systems, signals of the destination reflectors are obtained by summing the echo signal of each channel after a series of specific delays\cite{1,2}. To improve the lateral resolution of images, the technology of dynamic focusing is generally implemented by hardware systems directly; however, real-timely generating the focusing delays for each channel is always difficult to be implemented\cite{3}. In most of ultrasound systems, two mainstream methods are introduced to generate focusing delays. One mainly involves calculating real-time focusing delays directly, which depends on FPGA’s strong ability in calculation, and the other involves storing compressed focusing delay data. The former method generally uses the Coordinate Rotation Digital Computer (CORDIC) algorithm or recursive algorithm to avoid complex squaring and rooting operations. However, as the number of focusing channels increases, the method consumes a significant amount of FPGA resources\cite{4}. Obviously it will lead to the high cost of hardware. The latter method mainly generates focusing delays by decompressing compressed focusing delays which are pre-stored as reference data sets. These algorithms can avoid the direct storage of focusing delays and thus reduce memory requirements\cite{5}, but the approaches often lead the peripheral circuit of the latter algorithm to be very complicated, and correspondingly the hardware consumption would become very large when the dynamic focusing system has 64 or 128 channels\cite{6-9}. For example, the ultrasound system of the Philips HD7 series has taken pixel-level dynamic focusing technology and the fine ash-step point using point focusing technology, and the number of focusing channels reached up to 1,024 channels in Philips HD7 series products. In such case, high-quality digital beamforming is essential to high-end ultrasound systems, and the high-quality digital beamforming always requires the generation of focusing delays in a highly efficient and simple manner. Thus, real-time and highly efficient generation algorithms for multi-channel, high-precision focusing delays are worth studying.

This paper proposes a method for obtaining the least number of segmentations according to the given bounded error. Meanwhile, it presents a high efficient scheme for generating focusing delays. In order to test the effectiveness of the proposed method, focusing error analysis and point target imaging experiments are conducted.

The remainder of this paper is organized as follows. Section II analyzes the theorem of dynamic focusing. Section III proposes the segmentation algorithm of dynamic focusing based on the given bounded error and the schemes for compressing and generating focusing delays. Section IV shows the numerical analysis and experimental results. Section V draws a conclusion.

DYNAMIC FOCUSING

In the receiving process, focusing delays of dynamic focusing always change as focal depth increases\cite{10}. Ideally, all points of each scan line continue to serve as focal points in the process of dynamic focusing. To accomplish this task, the dynamic focusing system must trace targets along the scan line according to the speed of echo signals.

If reference origin O is the center of the linear subarray, then the coordinate of the \(i\)-th element of the linear subarray is

\[
x_i = \left(i - \frac{N + 1}{2}\right)d, \quad 1 \leq i \leq N
\]  

Where \(N\) is the number of linear subarray elements, \(i\) is the ordinal number of elements, and \(d\) is the center interval of array elements. Figure 1 presents the coordinate of \(i\)-th and \(j\)-th elements.

Figure 1: Calculation of focusing delays of the linear subarray
where \( F_k \) is focal depth at the focusing point \( P_k \), \( R_{i,k} \) is the distance from the \( i \)-th element to the focusing point \( P_k \), and the reference origin \( O \) is the center of the linear subarray. The acoustic-path difference \( \Delta R_{i,k} \) is

\[
\Delta R_{i,k} = R_{i,k} - F_k = \sqrt{F_k^2 + x_i^2} - F_k
\]

(2)

The delay data \( \tau_{i,k} \) can be obtained as follows

\[
\tau_{i,k} = \frac{\Delta R_{i,k}}{c} = \frac{1}{c} (\sqrt{F_k^2 + x_i^2} - F_k)
\]

(3)

where \( c \) is the ultrasound speed.

The delay and sum beamforming \( S_{DAS} \) at the focusing point \( P_k \) can be calculated as follows

\[
S_{DAS} = \sum_{i=1}^{N} s \left( \frac{F_k - \tau_{i,k}}{c} \right)
\]

(4)

The minimum interval between two adjacent focal points \( \Delta F \), focusing number \( N_p \), the necessary number of focusing delays \( M_s \), and the maximum data bandwidth \( B_w \) can be calculated as follows \([11]\)

\[
\begin{align*}
\Delta F &= \frac{c}{2f_s} \\
N_p &= \frac{2F_L}{f_s} / c \\
M_s &= \frac{mN_p}{2} = mF_Lf_s / c \\
B_w &= mf_s
\end{align*}
\]

(5)

where \( f_s \) is the sampling rate, \( m \) is the number of elements, and \( F_L \) is the depth of detection.

