Big ball era table tennis diameter and competition appreciation analog simulation applied research

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ABSTRACT

According to physical equation, it respectively calculates 38mm, 40mm diameters’ table tennis arc, strength, speed, rotation, drop point, time, clarity and its change rate. And then, use above factors to establish analytic hierarchy process model, athlete experience quality’s criterion layer is composed of arc, strength, speed, rotation, and drop point, and get that small ball has best experience quality to athlete while big ball experience quality is the worst. Similarly, audience’s criterion is composed of round numbers, clarity and time, and get big ball has best appreciation quality to audience, and small ball experience quality is the worst. Finally it gets table tennis ideal diameter is 41mm by analog simulation.

KEYWORDS
Table tennis diameter; Analytic hierarchy process; Analog simulation; Appreciation value; Mathematical model.

INTRODUCTION

In 2000, international table tennis federation increased international table tennis professional competition official ball diameter from 38mm to 40mm. The aim is to further increase ball’s air resistance during air running, slow down competition’s ball running speed, so that achieve the purpose of further increasing and enriching table tennis professional athletes hitting techniques and skills, and finally increase table tennis competitions’ overall appreciation. However, since incoming of table tennis “big ball era” up to now, dispute about ball diameter has never ceased. Chinese and foreign coaches and athletes from all walks of life have mixed. It is worth noting that due to professional athletes’ height, playing habit, gripping habit differences, their sensitivities to ball diameter changes are also different.

Table tennis diameter changes from original 38mm to current 40mm, the main reason is with athlete’s technique and skills being constantly skilled, small ball era table tennis running process flight speed and rotation become faster and faster, the change leads to competition process two parties’ athletes confronting moment round numbers reduce; even sometimes ball is like lightning, audience has not yet reacted while the result come out, ... change. Therefore, after diameter changing, table tennis ball speed will slow down, rotation will weaken, two parties’ athlete confronting moment round numbers increase, so that increase competition appreciation.
TABLE TENNIS TECHNICAL BIOMECHANICAL ANALYSIS

Loop appeared pressure difference due to rotation is called lift force $F_L$ that is Magnus force. It is related to sphere movement speed and rotational frequency, its expression is as following:

$$F_L = C_L \rho D^3 f v$$  \hspace{1cm} (1)

Table tennis suffered air resistance size is related to the maximum cross-section area and suffered wind speed. As following Figure 1 show, assume airflow hasn’t arrived at table tennis is called “state 1”, and airflow already arrives at table tennis is called “state 2”, $v_2 = 0$, and can get by Bernoulli principle:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$ \hspace{1cm} (2)

Figure 1: Table tennis suffered air resistance schematic diagram

$p$ is unit volume airflow pressure energy, from which $v_1 = v$, $v_2 = 0$, and then $P_1 - P_2 = \frac{1}{2} \rho v^2$ in case it is thought that table tennis another side suffered air pressure is equal to front state 1 moment pressure $P_1$, multiply by the maximum cross-section area $A$, and get resistance $F_d$:

$$F_d = (P_2 - P_1)A = \frac{1}{2} \rho Av^2$$ \hspace{1cm} (3)

But considering other factors influence, introduce resistance coefficient $C_d$, and can get final formula:

$$F_d = \frac{1}{2} \rho Av^2$$ \hspace{1cm} (4)

Orbit analysis without considering gravity effects, table tennis force schematic diagram is as Figure 2 show. Thereupon, it can get dynamic equation as following:

$$m \frac{dv}{dt} = \frac{1}{2} C_d \rho Av^2$$ \hspace{1cm} (5)

Figure 2: Table tennis force schematic diagram

Transform and sort the formula use diameter $D$ to express the maximum cross-section area $A$, and let $v = v(t)$, it can get curvature radius formula:

$$R = R(t) = \frac{m}{C_d \rho D} v(t)$$ \hspace{1cm} (7)

Make orthogonal decomposition of table tennis force status, and get Figure 3.

