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Basketball shooting angle and movement trajectory correlation research based on differential model

Guang Lu

Wuhan Institute of Physical Education, Wuhan 430079, Hubei, (CHINA)

ABSTRACT

Basketball competition is intense, shooting technique is the key to a team scoring. This paper applies differential equation modelling knowledge to make research on basketball movement rules in the air, and gets basketball air movement trajectory formula. According to trajectory formula, it gets rim any distance status best release speed, angle. Combine with fluid mechanics knowledge, it carries out force analysis on basketball air movement, and it finds out basketball speed and air resistance function relationships by combing differential equation modelling knowledge. Then it carries out basketball force analysis and gets basketball air movement trajectory formula, so that finds out basketball air movement rules to make contributions to basketball players' shooting technique improvement.

KEYWORDS

Differential equation;
Basketball trajectory;
Air resistance;
Fluid mechanics.

INTRODUCTION

Basketball is founded in January,1892, though it has been ignored by people in the latte half century, after Berlin Olympic Games in 1936, it is fast popular all around the world, while in modern times, basketball national level competitions mainly are China CBA, America NBA. International competitions mainly include basketball world championship, Olympic Games, Stankovic Cup. With era development, basketball rules are constantly perfecting, technique is constantly improving, and people around the world more and more love it. Each basketball team requests on shooting technique also get higher, therefore, this paper makes research on basketball air movement rules with an aim to make suggestions for shooting technique.

For basketball shooting technique, lots of people have made researches, and put

Forward their own thoughts: Wang Ke-Hai(2001) combined with sports biomechanics analysis, made research on shooting technical motions' shooting different hands, waist as well as lower limbs motions and shooting stability relationships, and got some rules that moment of momentum theorem and moment of momentum conservation law applying into basketball shooting technique, it made suggestions for shooting techniques[1]; Liu Xin-Xing (2008)combined mechanical analysis to research on shooting technical motions and field goal percentage relationships, it stated shooting technical motions importance on improving field goal percentage. And pointed out improve shooting motions methods that was to strengthen receiving –holing motions-aiming-shooting motion transformation coordination[2]; Yuan Feng-Sheng (2007) applied sports mechanics principle to carry out research relative comprehensive, systematical analysis of shooting technical structure, and made research on projection angle and shooting angle rules when shooting air ball, and got different shooting distances best incident angle and projection angle[3].

This paper, on the basis of previous research, makes research on basketball air movement rules, and utilizes differential equation establishing basketball movement trajectory model. Finds out different shooting distances best release speed and angle with an aim to make contributions to shooting technique and propel to basketball development.

BASKETBALL FORCE ANALYSIS

After athletes' releasing, basketball belongs to resistance oblique projectile movement. Basketball oblique projectile force analysis is as Figure 1 show.

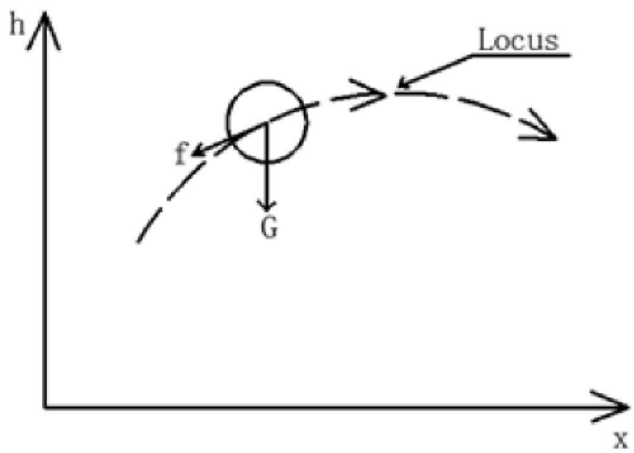

Figure 1 : Basketball oblique projectile force analysis chart

From Figure 1, it is clear that basketball suffers effects from gravity G and air resistance f, gravity G size is constant, $G = mg$. Direction is constant that vertical and turns downwards, but air resistance is influenced by multiple factors, and it changes with basketball speed changing, is a changing quantity at any moment, is very complicated.