When the detection depth \( F_L \) is 240 mm and the sample rate \( f_s \) is 50 MHz, the number of focusing delays \( N_p \) is 1.56\times10^4. For a modest ultrasound system with a 32-channel, 128-element transducer, the symmetry in the beamforming can be exploited, such that the number of values is halved. In this case, if a focusing delay serves as 2 bytes, the necessary number of focusing delays \( M_s \) can reach up to 4.98\times10^3 bytes and the maximum data bandwidth \( B_w \) is 1,600 Mbytes/s. In such cases, a single-chip such as FPGA with limited memory could hardly fit with the dynamic focusing circuits \([12]\).

**DYNAMIC FOCUSING SEGMENTATION AND FOCUSING DELAYS OPTIMIZATION**

**Dynamic focusing segmentation**

**Segmentation algorithm of dynamic focusing based on bounded error**

The linear subarray is shown in Figure 1. The delay profiles are symmetrical and thus allow the subarray to be halved. Moreover, only the elements \( N/2+1 \) to \( N \) (\( N \) is even) are considered.

According to (2) and (3), when the focal depth changes from \( F_k \) to \( F_{k+1} \), the deviations of focusing delays \( \Delta \tau_i \) and \( \Delta \tau_j \) can be calculated as follows.

\[
\begin{align*}
\Delta \tau_i &= \frac{1}{c} [\sqrt{F_k^2 + x_i^2} - \sqrt{F_{k+1}^2 + x_i^2}] - (F_k - F_{k+1}) \\
\Delta \tau_j &= \frac{1}{c} [\sqrt{F_k^2 + x_j^2} - \sqrt{F_{k+1}^2 + x_j^2}] - (F_k - F_{k+1})
\end{align*}
\]

(6)

According to (6), the relations between \( \Delta \tau_i \) and \( \Delta \tau_j \) can be obtained.

\[
|\Delta \tau_i| > |\Delta \tau_j|, j > i \quad \text{and} \quad i, j \in [N/2+1, N]
\]

(7)

Where \( i, j \in [N/2+1, N] \), \( j > i \), and \( x_j > x_i \)

According to (7), in the range of focal depth \( [F_k, F_{k+1}] \), if \( |\Delta \tau_i| < \delta \), (8) can be established.

\[
|\Delta \tau_{N/2+1}| < \cdots < |\Delta \tau_i| < \cdots < |\Delta \tau_j| < \cdots < |\Delta \tau_N| < \delta
\]

(8)
In particular, if the focusing delay error $\Delta \tau_N$ is under the given bounded error $\delta$, the focusing delay errors of other elements will not exceed $\delta$. Therefore, in the detection range $[F_0, F_L]$, segmentation positions or focusing points can be calculated according to (9).

\[
\begin{cases}
\tau_{N,F_k} - \tau_{N,F_{k+1}} = \delta \\
\tau_{N,F_k} - \tau_{N,F_{k+1}} = \delta \\
F_{k-1} < F_k < F_{k+1} \\
k \geq 1 \\
F_{k+1} \leq L
\end{cases}
\]

where $\tau_{N,F_k}$ is the focusing delay of $N$-th element at focal point $P_k$. Figure 1 shows that if each channel is beamformed according to focusing delays $[\tau_{1,F_1}, \tau_{2,F_2}, ..., \tau_{N,F_N}]$ at focal point $P_k$, then the focusing error will be less than $\delta$ in the detection range $[F_{k-1}, F_{k+1}]$.

According to (3) and (9), $F_k$ can be derived as follows

\[
F_k = \frac{x_N^2 - l^2}{2l} \quad (3)
\]
\[
l = c \cdot (\tau_{N,F_{k+1}} - \delta) \quad k = 1, 2, ..., K
\]

where $\tau_{N,F_k}$ can be calculated using (3). According to (10), $F_1, F_2, ..., F_k$ can be sequentially calculated in the detection range $[F_0, F_L]$, where $F_0, F_2, F_4, ..., F_K$ are segmentation boundaries, and $F_1, F_3, F_5, ..., F_{K-1}$ are the focusing points of the corresponding segmentations.