Vertical direction force analysis:

Vertical direction initial speed is $v \sin \theta$, suffered upward resultant force $F_d \sin \theta + G$, distance from ground

Figure 3: Table tennis force status orthogonal decomposition
When table tennis moves upward:  
\[ v_{\text{vertical}} = v \sin \theta - a_{\text{vertical}} \cdot t, \]

\[ a_{\text{vertical}} = \frac{F_g \sin \theta + G}{m}. \]

It can get lift time:  
\[ t_1 = \frac{v \sin \theta \cdot m}{F_g \sin \theta + G}. \]

Lift displacement:  
\[ S_1 = \frac{1}{2} \frac{v^2 \sin^2 \theta m}{F_g \sin \theta + G}. \]

Vertical direction total displacement:  
\[ S = S_0 + S_1. \]

When table tennis moves downward:  
\[ S = \frac{1}{2} at_2^2. \]

It can get reduction time:  
\[ t_2 = \sqrt{\frac{2mS_y}{F_g \sin \theta + G} + \frac{v^2 \sin^2 \theta m^2}{(F_g \sin \theta + G)^2}}. \]

That table tennis air movement time is:  
\[ t = t_1 + t_2. \]

Horizontal direction force analysis:  
When table tennis takes horizontal movement:  
\[ v_{\text{horizontal}} = v \cos \theta - a_{\text{horizontal}} \cdot t, \]

\[ a_{\text{horizontal}} = \frac{F_g \cos \theta}{m}, \quad t = t_1 + t_2. \]

Horizontal movement displacement:  
\[ S_{\text{horizontal}} = v \cos \theta \cdot t - \frac{1}{2} \frac{F_g \cos \theta}{m} \cdot t^2. \]

By above, it can get result as following TABLE1.

**TABLE 1: Big ball and small ball parameters comparison**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Arc/ $R$</th>
<th>Strength</th>
<th>Speed/ $v$</th>
<th>Rotation</th>
<th>Drop point/ $S$</th>
<th>Time/ $t$</th>
<th>Clarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big ball</td>
<td>31.28652</td>
<td>0.12218</td>
<td>20</td>
<td>16.552</td>
<td>2.426577</td>
<td>0.589193</td>
<td>1922.37</td>
</tr>
<tr>
<td>Small ball</td>
<td>29.83818</td>
<td>0.13608</td>
<td>18</td>
<td>20.69</td>
<td>2.226201</td>
<td>0.568804</td>
<td>1733.15</td>
</tr>
<tr>
<td>Change rate</td>
<td>5%</td>
<td>11%</td>
<td>11%</td>
<td>25%</td>
<td>9%</td>
<td>4%</td>
<td>11%</td>
</tr>
</tbody>
</table>

**TABLE 2: Scale description**

<table>
<thead>
<tr>
<th>Scale</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Indicates two factors have equal importance by comparing</td>
</tr>
<tr>
<td>3</td>
<td>Indicates the former is slightly more important than the later by comparing two factors</td>
</tr>
<tr>
<td>5</td>
<td>Indicates the former is obviously more important than the later by comparing two factors</td>
</tr>
<tr>
<td>7</td>
<td>Indicates the former is intensely more important than the later by comparing two factors</td>
</tr>
<tr>
<td>9</td>
<td>Indicates the former is extremely more important than the later by comparing two factors</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Indicates middle value of above adjacent judgment</td>
</tr>
</tbody>
</table>

Reciprocal: If importance ratio between element $i$ and $j$ is $a_{ij}$, then importance ratio between $j$ and $i$ is $1/a_{ij}$.

**ANALYTIC HIERARCHY PROCESS MODELS**

By analyzing, it is clear that table tennis technical factor is extremely important to athlete, so analyze table tennis techniques to athlete’s experience quality influences, consult information and can know arc, strength, speed, rotation, and drop point these aspects influences on athlete’s experience quality are larger, but to audience, competition is splendid or not that decides its appreciation qualities scale, and competition splendidness is up to round numbers, clarity, each time, establish their and table tennis hierarchical structure.