Basketball oblique movement air resistance analysis

Air resistance is affected by multiple factors, according to fluid mechanics; it is clear that basketball air movement mainly affected by pressure drag, Reynolds number, effective windward area, air resistance coefficient, as well as movement speed.

1) At first consider round plate suffered pressure drag when it moves in the air, as Figure 2 shows:

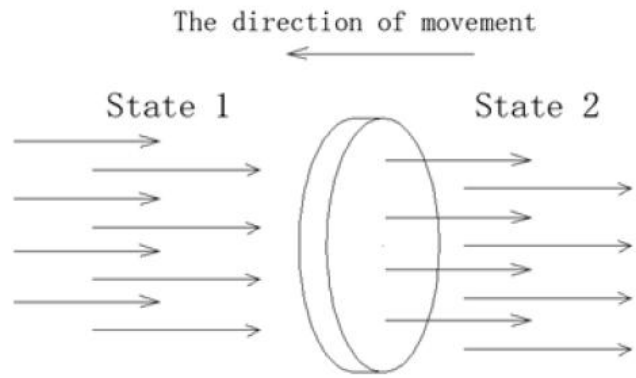

Figure 2 : Plate and airflow state relationships

Assume that airflow not yet arrives at plate is called as " state 1" and airflow already arrives at plate is called "state 2", and first assume airflow speeds all turn into zero after colliding in plate that $v_2 = 0$, then according to Bernoulli principle, it has:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^{\ 2} \quad \textbf{(1)}$$

P is unit volume airflow pressure, from which $v_1 = v$ (that is plate movement speed), $v_2 = 0$, then $P_2 - P_1 = \frac{1}{2}\rho v^2$, if it thinks that plate back pressure is equal to front state 1 pressure P_1, then, plate two sides pressure difference multiples plate area A, it can get resistance F_d :

$$F_d = (P_2 - P_1)A = \frac{1}{2}\rho A v^2 \quad \textbf{(2)}$$

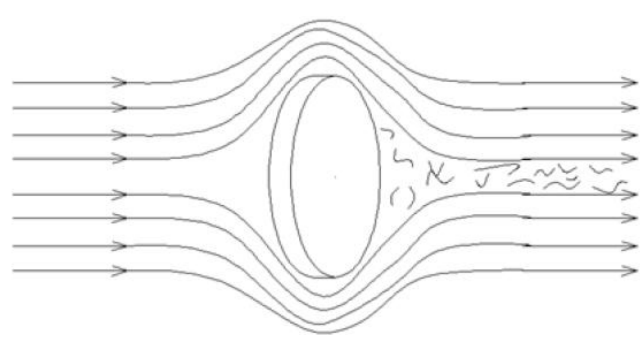

Figure 3 : Air flowing schematic chart

It is not the correct expression, because even plate, not all gaseous mass fully loses speed when it flows to plate and would be more possible as Figure 3 shows flows to the back surrounding plate, obviously at this time, P_2 should reduce, plate back P_1 should also change.

And it will appear friction resistance as well as more complicated other air resistance forms when gas flows to surface, because relative correct general resistance expression should bring into a resistance coefficient C_d, let it change into:

$$F_d = \frac{1}{2} C_d \rho A v^2 \qquad (3)$$

C_d significance has two points: the first, it expresses actual resistance and above plate extreme case, the second, C_d is total effect that includes pressure drag, friction resistance and other resistance. That $C_d = C_d pressure + C_d friction + C_d others$, in concrete case, due to object shapes, movement speed sizes and object surface evenness differences all would led to each concrete resistance proportion in total resistance appearing greater differences, that:

$$C_d pressure / C_d + C_d friction / C_d + \cdots = 1 \quad (4)$$

Formula(3) is present theory generally used air resistance expression, from which A is movement object cross sectional area (it is $\frac{1}{4}\pi D^2$ to sphere), the resistance coefficient C_d of them is related to shape factor and also speed factor, therefore, it cannot generally think air resistance in formula(3) is in direct proportion to v^2, it needs to specific define.

