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Behavior Study of Electromagnetic Faraday Rotation for Multiple Reflection of an Electromagnetic Wave from Successive Impulsive Gravitational Waves in General Relativity

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Abstract

We have analyzed the exact behavior of the polarization vector for multiple reflection of an electromagnetic wave from successive impulsive gravitational wave, by using Einstein's theory of general relativity. The Faraday rotation in the polarization vector of the electromagnetic field is induced in this nonlinear process. We show that the Faraday's angle highly depends on the electromagnetic parameter, gravitational parameter, the width and the number of the gravitational wave.

Introduction

The gravitational colliding plane waves was first studied as an exact solution of Einstein's theory by Szekeres [1] and Khan and Penrose [2] in their two pioneering papers and have received much attention since then. By now the colliding waves in general relativity (GR) is a well-known subject [3,4]. The collision and reflection of electromagnetic (em) and gravitational waves (GWs) is an important topic in GR since the theory predicts that there will be a non-linear interaction between such waves. Therefore when we consider em wave reflection in GR we have to take into account that both the reflected and the reflector waves modified. However in classical theory, electromagnetic waves is partly reflected and transmitted since the reflector remains unaffected.

Gravitational wave detectors have been under development since the pioneering work of Weber [5]. The long and painstaking research effort has yielded enormous improvements in detector sensitivity. Astronomical observations of binary pulsar systems have confirmed the existence of gravitational radiation. Direct detection is inevitable once planned detectors reach sensitivity goals. Though the Hulse-Taylor [6] observations were very important, they give only indirect evidence for gravitational waves. A more conclusive observation would be a direct measurement of the effect of a passing gravitational wave, which could also provide more information about the system which generated it. Any such direct detection is complicated by the extraordinarily small effect the waves would produce on a detector. The race is onto detecting ripples from the most massive events in the universe: spinning, orbiting, exploding or colliding ultra-dense objects like black holes and neutron stars.

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A method is given which renders indirect detection of strong gravitational waves possible. This is based on the reflection of a linearly polarized em wave from a cross polarized GW which induces a detectable Faraday rotation in its polarization vector [7,8]. The importance of detectable electromagnetic Faraday rotation as an outcome of the non-linear interaction of GWs with em waves lead us for detecting strong GWs.

In physics, the Faraday effect or Faraday rotation is a magneto-optical phenomenon, or an interaction between light and magnetic field in a medium. The rotation of the plane of polarization is proportional to the intensity of the component of the applied magnetic field in the direction of the beam of light. In GR the Faraday effect or the rotation of the plane of polarization of em waves has been studied extensively. For example [8] shown that the Faraday rotation angle of the plane of em waves is a boundary effect which vanishes for localized astrophysical GWs and is non-zero for cosmological GWs. Recall that the Faraday rotation has been introduced also for the GWs [9,10] It is interesting to study and analysis the behavior of the Faraday effect in colliding waves in GR [11]. Physically, we interpret the collision of em shock wave with GWs as; the linear mode of the em wave rotates due to interaction with a GW giving rise to the Faraday effect in the em wave.

In this paper we will consider the exact solution for multiple reflection of an electromagnetic wave from successive impulsive gravitational waves using Einstein's theory of General Relativity [12]. As a result of this non-linear collision we expect that Faraday rotation in the polarization vector of the electromagnetic field will be induced. We will study the exact behavior of the electromagnetic Faraday rotation by showing that Faraday angle highly depends on the frequencies of both electromagnetic and gravitational waves and the number of successive impulsive gravitational waves.

Reflection of em wave from a succession of gravitational impulsive waves

The general metric for colliding plane waves is represented by J.B. Griffiths [13]

$$(ds)^2 = 2e^{-M} dudv - e^{-U}\left\{\left[e^{V}dx^2 + e^{-V}dy^2\right]\cosh W - 2\sinh W dxdy\right\} \qquad (1)$$

In which the metric functions depend at most on the null coordinate's *u and v*.