The focusing number $F_{N,s}$ at the $s$-th segmentation boundaries can be obtained as follows

\[
F_{N,s} = 2F_{2s}/c, (s=0,1, ..., K/2)
\]

The least segmentations number $S_{\text{min}}$ in the detection range $[F_0, F_L]$ can be calculated as follows.

\[
S_{\text{min}} = (\tau_{N,F_{K-1}} - \tau_{N,F_k}) / 2\delta
\]

Where $\delta$ is the bounded error. If the segmentation number is less than $S_{\text{min}}$, the expression (9) cannot be established. Thus, the number of segmentations derived by (12) will be the least if the bounded error and detection range are given.

**Optimal approximating algorithm of the dynamic focusing delays**

In dynamic focusing systems, the element closer to the focusing center line has a stronger influence on the echo signal on the beamforming. According to (3) and (4), when the focusing depth is the same and the bounded error is given, the closer the focusing channels to the focusing center line are, the smaller the delays value of the channels are and the larger the relative errors are. To improve the accuracy of beamforming further, the dynamic approximation unit $\lambda_i$ is introduced for focusing channels as follows.

\[
\begin{cases}
\lambda_i = \delta \quad \left( \frac{3N}{4} < i \leq N \right) \\
\lambda_i = \frac{\delta}{2} \quad \left( \frac{5N}{8} < i \leq \frac{3N}{4} \right) \\
\lambda_i = \frac{\delta}{4} \quad \left( N / 2 + 1 \leq i \leq \frac{5N}{8} \right)
\end{cases}
\]

**Dynamic focusing segmentation and focusing delays optimization**

The detection depth is 2 mm–240 mm, according to (1), (3), (9), and (10). The segmentation number $S$ is 82.

Figure 2 (a) shows the stair-step approximation of the segmentation focusing delays, which is based on the unified approximation unit. Figure 2 (b) is the high-precision approximation for focusing delays, which is based on dynamic approximation unit.
Figure 2 : The comparison of approximation for focusing delays

Figure 2 (a) and (b) show that the ideal focusing delays are always surrounded by approximate focusing delays, and the maximum error is limited within the same approximation unit. Compared with the upper right corner of Figure 2 (a) and (b), the introduction of dynamic approximation unit $\lambda_i$ can minimize the relative errors for the focusing channels, which are closer to the focusing center line. This innovation is beneficial for improving the quality of scanning lines.

Compression storage of the focusing delays

The focal depth increases, ideal focus delays become a smooth curve. To approximate the ideal focusing delays, the $i$-th channel and $s$-th segmentation focusing delays $T_{i,s}$ can be corrected according to (14) in real time.

\[
T_{i,s} = \begin{cases} 
T_{i,s-1} + |T_{i,s-1} - \tau_{i,s}| \leq \lambda_i \\
T_{i,s-1} - \lambda_i, & |T_{i,s-1} - \tau_{i,s}| \geq \lambda_i
\end{cases} \quad s = 1, 2, ..., S
\]

Figure 3 shows the relationship between the stair-step approximation and the accuracy focusing delays curve of the $i$-th channel.

Figure 3 : The stair line approximates to focusing delays

In the graph, the horizontal axis is the segmentation number, the vertical axis is the value of focusing delays, and $\lambda_i$ is the approximation unit.
Figure 3 shows that the stair-step approximation of focusing delays $T_{i,s}$, which is corrected according to the approximation information $BIT_{i,s}$ (approximation information of $i$-th element and $s$-th segmentation) in real time and can also be presented as (15).

$$BIT_{i,s} = \begin{cases} 
1, & |T_{i,s} - \tau_{i,s}| \geq \lambda_i \\
0, & |T_{i,s} - \tau_{i,s}| < \lambda_i 
\end{cases}$$

(15)

When the difference between $T_{i,s}$ and $\tau_{i,s}$ is larger than $\lambda_i$, the corresponding approximation information $BIT_{i,s}$ can be represented as 1 and the stair-like line $T_{i,s}$ must be decreased by $\lambda_i$. Otherwise, $BIT_{i,s}$ is 0, and $T_{i,s}$ retains its value. Thus, according to (14) and (15), the focusing delays $T_{i,s}$ can be corrected by the $\lambda_i$ in real time to approximate the ideal focusing delays $\tau_{i,s}$.

