# 2014

# *BioTechnology*

## *An Indian Journal*

## Based on the least square method of table tennis robot motion planning research

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## ABSTRACT

Based on the table tennis robot strike table tennis as the research object, has carried on the force analysis of the physical, and it is concluded that the physical stress model has disadvantages, in order to further optimize model, put forward the movement trajectory of the least squares model, identification and prediction of the trend of table tennis sports. Finally through the table tennis the impact process, and out of the ball speed, speed and so on between the relationship, for the development of table tennis robot has played a positive role.

## KEYWORDS

Table tennis robot; Trajectory prediction model; Least square method, Identifying prediction.

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## PREFACE

For several years, with the development of table tennis career, the intelligent robot, multi-level information network gradually into the cause of sports, such as table tennis robot belongs to one of these, which it through a long course of development, and it towards machine turns to heavy structure and modularization in the direction of the exhibition, the virtual robot strike table tennis as well. There are many scholars are studied, including AnDanYang was used for about table tennis ball machine structure and key technology are analyzed, and finally it is concluded that in the face of different size and power consumption using one-way or two-way rotation of requirements, the size of the code and the trigger for testing through cooperate with light, so as to realize the closed loop control system.

This article is based on the above narrative, in view of the strategy and the table tennis table tennis hitting racket trajectory prediction analysis, time budget to speed and related analysis, so as to contribute to the development of table tennis robot.

## TABLE TENNIS MOTIONS' MECHANICAL ANALYSIS

### Predict trajectory

In order to let table tennis robot to be able to stroke table tennis, robot needs to move in advance and predict table tennis movement direction before it arriving at racket.

If table tennis during motion process, it conforms to air resistance and gravity influence, three directions in coordinate axis are mutual independent from each other, and table tennis speed and resistance are in direct proportion, then table tennis motion force is as following Figure 1 show:
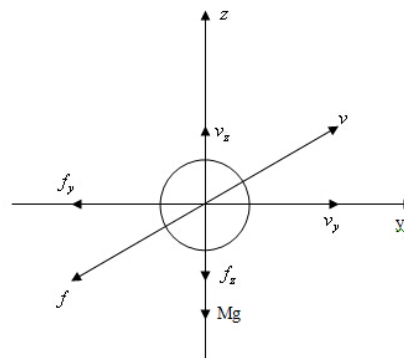


**Figure 1: Table Tennis force analysis process diagram**

By above Figure 1table tennis force, we can analyze it, in plumbed direction, it suffers air resistance and gravity that:

$$A_z = \frac{dV_z}{dt} = \frac{F_z}{m} = \frac{\mu}{m}V_z - g \tag{1}$$

In horizontal plane, it still suffers resistance effects, we take x as an example, that:

$$A_x = \frac{dV_x}{dt} = \frac{F_x}{m} = \frac{\mu}{m}V_x \tag{2}$$

Table tennis mass is using $m$ to express, and viscous resistance coefficient is using $\mu$ to express. Make analysis of table tennis motion process in horizontal plane:

In speed:

$$\begin{cases} V_x(t) = V_x e^{-kt} \\ V_y(t) = V_y e^{-kt} \\ V_z(t) = (V_z + \dfrac{g}{k})e^{-kt} - \dfrac{g}{k} \end{cases} \tag{3}$$

In displacement:

$$\begin{cases} S_x(t) = \displaystyle\int_0^t V_x(\tau)d\tau = \dfrac{V_x}{k}(1 - e^{-kt}) \\ S_y(t) = \displaystyle\int_0^t V_y(\tau)d\tau = \dfrac{V_y}{k}(1 - e^{-kt}) \\ S_z(t) = \displaystyle\int_0^t V_z(\tau)d\tau = \dfrac{V_z + \dfrac{g}{k}}{k}(1 - e^{-kt}) - \dfrac{g}{k}t \end{cases} \tag{4}$$

In above formula, $k$ needs to meet $k = \dfrac{\mu}{m}$.

Table tennis motion prediction is carrying on speed and displacement in the direction of $x, y, z$, but for practical status, considered speed and displacement effects on prediction are not quite obvious.

**Apply least square method identifying table tennis operation status prediction research**

By above force analysis model, we can see that due to model self constraints, it cannot correctly predict table tennis trajectory, so we introduces least square system identifying prediction model.