The interaction of a linearly polarized plane em wave with a succession of gravitational impulsive waves that propagate in the opposite direction in each of the incoming regions [12] is illustrated in Figure 1. The incoming region III ($v > 0$, $u < 0$) consists of a linearly polarized plane em wave described by the line element

$$(ds)^2 = 2dudv - H^2(v)(dx^2 + dy^2) \qquad (2)$$

Where,

$$H^2(v) = \cos av\theta(v) \qquad (3)$$

with $\theta(v)$ the unit step function and a is the energy (frequency) constant of the em wave described by the Ricci tensor component

$$\Phi_{22} = a^2\theta(v) \qquad (4)$$

The general form of the metric in region II, for parallel polarization can be taken as

$$(ds)^2 = 2dudv - F^2dx^2 - G^2dy^2 \qquad (5)$$

where F and G are only functions of u. For the single impulsive wave located at $u = 0$ we have

$$F = 1 + u\theta(u), \qquad G = 1 - u\theta(u) \qquad (6)$$

Now if we superimpose a second impulsive wave at the wave front $u = u_1 > 0$, our F and G functions become

$$F = 1 + u\theta(u) - \frac{2}{1-u_1}(u-u_1)\theta(u-u_1) \qquad (7)$$
$$G = 1 - u\theta(u) + \frac{2}{1+u_1}(u-u_1)\theta(u-u_1)$$

in which the coefficients are chosen deliberately such that

$$e^{-U} = FG = 1 - \theta^2(u) \qquad (8)$$

still holds. Similarly, for the 3-waves case we have

$$F = 1 + u\theta(u) - \frac{2}{1-u_1}(u-u_1)\theta(u-u_1) + \frac{2}{1-u_1}\frac{1+u_1}{1+u_2}(u-u_2)\theta(u-u_2) \qquad (9)$$
$$G = 1 - u\theta(u) + \frac{2}{1+u_1}(u-u_1)\theta(u-u_1) - \frac{2}{1-u_1}\frac{1-u_1}{1-u_2}(u-u_2)\theta(u-u_2)$$

which also satisfies (8) for $u_2 > u_1 > 0$. The remaining metric function V is given by

$$e^V = \frac{F}{G} \qquad (10)$$

Since it is the expression V_u that enters into the field equations we wish to give its form for different cases:

$$2-waves: V_u = \frac{2}{1-u^2}\left[\theta(u) - 2\theta(u-u_1)\right]$$
$$3-waves: V_u = \frac{2}{1-u^2}\left[\theta(u) - 2\theta(u-u_1) + 2\theta(u-u_2)\right] \qquad (11)$$
$$n+1-waves: V_u = \frac{2}{1-u^2}\left[\theta(u)2\sum_{i=1}^{n}(-1)^i\theta(u-u_i)\right]$$

The impulsive wave fronts u_i must obviously satisfy $0 \le u_i < 1$ and we have the ordering relation $u_i < u_j$ for $i < j$. In between each of the successive waves our space-time is naturally flat. We note that in taking $(V_u)^2$, which appears in the field equations we adopt the standard properties of the step functions, such as

$$\theta(u-u_i)\theta(u-u_j) = \theta(u-u_j) \qquad (12)$$

for $i < j$. As a result we obtain

$$(V_u)^2 = \frac{4\theta(u)}{(1-u^2)^2} \qquad (13)$$

which implies that this term effectively is equivalent to a single wave located at u_0. The Weyl components of the successive waves can be found easily. We have for

$$\begin{aligned}
&2-waves:\Psi_4=-\delta(u)+\frac{2}{1-u^2{}_1}\delta(u-u_1),\\
&3-waves:\Psi_4=-\delta(u)+\frac{2}{1-u^2{}_1}\delta(u-u_1)-\frac{2}{1-u^2{}_2}\delta(u-u_2) \qquad (14)\\
&n+1-waves:\Psi_4=-\delta(u)-\left[2\sum_{i=1}^{n}\frac{(-1)^i}{1-u^2{}_i}\delta(u-u_i)\right]
\end{aligned}$$

It is seen that in region III the em field develops a fold singularity at $v = \pi/2a$. Thus, in the interaction region ($u > u_0$, $v > 0$), we are confined in a region $av < \pi/2$, $u > u_1$ to study the Faraday rotation. Before going to the interaction region let us transform the *(x, y)* axes to align them along with *(x, y)* by rotating the axes by an angle α. As a result a cross polarization term will rise in Equation (5).