**Generation of dynamic focusing delays**

Considering the above approach, which uses stair-line to approximate the ideal focusing delays curve, Figure 4 presents the scheme of focusing generation delays for 32 channels.

![Figure 4: The scheme of focusing delays' generator](image)

Before the generation of focusing delays, the MCU loads the approximation information $BIT_{i,s}$ of the segments to the dual port RAM, the focusing number $FN_s$ ($s=0,1,…,81$) to the address generator, as well as the initial value $T_{i,0}$ ($T_{i,0}$ is the $i$-th channel’s focusing delay of the first segmentation) and the approximate unit $\lambda_i$ (the approximate unit of $i$-th channel) to the corresponding subtracter. When the dynamic focusing starts, the address generator counts the FOCUS_CLK. If the counted value is equal to the focusing number $FN_s$, the address generator increases the ADDR[6..0] by 1, and the dual port RAM outputs the approximation information $BIT_{i,s}$ of the next segment to the subtracters. If the $BIT_{i,s}$ is 1, $T_{i,s}$ is decreased by $\lambda_i$. Thus, $T_{i,s}$ can be generated in real time. Figure 3 shows the approximation process of $T_{i,s}$.

**ANALYSES AND EXPERIMENT**

**Memory requirement calculation and comparison**

The initial value of the focusing delays of each channel requires 2 bytes to store. Given the symmetry of the linear subarray, the memory requirement $M_1$ of focusing delays of $N$ channels is

$$M_1 = (N/2) \times 2 = 32 \text{bytes}$$

The memory requirement of $\lambda_i$ is a byte, and the memory requirement $M_2$ of $N$ channels is

$$M_2 = (N/2) \times 1 = 16 \text{bytes}$$

The memory requirement of each focusing number $FN_s$, which records the segmentation boundaries, is 2 bytes. The memory requirement $M_3$ of all focusing numbers is

$$M_3 = S \times 2 = 164 \text{bytes}$$
The memory requirement of the focusing delay approximation information $BIT_{i,s}$ of each channel in each segmentation is a binary bit. The memory requirement $M_4$ of $N$ channels is

$$M_4 = S \times N / 2 / 8 = 164\text{bytes}$$

Thus, only $M_1+M_2+M_3+M_4=376$ bytes can reconstruct the segmentation focusing delay $T_{i,s}$. TABLE 1 presents the comparison of memory requirement of different numbers of channels between the direct storage with the proposed algorithm in this study, where the bounded error $\delta$ is 20 ns.

**TABLE 1 : Comparison of memory requirement about different focusing methods**

<table>
<thead>
<tr>
<th>Focusing beamforming type</th>
<th>32-channel</th>
<th>64-channel</th>
<th>128-channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal dynamic focusing beamforming</td>
<td>494560</td>
<td>989120</td>
<td>1978240</td>
</tr>
<tr>
<td>Proposed dynamic focusing beamforming</td>
<td>376</td>
<td>1230</td>
<td>3472</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>1/1315</td>
<td>1/804</td>
<td>1/570</td>
</tr>
</tbody>
</table>

TABLE 1 shows that in 32-channel focusing system, the compression ratio can reach up to 1/1315. Although the compression ratio decreases as the number of channels increases, the compression ratio for 128-channel focusing system is 1/570. Apparently, the proposed segmentation method can save a significant amount of memory for dynamic focusing delays.

**Error analysis of optimized approximation**

The ultrasonic echo signal is defined as $s_0 = \sin(2\pi f_0 t + \phi_0)$, where $\phi_0$ is the initial phase. The ideal delay and sum beamforming for 32 channels is

$$s_i = 32s_0 = 32\sin(\omega t + \phi_0)$$

(16)

For ultrasonic echo signals with 3.5 MHz, the maximum phase error is 0.4398 rad when the bounded error $\delta$ is 20 ns. According to (13), the delay and sum beamforming for 32 channels based on the proposed algorithm is

$$s_2 = 2\sum_{z=17}^{20} \sin(\omega t + \phi_0 + \phi_z / 4) + \sum_{z=21}^{24} \sin(\omega t + \phi_0 + \phi_z / 2) + \sum_{z=25}^{32} \sin(\omega t + \phi_0 + \phi_z)$$

(17)

where $\phi_z$ is the random distribution during $[\phi, 2\phi]$. Figure 5 presents the waveforms of $s_1$ and $s_2$.