(1) Construct judgment matrix

Hierarchical structure reflects relations among elements, but a criterion hierarchy’s every criteria weight in target measuring is not always the same. This paper adopts comparison between two factors and establishes paired comparison matrix to factor $B$. Which takes two factors $B_i$ and $B_j$ every time, $a_{ij}$ shows the influence ratio that $B_i$ and $B_j$ cover $A$, all comparison results use matrix $C = \left( a_{ij} \right)_{n \times n}$ to express. Call $C$ is paired comparison judgment matrix between $A - B$. If $B_i$ and $B_j$ to $A$ influence ratio is $a_{ij}$, and then $B_j$ and $B_i$ to $A$ influ-
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**Definition 1** If matrix 
\[ A = \begin{pmatrix} a_{ij} \end{pmatrix} \] meets

(i) \( a_{ij} > 0 \), (ii) \( a_{ij} = \frac{1}{a_{ij}} (i, j = 1, 2, \ldots, n) \) then is called positive reciprocal matrix.

In order to define \( a_{ij} \) value, quote number 1-9 and its reciprocal as scale, following TABLE 2 lists description of 1-9 scale meanings.

By TABLE 1, it is clear that big ball and small ball arc, strength, speed, rotation, drop point change rate, change rate gets bigger; it shows it is more important, so fixed arc, strength, speed, rotation, and drop point weight is as TABLE 3, TABLE 4.

**TABLE 3**: Athlete experience quality criterion layer judgment matrix

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B₁</th>
<th>B₂</th>
<th>B₃</th>
<th>B₄</th>
<th>B₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>B₁</td>
<td>1</td>
<td>1/5</td>
<td>1/5</td>
<td>1/7</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>B₂</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1/3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>B₃</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1/5</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>B₄</td>
<td>7</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>B₅</td>
<td>3</td>
<td>1/5</td>
<td>1/5</td>
<td>1/5</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 4**: Athlete experience quality scheme layer judgment matrix

<table>
<thead>
<tr>
<th></th>
<th>B₁</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>B₂</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>B₃</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>1</td>
<td>1/2</td>
<td>1/3</td>
<td>C₁</td>
<td>1</td>
<td>1/3</td>
<td>1/5</td>
<td>C₁</td>
<td>1</td>
<td>1/3</td>
<td>1/5</td>
<td></td>
</tr>
<tr>
<td>C₂</td>
<td>2</td>
<td>1</td>
<td>1/3</td>
<td>C₂</td>
<td>3</td>
<td>1</td>
<td>1/3</td>
<td>C₂</td>
<td>3</td>
<td>1</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>C₃</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>C₃</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>C₃</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>B₁</td>
<td>C₁</td>
<td>C₂</td>
<td>C₃</td>
<td>B₂</td>
<td>C₁</td>
<td>C₂</td>
<td>C₃</td>
<td>B₃</td>
<td>C₁</td>
<td>C₂</td>
<td>C₃</td>
<td></td>
</tr>
<tr>
<td>C₁</td>
<td>1</td>
<td>1/5</td>
<td>1/7</td>
<td>C₁</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>C₁</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C₂</td>
<td>5</td>
<td>1</td>
<td>1/5</td>
<td>C₂</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>C₂</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C₃</td>
<td>7</td>
<td>5</td>
<td>1</td>
<td>C₃</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>C₃</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

(2) Hierarchical single arrangement and consistency test

Judgment matrix \( C \) corresponding maximum feature value \( \lambda_{\text{max}} \), feature vector \( \text{W} \), it is the priority weight of same hierarchy corresponding elements relative importance to last hierarchy some element after normalization, the process is called hierarchical single arrangement.