At, $u = 0 = v$ an incoming em wave from left encounters the succession of gravitational impulsive waves from right developing a space–time region described by line element given in Rosen form as in Equation (1)
With the solution given by

$$\begin{aligned}
&e^{-U}=\cos^2 bv-u^2, \qquad e^V=\frac{F}{G}\\
&e^{-M}=(\cos bv)\sqrt{1-u^2}\,e^{U/2}\\
&A_y=\sqrt{2}(\sin bv)e^{-V/2} \qquad (15)\\
&\Phi_0^{(0)}=-b\theta(v)(\cos bv)e^{U/2}\\
&\Phi_2^{(0)}=\left(\frac{V_u}{2}\right)(\sin bv)e^{U/2}
\end{aligned}$$

where *F* and *G* are given in terms of the impulsive waves in succession.

For 2-wave and 3-wave cases we have given them explicitly in (9) and (11). It is observed that although the Maxwell component Φ_0 remains unaffected by the successive waves, Φ_2 component changes sign each time when the em wave encounters a new wave front. This amounts to a phase change by 180° of the reflected Φ_2 component of the em wave. Overall effect is that if it crosses an even number of impulsive waves we have $\Phi_2 \rightarrow -\Phi_2$ while for an odd number of waves the sign remains unchanged.

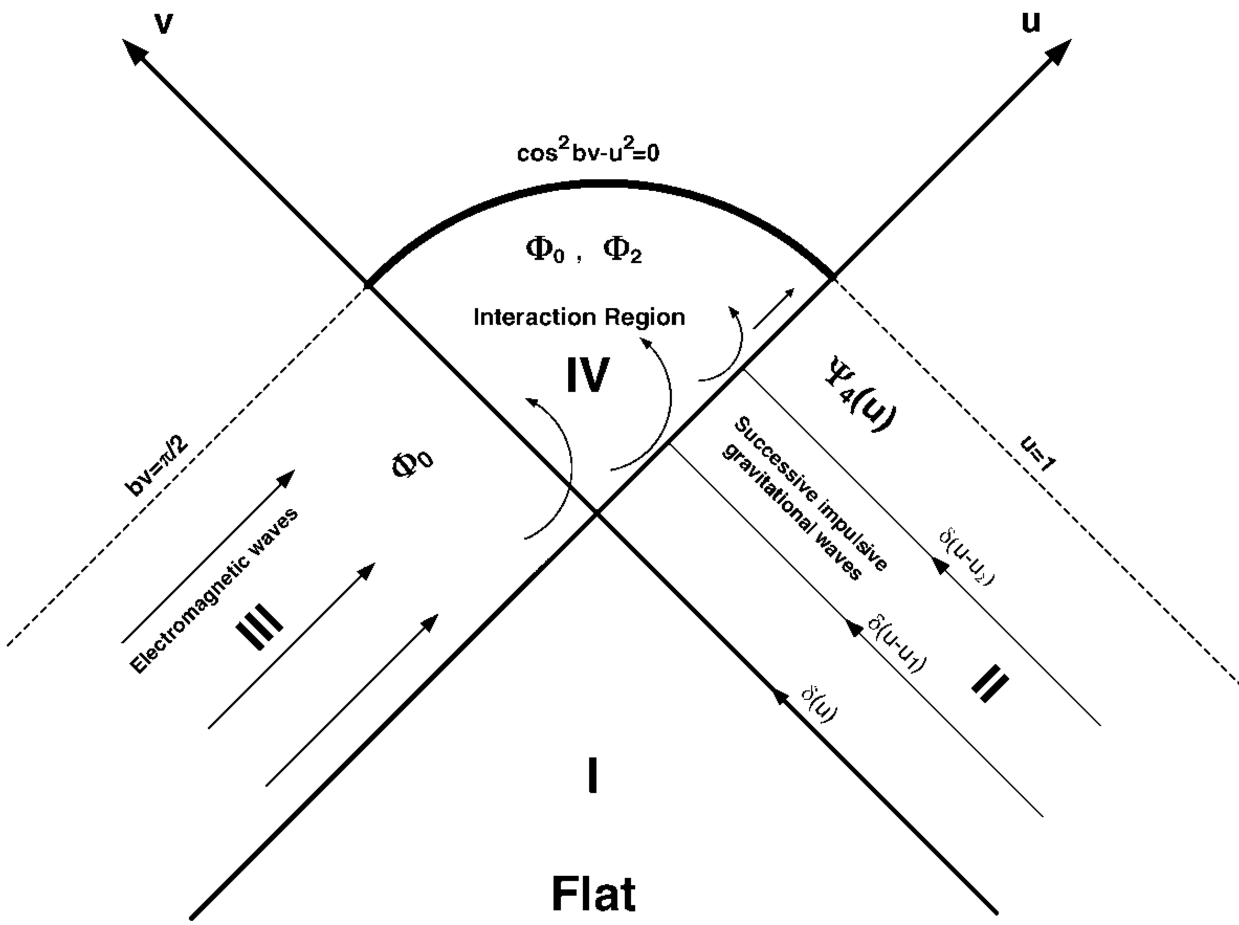

Figure 1: The space-time diagram describes the reflection process of an Em wave from a succession of three impulsive gravitational waves. The process results in a curvature singularity on the focusing surface $\cos^2 bv\text{-}u^2=0$.

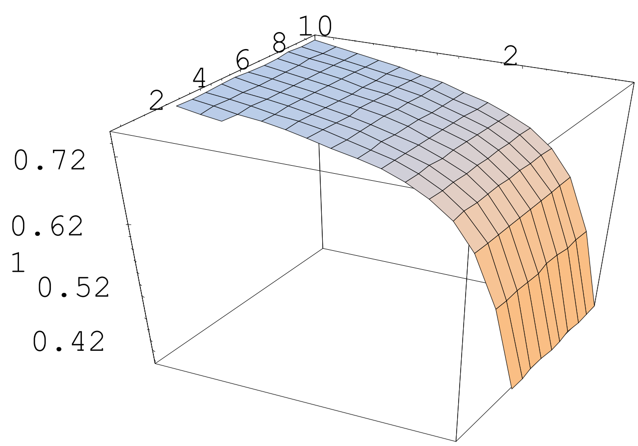

Figure 1(a): The tan θ plot when the electromagnetic wave encounter a gravitational wave with the *(×)* polarization angle $\alpha = \pi/4$, for width (a) $u_1 = \pi/4$ and (b) $u_1 = \pi/2$ *(a =b =* 1*)*.