**Figure 5 : Comparison of different beamforming methods**

Figure 5 shows that when the bounded error $\delta$ is 20 ns, signals $s_1$ and $s_2$ basically coincide. The further comparison indicates that the correlation coefficient of $s_1$ and $s_2$ is 0.9999, and the PRD (percentage root mean squared difference) of $s_1$ and $s_2$ is 0.0276%.

**Point target imaging experiment**

The simulation uses fixed-point emission and 32-channel and 128-element linear array transducers. The center interval $d$ of array elements is 0.44 mm. The emission frequency $f_0$ is 3.5 MHz, the emission focal point is at 50 mm, the sampling frequency $f_s$ is 50 MHz, and the ultrasound velocity $c$ is 1,540 m/s. The analysis and simulation are implemented using MATLAB 7.10.
The point scattering target experiment is implemented using Field II\textsuperscript{[14]}. Then, 60 dB Gaussian white noise is added to the ultrasonic signal\textsuperscript{[18]}, and the dynamic range of imaging is 60 dB. The six target scattering points are located at (0 0 30), (0 0 40), (0 0 50), (0 0 60), (0 0 70), and (0 0 80) mm, and the detection width is 10 mm. The simulation results are shown in Figure 6. Figure 6 (a) shows the imaging simulation of ideal dynamic focusing, and Figure 6 (b) is the imaging simulation of proposed segmentation dynamic focusing. Compared with Figure 6 (a), Figure 6 (b) shows that the images simulated using the proposed method and the ideal dynamic focusing are basically the same, and the structural similarity index Structural Similarity index (SSIM)\textsuperscript{[16]} in Figure 6 (a) and (b) is 0.948.

![Comparison of different imaging simulations](image)

(a) Ideal dynamic focusing (b) Segmentation dynamic focusing

Figure 6 : Comparison of different imaging simulations

<table>
<thead>
<tr>
<th>Number</th>
<th>Point targets/mm</th>
<th>$\rho$</th>
<th>PRD (%)</th>
<th>SSIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,0,30)</td>
<td>0.9862</td>
<td>2.5</td>
<td>0.9726</td>
</tr>
<tr>
<td>2</td>
<td>(0,0,40)</td>
<td>0.9879</td>
<td>2.4</td>
<td>0.9853</td>
</tr>
<tr>
<td>3</td>
<td>(0,0,50)</td>
<td>0.9891</td>
<td>2.1</td>
<td>0.9764</td>
</tr>
<tr>
<td>4</td>
<td>(0,0,60)</td>
<td>0.9977</td>
<td>0.4</td>
<td>0.9721</td>
</tr>
<tr>
<td>5</td>
<td>(0,0,70)</td>
<td>0.9967</td>
<td>0.6</td>
<td>0.9870</td>
</tr>
<tr>
<td>6</td>
<td>(0,0,80)</td>
<td>0.9789</td>
<td>2.6</td>
<td>0.9846</td>
</tr>
</tbody>
</table>

TABLE 2 : Performance of point targets

TABLE 2 shows that the correlation coefficients of the six target points are more than 0.97, the maximal PRD is less than 3\%, and the minimal SSIM is more than 0.970. The images simulated using the proposed method and ideal dynamic focusing are highly consistent.

CONCLUSION

(1) This paper proposes a segmentation method for dynamic focusing based on bounded error and the generation focusing delay scheme. The core idea of the proposed method is segmenting the detection depth based on the bounded error $\delta$ of focusing delays and constraining the error of focusing delays within the given bounded error $\delta$ in the whole detection range. The accuracy of delays of each focusing channel is enhanced by dynamically changing the approximation unit $\lambda_i$.

(2) The corresponding compressed storage method and real-time generation schemes for focusing delays are proposed in this paper. The proposed method can avoid complex multiplications and significantly improve the compression ratio of focusing delays. It also provides an efficient method to generate focusing delays for high-end digital ultrasonic imaging system.
(3) Error analysis, the point target imaging experiment, and similarity analysis show that the images simulated using
the proposed method and those simulated through ideal dynamic focusing are basically the same. In addition, to meet
different precision requirements, the proposed method can provide a good tradeoff between the compression ratio and
memory consumption by adjusting the given bounded error $\delta$.

(4) The proposed method avoids complex multiplications and significant reduces the storage capacity of the
focusing delays. These advantages make realizing a multi-channel and high-precision dynamic focusing system with a single
low capacity FPGA possible.

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