Above paired comparison judgment matrix construction method though can reduce other factors inference, relatively make objective reflection of a couple of factors influence difference. But when integrate all comparison results; it is hard to avoid containing inconsistency to some extent. If comparison result is wholly consistent, then matrix \( C \) element should also meet:

\[ a_{ij}a_{jk} = a_{jk}, \forall i, j, k = 1, 2, \ldots, n \]  

**TABLE 2** lists description of 1-9 scale meanings.
Theorem 1 Positive reciprocal matrix $C$ maximum feature root $\lambda_{\text{max}}$ is surely positive real number, its corresponding feature vectors’ all components are positive real numbers, $C$’s other feature values all seriously are less than $\lambda_{\text{max}}$.

Theorem 2 If $C$ is consistency matrix, then
(i) $C$ is surely positive reciprocal matrix.
(ii) $C$ transposed matrix $C^T$ is also consistency matrix.
(iii) $C$ any two lines are in proportion, proportion factor is above zero, so that rank($C$) = 1 (similarly, $C$ any two columns are also in proportion).
(iv) $C$ maximum feature value $\lambda_{\text{max}} = n$, from which $n$ is matrix $C$ order. $C$’s other feature roots are zero.
(v) If $C$ maximum feature value $\lambda_{\text{max}}$ corresponding feature vector is $W = (w_1, \cdots, w_n)^T$, then $a_{ji} = \frac{w_i}{w_j}$, $\forall i, j = 1, 2, \cdots, n$, that:

\[
C = \begin{bmatrix}
1 & \frac{1}{5} & \frac{1}{5} & \frac{1}{7} & \frac{1}{3} \\
5 & 1 & 1 & \frac{1}{3} & 3 \\
5 & 1 & \frac{1}{1} & \frac{1}{3} & 3 \\
7 & 3 & 3 & 1 & 5 \\
3 & \frac{1}{3} & \frac{1}{3} & \frac{1}{5} & 1
\end{bmatrix}
\]

By MATLAB calculating, it can get

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Arc</th>
<th>Strength</th>
<th>Speed</th>
<th>Rotation</th>
<th>Drop point</th>
<th>Total arrangement weight value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion layer weight value</td>
<td>0.04269</td>
<td>0.20099</td>
<td>0.20099</td>
<td>0.46910</td>
<td>0.86232</td>
<td>0.154601</td>
</tr>
<tr>
<td>Scheme layer single arrangement weight value</td>
<td>40mm</td>
<td>0.16342</td>
<td>0.10473</td>
<td>0.10473</td>
<td>0.10473</td>
<td>0.63699</td>
</tr>
<tr>
<td></td>
<td>39mm</td>
<td>0.29696</td>
<td>0.25828</td>
<td>0.25828</td>
<td>0.25828</td>
<td>0.33333</td>
</tr>
<tr>
<td></td>
<td>38mm</td>
<td>0.53961</td>
<td>0.63699</td>
<td>0.63699</td>
<td>0.63699</td>
<td>0.33333</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Round number</th>
<th>Clarity</th>
<th>Time</th>
<th>Total arrangement weight value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion layer weight value</td>
<td>0.63699</td>
<td>0.25828</td>
<td>0.10473</td>
<td>0.641043</td>
</tr>
<tr>
<td>Scheme layer single arrangement weight value</td>
<td>40mm</td>
<td>0.71849</td>
<td>0.57143</td>
<td>0.63699</td>
</tr>
<tr>
<td></td>
<td>39mm</td>
<td>0.21849</td>
<td>0.28571</td>
<td>0.25828</td>
</tr>
<tr>
<td></td>
<td>38mm</td>
<td>0.06680</td>
<td>0.14286</td>
<td>0.104795</td>
</tr>
</tbody>
</table>
By MATLAB calculating, it can get \( w_2 = (0.63699, 0.25828, 0.10473)^T \).

Theorem 3: n order positive reciprocal matrix \( A \) is consistency matrix when and only when matrix feature value \( \lambda_{\max} = n \), and when positive reciprocal matrix \( A \) is inconsistent, it surely has \( \lambda_{\max} > n \).