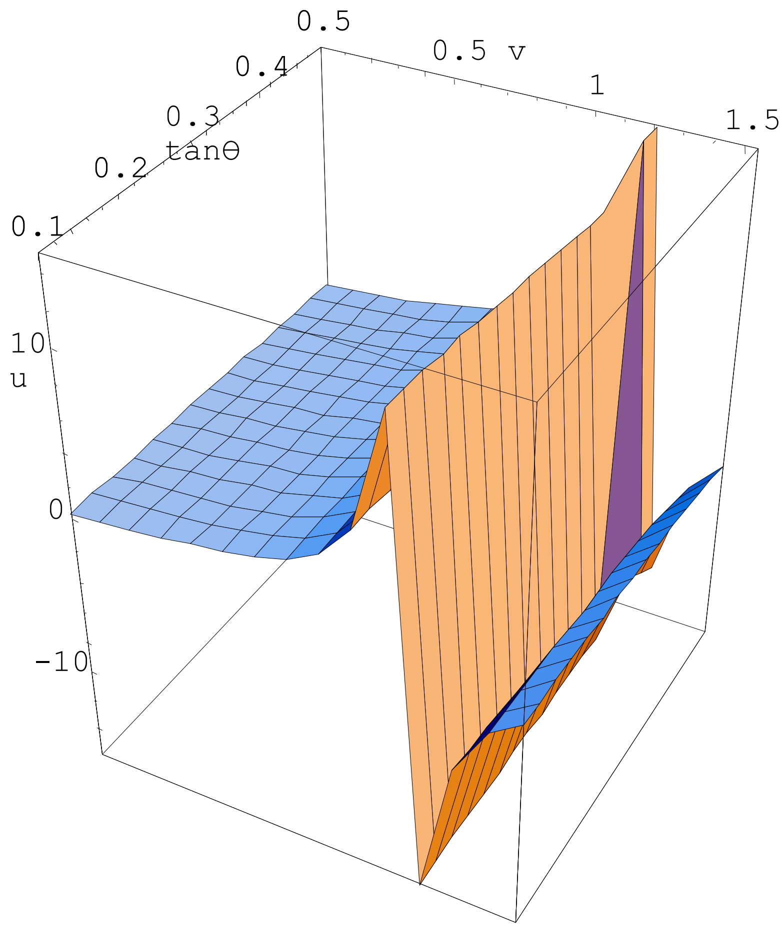

Figure 1(b)

The Faraday rotation

Physically we interpret the interaction of the em shock wave with a successive GW as, the linear (+) mode of the em wave rotates due to encountering a (×) GW giving rise to the Faraday effect in the em wave. Recall that the Faraday rotation angle θ can be determined by

$$\tan\theta = \frac{\mathrm{Im}(\Phi_0 + \Phi_2)}{\mathrm{Re}(\Phi_0 - \Phi_2)} \qquad (16)$$

Let us define the electric and magnetic field components [14]

$$\begin{aligned} E_x &= F_{02} = \mathrm{Re}(\Phi_0 - \Phi_2) \\ B_y &= F_{12} = -\mathrm{Re}(\Phi_0 + \Phi_2) \\ E_y &= F_{03} = \mathrm{Im}(\Phi_0 + \Phi_2) \\ B_x &= F_{31} = \mathrm{Im}(\Phi_0 - \Phi_2) \end{aligned} \qquad (17)$$

in which Φ_0 and Φ_2 are given in Equation (15). Substituting Equation (15) into Equation (16) we obtain the Faraday angle in terms of the metric functions as

$$\tan\theta = \frac{W' + \left(V'\cosh W - 2a\cot av\right)\tan\beta}{W'\tan\beta - V'\cosh W - 2a\cot av} \qquad (18)$$

In terms of the functions F and G Equation (18) with a particular angle $\alpha = \pi/4$ becomes

$$\frac{1}{4}\tan\theta = \frac{(FG)^2\left(GF' - FG' - \left[D(u) + (F^2 + G^2)\right]a\cot av\right)}{(GF' - FG')\left[D(u) + (F^2 + G^2)\right] - 2a(FG)^2\cot av} \qquad (19)$$

Where

$$D(u) = \sqrt{F^4 + G^4 + 6F^2G^2} \qquad (20)$$

Now, we shall study the behavior of this expression and see what is the effect of the parameters, em a and gravitational b, and the width of the GW u_1 on the Faraday's angle. Note that the curvature singularity is at

$$FG + \cos^2 av\theta(v) = 1 \qquad (21)$$

which implies that we have to choose $0 < FG < 1$ and $0 < av < \pi/2$ such that $FG + \cos^2 av\theta(v) < 1$. This limitation on v prevents us from obtaining a periodic behavior in tanav term. Furthermore, the value of u_1 must be selected according to the condition $u_1 < \pi/2a$.

Let us first study the effect of the width (u_1) of the GW on Faraday's angle by plotting the tan θ expression. Equation (19) for different values of u_1. We will study this effect by taking narrow and wide GWs. Figures 2a and 2b represent the GW for widths $u_1 = \pi/4$ (narrow) and $u_1 = \pi/2$ (wide), respectively with $(a = b = 1)$. Note that, in the interaction region ($u > u_1$, $v > 0$), we are confined in a region $v < \pi/2a$, $u > u_1$ to plot the figures. By studying the Figures 2a and 2b we notice the effect of the width of the GW on the Faraday rotation is that for wider GW the em wave undergoes clear and obvious rotation whereas for narrower width the effect is small on the rotation (tan θ). Next, the effect of the em parameter a and $b$on tan θ has already been described in M. Halilsoy and O. Gurtug [6] and A. Al-badawi [11] respectively. For this reason we consider only the tan θ plot for different values of the successive gravitational waves (1-wave, 2-wave and 3-waves) which are exposed in Figures 3a–3c. It is seen clearly from these plots that as the number of waves increase the polarization vector (tan θ) rotates by a wider angle.