2) Reynolds number

Reynolds number is fluid flow critical exponent describing airflow flow state laminar flow, turbulence flow, it records as R_e. Laminar flow or turbulence flow state air viscous resistance differences is quite bigger, therefore Reynolds number

$$R_e = \rho v D / \eta \quad (5)$$

Among them, v is flow speed, D is jet diameter, the subject is sphere air movement, then 0 are respectively ball speed and ball diameter; η is air viscosity coefficient.

3)Sphere C_d and R_e relationship

According to each sphere air flight moment resistance coefficient testing experiment (Figure 4), it finds: C_d and R_e relationships are up to sphere surface evenness degree, it has nothing to do with sphere diameter. From Figure 4, it is known that C_d has great relationship with sphere surface.

Smooth sphere moves in the air, it can regard sphere moving front and back airflow movement as "pipe flow". When movement speed is not big, "pipe flow" is in laminar flow state, current line distribution of laminar flow condition can refer to Figure 5.

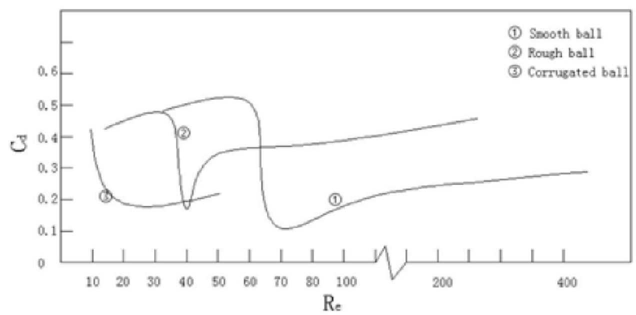

Figure 4 : C_d and R_e relational graph

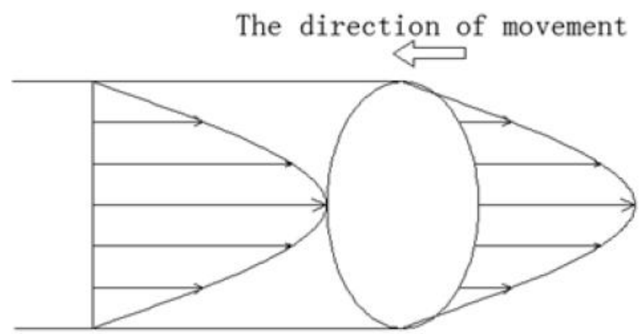

Figure 5 : Laminar flow condition current line distribution graph

Pipe center (that is sphere center) speed is the maximum that is ball speed; "pipe" wall (ball side) speed is zero. At this time, sphere front airflow will not turn towards back. Then pressure drag dominates. Similar to plate extreme pressure drag deduction, only ball front and back airflow show change rules as:

$$v(y) = v\left(1 - \frac{y}{a}\right) \qquad (6)$$

Among them, a is ball radius, y is ball center line distance. The pressure drag calculation is as following:

$$\mathrm{d}F_a = \frac{1}{2}\rho v^2(y)\mathrm{d}A \tag{7}$$

$$\mathrm{d}A = 2\sqrt{a^2 - y^2} * \mathrm{d}y \tag{8}$$

Among them, dy expressed area is as Figure 6 show.