For observed value as $\{u(k), y(k)\}, k = 1, 2, \cdots, N + n$ output and input $N$ pieces of values, corresponding system can be expressed as:

$$Y = \eta\theta + e \tag{5}$$

In above formula, each parameter should meet:

$$\theta = [a_1, a_2, \cdots, a_n; b_1, b_2, \cdots, b_n]^T \tag{6}$$

$$Y = [y(n+1), y(n+2), \cdots, y(n+N)]^T \tag{7}$$

$$e = [e(n+1), e(n+2), \cdots, e(n+N)]^T \tag{8}$$

$$\eta = \begin{bmatrix} -y(n) & -y(n-1) & \cdots & -y(1) & u(n) & \cdots & u(1) \\ -y(n+1) & -y(n) & \cdots & -y(2) & u(n+1) & \cdots & u(2) \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ -y(n+N) & -y(n+N+1) & \cdots & -y(N) & u(n+N) & \cdots & u(N) \end{bmatrix} \tag{9}$$

After that, according to least square method principle, we take positives, and then corresponding function is:

$$J(\theta) = \sum_{i=1}^{N} (Y - \eta\theta)^T (Y - \eta\theta) \tag{10}$$

So as to let $J(\theta)$ appear minimum value, then:

$$\frac{\partial J}{\partial \theta} = \frac{\partial}{\partial \theta}[(Y - \eta\theta)^T (Y - \eta\theta)] = -2\eta^T (Y - \eta\theta) = 0 \tag{11}$$

And corresponding estimated value is:

$$\theta = (\eta^T Y)^T \eta^T Y \tag{12}$$

Now table tennis motion trajectory that needs to meet is:

$$p = \theta \cdot t \tag{13}$$

In above formula, $\theta$ is parameter, and is also identified.
If regard coordinates axis direction as time function relative curve, then it has:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_x & v_x & p_x \\ a_y & v_y & p_y \\ a_z & v_z & p_z \end{bmatrix} \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} \tag{14}$$

In above formula, $a_x$, $v_x$, $p_x$, $a_y$, $v_y$, $p_y$, $a_z$, $v_z$, $p_z$ property is the same as $\theta$.
For randomly collected one point, we can express it as:

$$p_i = [x_i \ y_i \ z_i \ t_i] \tag{15}$$

In formula, time for the first measurement point time $t_0$ is using $t_i$ to express, and measurement point position is using $(x_i, y_i, z_i)$ to express.
In the following, collect two samples, and then matrix form is:

$$\begin{bmatrix} t_i^2 & t_i & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_i^2 & t_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_i^2 & t_i & 1 \\ t_{i+1}^2 & t_{i+1} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_{i+1}^2 & t_{i+1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_{i+1}^2 & t_{i+1} & 1 \\ t_{i+2}^2 & t_{i+2} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_{i+2}^2 & t_{i+2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_{i+2}^2 & t_{i+2} & 1 \end{bmatrix} \begin{bmatrix} a_x \\ v_x \\ p_x \\ a_y \\ v_y \\ p_y \\ a_z \\ v_z \\ p_z \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \\ z_i \\ x_{i+1} \\ y_{i+1} \\ z_{i+1} \\ x_{i+2} \\ y_{i+2} \\ z_{i+2} \end{bmatrix} \tag{16}$$

By utilizing $Ax = b$ form to solve, but it has larger errors, to reduce the errors, we optimize them, increase selected point's number $3j$, then it can get:

$$Ax = b \tag{17}$$

Among them, $A$、 $x$、 $b$ can be expressed as:

$$A = \begin{bmatrix} t_i^2 & t_i & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_i^2 & t_i & 1 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & t_{i+j-1}^2 & t_{i+j-1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & t_{i+j-1}^2 & t_{i+j-1} & 1 \end{bmatrix}$$

$$x = [a_{x_i} \ v_{x_i} \ p_{x_i} \ a_{x_i+j-1} \ v_{x_i+j-1} \ p_{x_i+j-1}]^T$$

$$b = [x_i \ y_i \ z_i \ x_{i+j-1} \ y_{i+j-1} \ z_{i+j-1}]^T$$

And after solving identification parameter x, it is $3j \times 1$, A、 b is also $3j \times 3j$ matrix.