According to theorem 3, we can test whether judgment matrix \( A \) is consistent matrix by defining whether \( \lambda_{\max} \) is equal to \( n \) or not. Due to feature root continuously relies on \( a_{ij} \), so \( \lambda_{\max} \) is far bigger than \( n \), \( A \) inconsistency degree would be more serious, \( \lambda_{\max} \) corresponding standardized feature vector would not really reflect \( X = \{x_1, \ldots, x_n\} \) ratios in element \( Z \) influences. So, it is necessary to make one time consistency test on judgment matrix that provided by decision maker so as to decide whether it is acceptable or not.

Steps for judgment matrix consistency test are as following:

(i) Calculate consistency indicator \( CI \)

\[
CI = \frac{\lambda_{\max} - n}{n - 1} \tag{12}
\]

(ii) Look up corresponding average random consistency indicator \( RI \). To \( n = 1, \ldots, 9 \), it provides \( RI \) values, as TABLE 5 show.

\( RI \) value is got in this way that construct 500 sample matrix by random method, random select numbers from 1 to 9 as well as its reciprocals to construct positive reciprocal matrix, and determine average value of maximum feature root \( \lambda'_{\max} \), and define:

\[
RI = \frac{\lambda'_{\max} - n}{n - 1} \tag{13}
\]

(iii) Calculate consistency proportion \( CR \)

\[
CR = \frac{CI}{RI} \tag{14}
\]

When \( CR < 0.10 \), it is thought that judgment matrix consistency is acceptable, otherwise it should make proper correction to judgment matrix.

(3) Hierarchical total arrangement and consistency test

In above, we achieved is a group element to its previous layer one element weight vector. We finally should get each element, especially bottom layer each scheme to target arrangement weight, so that carry out scheme selection. Total arrangement weight carries out composite on single criterion’s weight from top to bottom as TABLE 6.

Set previous layer (A layer) includes \( A_1, \ldots, A_m \) total \( m \) pieces of factors, their hierarchical total arrangement weight are respectively \( a_1, \ldots, a_m \). And set its next layer (B layer) includes \( n \) pieces of factors \( B_1, \ldots, B_n \), their hierarchical single arrange weight about \( A_i \) are respectively \( b_1, \ldots, b_n \) (when \( B_i \) and are uncorrelated, \( b_{ij} = 0 \)). Now solve B layer each factor weight regarding total target that is to solve B layer each factor hierarchical total arrangement weight \( b_1, \ldots, b_n \), calculation carries out according to TABLE 6 showed ways, that

\[
b_i = \sum_{j=1}^{m} b_{ij} a_j, \quad i = 1, \ldots, n \tag{15}
\]

To hierarchical total arrangement, it also needs to do consistency test, the test is still like hierarchical total arrangement, and it proceeds gradually from high layer to low one. It is because though each layer has gone through hierarchical single arrangement test, each paired comparison judgment matrix has relative satisfied consistency. But when comprehensive investigates it, each layer inconsistency still may accumulate, and lead to final analysis result relative serious inconsistency.

Set B layer \( A_j \) correlated factors paired comparison judgment matrix go through consistency test in single arrangement, and solve single arrangement con-
sistency indicator is \( CI(j), \) (\( j = 1, \ldots, m \)), corresponding average consistency indicator is \( RI(j)(CI(j)) \), \( RI(j) \) already solved in hierarchical single arrangement), then layer total arrangement random consistency proportion is:

\[
CR = \frac{\sum_{j=1}^{m} CI(j)}{\sum_{j=1}^{m} RI(j)}
\]

(16)

When \( CR < 0.10 \), it is thought that hierarchical total arrangement result has an relative satisfying consistency, and accept the analysis result.

Use MATLAB software to calculate athlete experience quality \( CR_1 = 0.028 < 0.1 \), so comparison matrix \( C \) meets consistency test, \( w_1 \) can be used as weight vector. Similarly, audience appreciation quality \( CR_2 = 0.033 < 0.1 \), so comparison matrix \( C \) meets consistency test, \( w_2 \) can be used as weight vector.

By TABLE 7, it is clear that 38mm table tennis its total arrangement weight value is the highest, 40mm ball total arrangement weight value is lowest, that small ball is the best for athlete experience quality, and big ball experience quality is the worst.