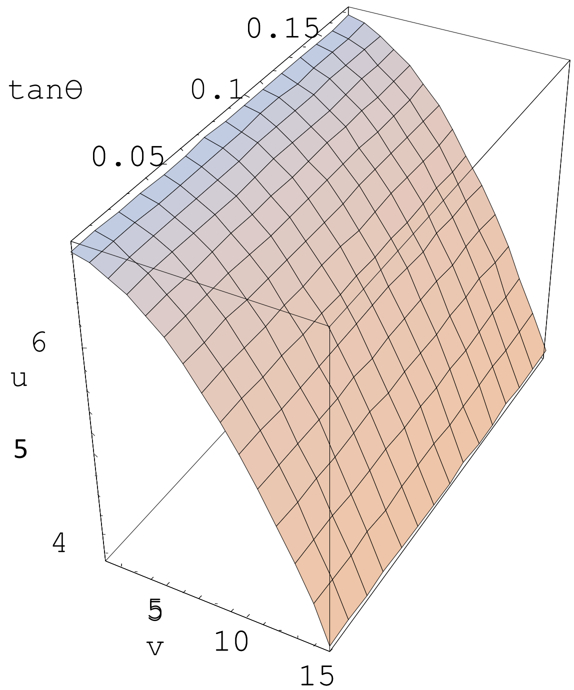

Figure 2(a): The tan θ plot when the em wave with constant frequency parameter a = 1 encounter a GW with the *(×)* polarization angle for $\alpha = \pi/4$, and width $u_1 = u_2 = u_3 = \pi/4$ for the GW frequency (a) 1-wave, (b) 2-waves and (c) 3-waves.

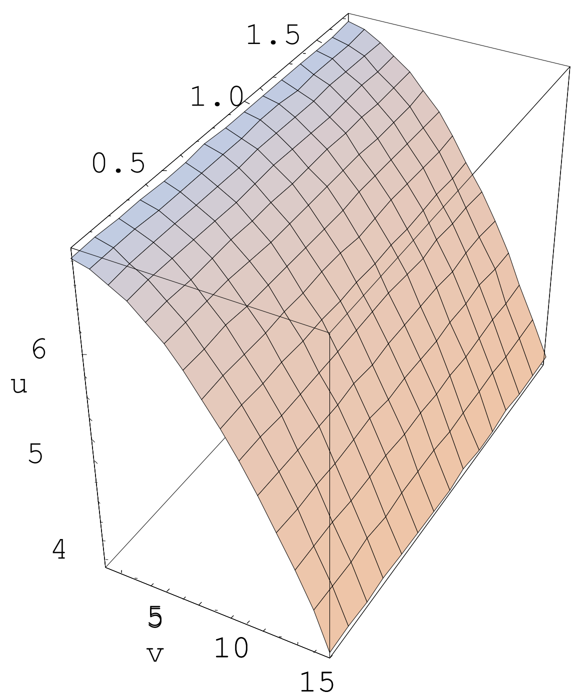

Figure 2(b)