## Ball landing model

If ball after falling is a kind of elastic collision process, regarding energy loss, we can define $z$ to be speed reverse direction lose, $x$、 $y$ positive direction lose, then total direction lose can be expressed as:

$$k = [k_x \ k_y \ -k_z] \tag{18}$$

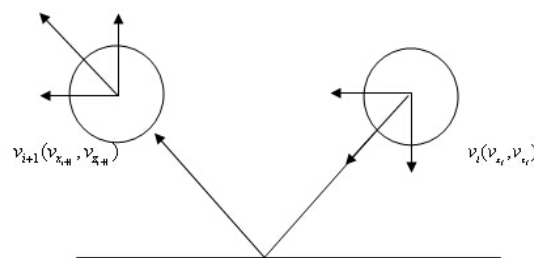Corresponding table tennis landing process is as following Figure 2 show:



$v_{i+1}(v_{x_{i+1}}, v_{z_{i+1}})$ $v_i(v_{x_i}, v_{z_i})$

**Figure 2 : Schematic collision course with the desktop**

In $t_1$ moment, predict ball release positions that can be expressed as $p'_x$, $p'_y$, $p'_z$ forms, then after colliding matrix can be expressed as:

$$\theta' = \begin{bmatrix} a_x & k_x v_x & p'_x \\ a_y & k_y v_y & p'_y \\ a_z & -k_z v_z & p'_z \end{bmatrix} \tag{19}$$

Among them, $t_1$ solving is using following formula:

$$p_z = a_z t^2 + v_z t + p_z = 0 \tag{20}$$

In above formula, energy lose can be solved according to experiment.

## TABLE TENNIS SPEED CONTROL MODEL

If table tennis robot under cooperating of robotic arms, during table tennis and racket colliding process, speed is $\vec{v}_k$, projectile speed is $\vec{v}_f$, and incident speed is $\vec{v}_0$, as following Figure 3 show:
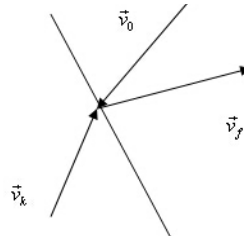


**Figure 3: The impact of table tennis ball and racket**

According to above figure, it can solve that on racket tangent line, it has:

$$^T v_{f\tau} = {}^T v_\tau + ({}^T v_{h\tau} - {}^T v_\tau)\alpha \tag{21}$$

And friction coefficient is using $\alpha$ to express, inelastic collision process condition that needs to meet is:

$$^T v_{fn} = {}^T v_{hn} - {}^T v_n \beta \tag{22}$$

Rebound coefficient is using $\beta$ to express, using $^T R$ to represent transformation matrix, then it has:

$$^T v = {}^T R \cdot v \tag{23}$$

From the perspective of racket, under its coordinate system, normal direction and collision direction are different, then it has:

$$\begin{cases} ({}^T R v_f)_x = ({}^T R v)_x + ({}^T R v_h - {}^T R v)_x \alpha \\ ({}^T R v_f)_y = ({}^T R v_h)_y - ({}^T R v)_y \beta \\ ({}^T R v_f)_z = ({}^T R v)_z + ({}^T R v_h - {}^T R v)_z \alpha \end{cases} \tag{24}$$

After sorting above formula, it can get:

$$\begin{cases} ({}^T R v_f)_x = ({}^T R v_h)_x \partial + ({}^T R v)_x (1 - \alpha) \\ ({}^T R v_f)_y = ({}^T R v_h)_y - ({}^T R v)_y \beta \\ ({}^T R v_f)_z = ({}^T R v_h)_z \alpha + ({}^T R v)_z (1 - \alpha) \end{cases} \tag{25}$$

If $^T R = (r_x \ r_y \ r_z)^T$ , then it can write as:

$$\begin{cases} r_x v_f = r_x [v_h \alpha + v(1-\alpha)] \\ r_y v_f = r_y v_h - r_y v \beta \\ r_z v_f = r_z [v_h \alpha + v(1-\alpha)] \end{cases} \tag{26}$$