By TABLE 8, it is clear that 40mm table tennis its total arrangement weight value is the highest, 38mm table tennis its total arrangement weight value is the lowest, that big ball is best for audience appreciation quality, and small ball experience quality is the worst.

Conclusions: 40mm table tennis compares to 38mm table tennis, it reduces athlete experience quality but improves audience appreciation quality.

BEST DIAMETER ANALOG SIMULATIONS

ITTF official data, table length, width and height(2.74, 1.525, 0.76), horizontal distance between people service and table is 0.67m, service height 0.7m, distance with net is 2m)

(1) Force analysis

Table tennis flight first trajectory refers to table tennis flying away from racket to just contacting table such phase movement process. Table tennis flight process mainly suffers vertical and downward gravity, vertical and upward buoyancy, and air resistance opposite to movement direction. In general, table tennis flight process will accompany with axis surrounding rotating, so, it should also consider Magnus force.

Assume that table tennis only moves in plane \( xoy \). Ball speed as \( v \) and horizontal line form into \( \theta \) angle,
rotational speed is $\gamma = \frac{\omega}{2\pi}$, rotation axis and vertical plane.

Buoyancy $F_j = \frac{1}{6} \rho g \pi D^3$, resistance $F_d = \frac{1}{2} C_d \rho A v^2$, $A = \frac{\pi D^2}{4}$ is table tennis cross-section area. Magnus force $F_m = C_m \rho D^3 f v$

Carry out force analysis according to Figure 4, decompose force along $x$ and $y$ directions, and can list kinematic equation.

$$m \frac{d^2x}{dt^2} = C_1 \rho D^3 v \sin \theta + \frac{1}{2} C_2 \rho A v^2 \cos \theta$$

$$m \frac{d^2y}{dt^2} = mg - \frac{1}{6} \rho g \pi D^3 + C_1 \rho D^3 y v \cos \theta + \frac{1}{2} C_2 \rho A v^2 \sin \theta$$

(17) (18)

(2) Maximum visual speed

There are two self-opposite factors decide table tennis competition watching interestingness. At first competition rhythm should be fast, so competition should be exciting and let athlete always in human reaction time extreme, however, competition should not be so fast that let athlete cannot make prediction by table tennis movement trajectory. The aim is to select table tennis diameter that can let competition speed arrive at maximum and still ensure audience has watching comfortable degree in whole competition.

To solve the problem, at first judge which speed table tennis arrives at will not ensure audience eyes catch up with table tennis movement. Assume the trail is smooth pursuit, which are also eyes continuous tracking moving objects. Maximum value of angular speed that human can make smooth pursuit is 90.9 degree/s. Human can use urgent tracking method to follow moving objects but it let appreciation difficulty enlarge.

In order to define human eyes traceable table tennis maximum speed, it needs to make clear video angle and sphere actual movement relations. Assume two athletes distance is 4m, distance between video and table is 13m, which conforms to ITTF suggestions. The setting can refer to Figure 5, video distance is far enough that ball movement approximates to straight line (though table tennis movement trajectory is arc, due to video and table vertical distance is farther, arc approximates to straight line), so it can combing speed $x$ direction that $v_x$ with eyes watching TV angular speed, then it has:

$$\theta = 2 \tan^{-1} \left( \frac{L/2}{d} \right)$$

$$V_{max} = L \frac{\theta_{max}}{\theta}$$

(19)

By above formula deduction, it gets that maximum speed is 19.2m/s, audience can clearly watch ball movement.