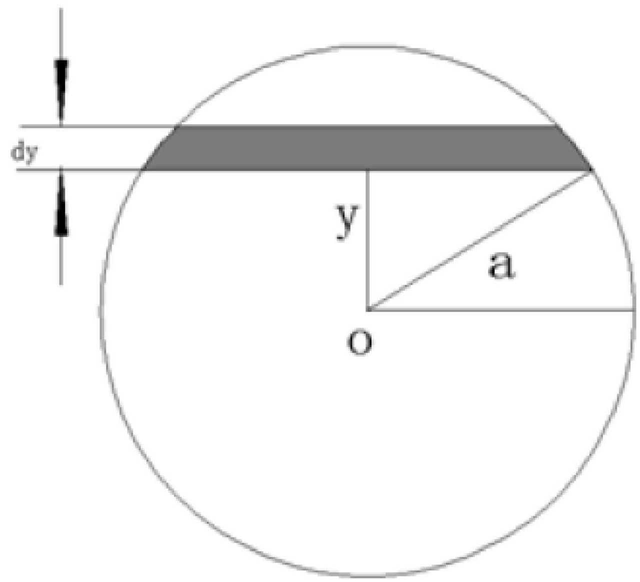

Figure 6 : dy expressed area

Analyze Figure 6, and then it has:

$$F = \int \mathrm{d}F_a = 2\int_0^a \frac{1}{2} 2\rho v^2 \left(1 - \frac{y}{a}\right)^2 \sqrt{a^2 - y^2}\,\mathrm{d}y \tag{9}$$

$$= \frac{1}{2}\rho v^2 4a^2 \int_0^a \left(1 - \frac{y}{a}\right)^2 \sqrt{1 - \left(\frac{y}{a}\right)^2}\,\mathrm{d}\left(\frac{y}{a}\right)$$

$$= \frac{15\pi - 32}{48}\left(\frac{1}{2}\rho v^2 4a^2\right)$$

$$F = 0.4\frac{1}{2}\rho v^2 A \tag{10}$$

Among them, $A = \pi a^2 = \dfrac{\pi D^2}{4}$ is sphere cross section.

When movement speed increases to turbulence state, current line changes into Figure 7 state, at this time front and back gas links, pressure drag precipitously gets small, C_d dramatic reduces, when turbulence state has already thoroughly formed, pressure drag doesn't dominant, friction resistance will increase with speed increasing, shows the slow rising state.

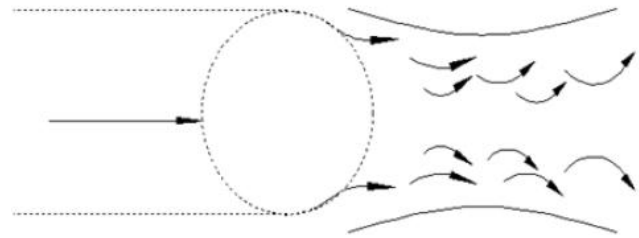

Figure 7 : Turbulence state current line

Rough sphere and smooth sphere has similar results, only maintaining laminar flow condition speed range is more small, therefore it has Figure 4 curve

②case, as for corrugation sphere, because of no laminar flow conditions, it has no curve ①, ②the first phase flat straight line, it appears curve③ case. Basketball belongs to rough ball, therefore basketball speed and resistance functional expression is:

$$F = 0.4\frac{1}{2}\rho v^2 A \tag{10}$$

BASKETBALL MOVEMENT TRAJECTORY CALCULATION

According to Figure 1 force analysis, it can decompose basketball movement into vertical direction and horizontal direction two segmental movements.

Vertical direction segmental movement solution

Assume athlete shooting moment rim horizontal distance is known that is x_0, shooting release speed is v_0, release angle is θ_0, basketball mass is m, when basketball is releasing, its height from ground is h_0. Combine with formula (10), it can solve vertical direction accelerated speed is:

$$a_1 = \frac{F\sin(\theta_0) - G}{m} \tag{11}$$

Then basketball vertical direction height expression following time changes is:

$$h = v_0\sin(\theta_0)t + \frac{1}{2}a_1 t^2 + h_0 \tag{12}$$

For formula(10) speed is also changing with time changes, its calculation formula is:

$$\mathbf{v} = \int_0^t \sqrt{\mathbf{a}_1^2 + \mathbf{a}_2{}^2}\ \mathbf{dt} \tag{13}$$