By simplifying, it can get:

$$\begin{cases} r_x [v_f - (1-\alpha)v - \alpha v_h] = 0 \\ r_y (v_f + \beta v - v_h) = 0 \\ r_z [v_f - (1-\alpha)v - \alpha v_h] = 0 \end{cases} \tag{27}$$

Matrix after rotating, it can get three coordinate systems projection as $r_x$, $r_y$, $r_z$, by above formula, we can get racket motion form. In practical sports process, plumbed direction doesn't affect racket stroking ball, if it has $z$ coordinate projected racket, then corresponding formula is:

$$r_y' = v_n = \frac{(v_{nx} \ v_{ny} \ v_{nz})}{|(v_{nx} \ v_{ny} \ v_{nz})|} = (v_{nx} \ v_{ny} \ v_{nz}) \tag{28}$$

$$r_z' = (0 \ 0 \ 1) - \frac{(0 \ 0 \ 1) \cdot (v_{nx} \ v_{ny} \ v_{nz})}{|(v_{nx} \ v_{ny} \ v_{nz})|} (v_{nx} \ v_{ny} \ v_{nz}) = (-v_{nx}v_{nz} \ -v_{ny}v_{nz} \ 1-v_{nz}^2) \tag{29}$$

$$r_x' = (R_T)_y \times (R_T)_z = \begin{vmatrix} i & j & k \\ v_{nx} & v_{ny} & v_{nz} \\ -v_{nx}v_{nz} & -v_{ny}v_{nz} & 1-v_{nx}^2 \end{vmatrix} = (v_{ny} \ -v_{nx} \ 0) \tag{30}$$

We make normalization processing with $r_x'$, $r_y'$, $r_z'$, and then it can get:

$$^T R = \begin{pmatrix} \dfrac{v_{ny}}{\sqrt{v_{nx}^2 + v_{ny}^2}} & -\dfrac{v_{nx}}{\sqrt{v_{nx}^2 + v_{ny}^2}} & 0 \\ -\dfrac{v_{nx}v_{nz}}{\sqrt{1-v_{nz}^2}} & -\dfrac{v_{ny}v_{nz}}{\sqrt{1-v_{nz}^2}} & \dfrac{v_{nz}}{\sqrt{1-v_{nz}^2}} \end{pmatrix} \tag{31}$$

$$R_T = (^T R)^T = \begin{bmatrix} \dfrac{v_{ny}}{\sqrt{v_{nx}^2 + v_{ny}^2}} & v_{nx} & -\dfrac{v_{nx}v_{nz}}{\sqrt{1-v_{nz}^2}} \\ -\dfrac{v_{nx}}{\sqrt{v_{nx}^2 + v_{ny}^2}} & v_{ny} & -\dfrac{v_{ny}v_{nz}}{\sqrt{1-v_{nz}^2}} \\ 0 & v_{nz} & \sqrt{1-v_{nz}^2} \end{bmatrix} \tag{32}$$

By $R_T$, it deduces reference coordinate system RPY angle contained relation is:

$$\begin{cases} \beta = a\tan 2(-r_{31}, \sqrt{r_{11}{}^2 + r_{11}{}^2}) = a\tan 2(0, \sqrt{v_{nx}{}^2 + v_{ny}{}^2}) = 0 \\[4mm] \alpha = a\tan 2(r_{21}, r_{11}) = a\tan 2(--\dfrac{v_{nx}}{\sqrt{v_{nx}{}^2 + v_{ny}{}^2}}, \dfrac{v_{ny}}{\sqrt{v_{nx}{}^2 + v_{ny}{}^2}}) \\[4mm] \gamma = a\tan 2(r_{32}, r_{33}) = a\tan 2(v_{nz}, \sqrt{1 - v_{nz}{}^2}) \end{cases} \tag{33}$$

So, ball and racket can be solved according to above formula.
From the perspective of racket, its force has:

$$v_\tau = v_f + \beta v - v_h \tag{34}$$

$v_n$ and $v_\tau$ are a kind of orthogonal way, then;

$$v_n{}^T \cdot v_\tau = 0 \tag{35}$$

Expand above formula, speed relationship has:

$$\begin{aligned} v_n{}^T \cdot v_\tau &= [v_{fx} - (1-\alpha)v_x - \alpha v_{hx}](v_{fx} + \beta_{vx} - v_{hx}) + [v_{fy} - (1-\alpha)v_y - \alpha v_{hy}](v_{fy} + \beta_{vy} - v_{hy}) + \\ &\quad [v_{fz} - (1-\alpha)v_z - \alpha v_{hz}](v_{fz} + \beta_{vz} - v_{hz}) \\ &= 0 \end{aligned}$$

By above formula, we can get table tennis robot relative stroking process each kind of parameters.