(3) Time ratio

Problem’s key element is: audience can clearly watch table tennis movement trajectory time proportion in whole movement process that is time ratio (clarity time and total time ratio). To solve the problem, record two groups of data, the first group is: $x$ direction component speed (that is $v_x$) initially lower than maximum watch able speed time, the second group is time that ball from departing to arriving at opponent athlete racket, it can get by kinematic differential equation

| TABLE 9: Time ratio and ball diameter changes |
|-----------------|---|---|---|---|---|---|---|---|---|---|
| Diameter/mm    | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 |
| Time ratio/%   | 0  | 20.64 | 40.16 | 58.56 | 75.84 | 92 | 99.66 | 99.66 | 99.66 | 99.66 |
Big ball era table tennis diameter and competition appreciation analog

\[ v_x = \frac{14}{11 - 37D/(4e^{0.0371})} \quad (20) \]

Set table tennis diameter changes between 30 and 48 mm, the changes affect kinematic equation and further will affect \( v_x \). It not only just relies on a trajectory to decide these time ratios. It allows every direction angular speed change between -20 rad/s and 20 rad/s. Figure 6 and Figure 7 show \( x \) direction component speed (that \( v_x \)) and trajectory time relations.

Figure 6 shows when table tennis changes between 30 and 38 mm, \( v_x \) changes, different colors curves corresponding to ball different trajectories, time that ball arrives at opponent is between 1 ms and 1.5 ms (by simulation trajectory).

Figure 7 shows when table tennis changes between 38 and 48 mm, \( v_x \) changes, different colors curves corresponding to ball different trajectories, it notices that when ball diameter is above 41 mm, curve never goes beyond maximum appreciation comfortable degree. Time that ball arrives at opponent is between 0.25 and 0.3 s (by trajectory).

In order to get precise time ratio, by changing diameters, averagely different trajectories get different results. By Figure 8, it can easily observe when ball diameter changes between 30 and 48 mm, appreciation comfortable speed’s time accounts for proportional changes in movement time.

By TABLE 9, it is clear that, ITTF previous regulated competitions with 38 mm diameter tablet tennis only has 75.84% ratio, when diameter changing to 40 mm, ratio arrives at 92%. Any ball diameter that larger than 41 mm can meet sphere movement speed lower than observe comfortable speed. Therefore, best table tennis diameter length is 41 mm.

CONCLUSIONS

According to physical formulas, respectively calculate 38 mm, 40 mm diameter table tennis arc, strength, speed, rotation, drop point, time, clarity and their change rates. And then, use above factors to establish analytic hierarchy process model, athlete experience quality criterion layer is composed of arc, strength, speed, rotation, drop point, their weights are respectively 4.27%, 20.10%, 20.10%, 46.91%, 8.62%, consistency proportion \( CR \) is 0.028 < 0.10, consistency test passes, 38 mm, 40 mm total arrangement weights are respectively 0.5723, 0.1546, which shows small ball is best for athlete experience quality, and big ball experience quality is the worst. Similarly, audience appreciation criterion layer is composed of round numbers, clarity and time, their weights are respectively 63.70%, 25.83%, 10.47%, consistency proportion is 0.0330.10, consistency test passes, 38 mm, 40 mm total arrangement weights are respectively 0.1048, 0.6410 that big ball is best for audience appreciation quality, and small ball experience quality is the worst. Conclusions: 40 mm table tennis compares to 38 mm table tennis, it reduces athlete experience quality but improves audience appreciation quality.

Two main factors affect best diameter defining: visibility and ball speed. Good competition can let ball speed arrive at maximum and let competition speed and competition challenge arrives at maximum, of course it also suffers observability constraint. By calculating, maximum visual speed is 19.2 m/s, that audience can clearly appreciate ball movement maximum speed. Audience clearly watchable table tennis movement trajectory time’s proportion in the whole movement process is time ratio. By kinematic differential equation, it can get direction component speed and trajectory time relations, take 10 diameters from table tennis diameter 38 and 48 mm, by computer analog simulation, it gets that ITTF previous regulated competitions with 38 mm diameter tablet tennis only has 75.84% ratio, when diameter changing to 40 mm, ratio arrives at 92%, diameter 41 mm time ratio is 99.66%, use any ball diameters that larger than 41 mm can meet sphere movement speed lower than maximum visibility. Therefore, best table tennis diameter length is 41 mm.

REFERENCES