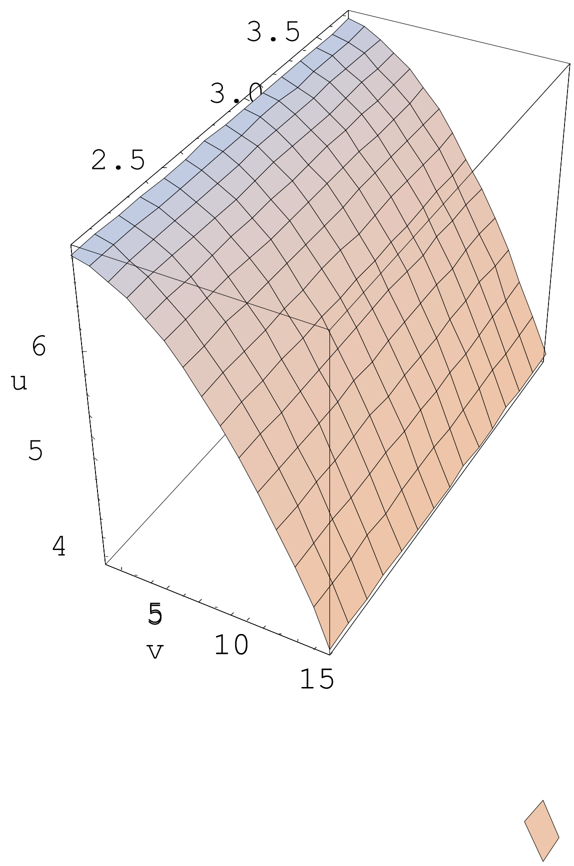

Figure 2(c)

Conclusion

In this paper, we have analyzed the exact behavior of the polarization vector of a linearly polarized em shock wave upon encountering with successive GWs. We showed that the Faraday's angle highly depends on the width and the number of GWs. For the effect of the width it is found that wide thickness gives rise to a clear Faraday rotation than for short one. For the effect of em frequency p*a*rameter a and the gravitational parameter b, it is known from [15-17] that highly energetic em waves (large a) undergoes rotation shortly after their encounter with GW and from [11] that for higher gravitational frequency (i.e. b), the Faraday rotation is larger. Here we have shown that for more GWs, the Faraday rotation (tan θ) is increases. In conclusion, the rotation of polarization vector depends both on the width u_1 of the GW, the parameters a and b and the number of GWs exhibits these behaviors clearly.

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References

1. Szekeres P (1970) Colliding gravitational waves. Nature 228: 1183-1184.

2. Khan KA, Penrose R (1971) Scattering of two impulsive gravitational plane waves. Nature 229: 185-186.

3. Griffths JB (1991) Colliding plane waves in general relativity. Oxford University Press, Oxford, United Kingdom.

4. Weber J (1960) Detection and generation of gravitational waves. Phys Rev 117: 306.

5. Weisberg JM, Taylor JH (2004) Relativistic binary pulsar B1913+16: Thirty years of observations and analysis.

6. Halilsoy M, Gurtug O (2007) Search for gravitational waves through the electromagnetic Faraday rotation Phys Rev D 75: 124021.

7. Barrabes C, Hogan PA (2004) Singular null hyper surfaces in general relativity. Word Scientific, Singapore.

8. Faraoni V (2008) The rotation of polarization by gravitational waves. New Astronomy 13, 178-181.

9. Tomimatsu A (1989) The gravitational Faraday rotation for cylindrical gravitational solitons. Gen Relativ Gravit 21: 613-621.

10. Wang A (1991) Gravitational Faraday rotation induced from interacting gravitational plane waves. Phys Rev D Part Fields 44: 1120-1131.

11. Al Badawi A (2010) The exact behavior of electromagnetic Faraday rotation in crossing a gravitational sandwich wave in general relativity. Physics Letters B 683: 50-54.

12. Weber J (1969) Evidence for discovery of gravitational radiation. Phys Rev Lett 22: 1320.

13. Faraoni V (1993) Gravitational lenses, Proceedings. In: Hamburg R, Kayser T, Schramm & S. Refsdal (Eds.). Springer, Berlin.

14. Surpi GC, Harari DD (1999) Weak lensing by large-scale structure and the polarization properties of distant radio sources. APJ 515: 455.

15. Cruise AM, Ingley RMJ (2006) A prototype gravitational wave detector for 100 MHz. Class Quantum Grav 23: 6185

16. Gurtug O, Halilsoy M, Unver O (2006) Phys Rev D 74, 044020

17. Halilsoy M, Math (1990) J Phys (N.Y.) 31 2694.