## ROBOT STROKING PLANNING

In table tennis motion process, it should meet that robot works in the space, and no collision happens, by practical investigating and measuring, we regulate robot motion system coordinate range is nearly:

$$\begin{cases} -400 \le x \le 400 \\ -1470 \le y \le -1570 \\ 110 \le z \le 450 \end{cases} \tag{36}$$

In robot table tennis playing process, if table tennis within above motion ranges, then it is safety region. Respectively input x、y、z into above formula, then it has:

$$\begin{cases} -400 \le x \le 400 \\ -1470 \le y \le -1570 \\ 110 \le z \le 450 \end{cases} \tag{37}$$

By calculating, it has:

$$\begin{cases} t_x \in [t_{x_1}, t_{x_2}] \\ t_y \in [t_{y_1}, t_{y_2}] \\ t_z \in [t_{z_1}, t_{z_2}] \end{cases} \qquad (38)$$

By above formula solved union set, after that select intermediate point $t = \dfrac{t_1 + t_2}{2}$, it can get speed $v = (v_x, v_y, v_z, t)$ and trajectory coordinate $p(x, y, z, t)$, racket stroking speed is using $v_h(v_{h_x}, v_{h_y}, v_{h_z})$ to express and self posture is using $(\alpha, \beta, \gamma)$ to express, $t$ represents stroking time, $v_h(v_{h_x}, v_{h_y}, v_{h_z})$ represents racket stroking speed.

## CONCLUSIONS

The paper uses kinematics to analyze table tennis stroking process, it solves which speed and what kind of postures that robot strokes table tennis should have and makes analysis, uses system trajectory and physical trajectory model to predict, in addition, it also researches on robot stroking table tennis collision model, therefore it fulfills table tennis trajectory planning problems, and the model has widely utilities.

## REFERENCES

[1] Alireza Fadaei Tehrani, Ali Mohammad Doosthosseini, Hamid Reza Moballegh, Peiman Amini, Mohammad Mehdi DaneshPanah; A New Odometry System to Reduce Asymmetric Errors for Omnidirectional Mobile Robots[J], RoboCup, 600- 610 (**2003**).
[2] R.E.Kalman; A New Approach to Linear Filtering and Prediction Problems [J], Transaction of the ASME - Journal of Basic Engineering, **82**, 35- 45 (**1960**).
[3] Carlos F.Marques, Pedro U.Lima; A Localization Method for a Soccer Robot Using a Vision- Based Omni- Directional Sensor [J], RoboCup, 96- 107 (**2000**).
[4] S.Thrun, D.Fox, W.Burgard, and F.Dellaert; Robust Monte Carlo localization for mobile robots [J], Artificial Intelligence Journal, **128**, 99- 41 (**2001**).
[5] Kan Li-ping; Evaluation on Technical data of Free Kick in Impose Fine Region in Football Game[J], Bulletin of Sport Science & Technology, **19(3)**, 19-20 (**2011**).
[6] Zheng Wei; On the Training of Football Shooting[J], Sport Science and Technology, **3,** 23-26, 33 (**2000**).
[7] Yang Jilin et al.; Research on shooting in the 17th World Cup football semi-finals[J], Journal of Shandong Physical Education Institute, **18(3)**, 51-53 (**2002**).
[8] WANG Xin. Analysis on the best region of shoot [J], Journal of Nanjing Institute of Physical Education, 2002, 16(5): 96-97.
[9] Zhang Ji, xiang; A Study on Effect of Application of the Skills of Side-to-middle Court Passing of Chinese Football Team[J], Journal of Hubei Sports Science, **21(1)**,74-75, 79 (**2002**).
[10] Li Ning, Zhou Jiandong; Statistical Analysis of Goals at 19th FIFA World Cup [J], Journal of Jilin Institute of Physical Education, **27(3)**, 45-47 (**2011**).