Therefore, according to formula(10)(11)(12)(13),it can get h and t functional relationship as following:

$$\mathbf{h} = \mathbf{v}_0 \sin(\theta_0)\mathbf{t} + \frac{\frac{1}{5}\rho\left(\int_0^t \sqrt{\mathbf{a}_1{}^2 + \mathbf{a}_2^2}\,\mathbf{dt}\right)^2 \mathbf{A}\sin(\theta_0) - \mathbf{G}}{2\mathbf{m}} + \mathbf{h}_0 \tag{14}$$

Horizontal direction segmental movement solution

Horizontal direction accelerated speed is:

$$\mathbf{a}_2 = \frac{\mathbf{F}\cos(\theta_0)}{\mathbf{m}} \tag{15}$$

Horizontal distance x functional relationship with time changes is:

$$\mathbf{x} = \mathbf{v}_0 \cos(\theta_0)\mathbf{t} + \frac{1}{2}\mathbf{a}_2\mathbf{t}^2 \tag{16}$$

Then according to formula (10)(11)(15)(16), it can get x and t functional relationship :

$$\mathbf{x} = \mathbf{v}_0 \cos(\theta_0)\mathbf{t} + \frac{\frac{1}{5}\rho\left(\int_0^t \sqrt{\mathbf{a}_1{}^2 + \mathbf{a}_2^2}\,\mathbf{dt}\right)^2 \mathbf{A}\sin(\theta_0)}{2\mathbf{m}} \tag{17}$$

According to formula(14)(17),it can get basketball movement trajectory when it is given shooting, rim horizontal distance x_0, shooting release speed v_0, release angle θ_0, basketball mass m, basketball releasing moment its height from ground h_0. To improve field goal percentage, then it needs to select proper incident angle, so that according to shooting angle combining with movement trajectory formula, define best shooting release speed, release angle.

ANALYSIS OF MOVEMENT TRAJECTORY AND SHOOTING ANGLE

Shooting angle refers to sphere entering rim previous instantaneous movement trajectory reflected curve as well as the point tangent line and rim plane formed included angle. The bigger athlete shooting moment projection angle is, the higher sphere flight arc is, the bigger shooting angle and shooting section surface would be; with permissible error range increasing, it is also helpful for improving field goal percentage. If in competition, athletes shooting arc is low, the shooting section and permissible error range would be reduced, it will surely reduce field goal percentage.

Basketball diameter $R = 24cm$, rim diameter $L = 45cm$, given θ_1 to be minimum shooting angle when making the basket, then minimum shooting angle when making the basket is equal to basketball diameter and rim diameter ratio arcsine function, that:

$$\theta = \mathbf{arcsin}\frac{\mathbf{R}}{\mathbf{L}} \tag{18}$$

According to formula (18), it is clear when shooting angle reduces to 32°393 , basketball section will equal to basketball diameter, at this time, permissible error range has already reduced to zero, if shooting angle reduces again, and ball will inevitably touch rim forward side that cause field goal percentage reduces. Therefore, shooting angle selection surely should above 32°393 . To different shooting distances, required shooting angles are different, shooting angle is too large that will cause arc being too large and reduce shooting stability, shooting angle is too small that it is easier to touch rim forward side so that field goal percentage reduces. Therefore, it should select reasonable shooting angle, for best shooting angle defining, lots of people have made their suggestions, is around 45°, the paper will not further state.

According to shooting angle and movement trajectory formula(14)(17),so that define different shooting distances best release speed v_0, best angle θ_0.

CONCLUSIONS

This paper, for basketball air movement conditions, combining with fluid mechanics knowledge to make force analysis, applies differential equation establishing air resistance and speed, as well as mass, surface rough degree and other factors functional relation expressions; it discusses basketball shooting angle. It gets that shooting angle selection should above 32°393 , shooting angle selection and optimal basketball movement trajectory are closely correlated; it establishes basketball movement trajectory and time functional relationship, according to the relationship as well as optimal shooting angle, it can work out rim any distances' optimal release angle and speed.

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