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Based on the differential equation of shooting biomechanics parameter correlation analysis

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ABSTRACT

Score in the game of basketball, the key is to the basketball into the net, which involves a lot of kinematic model. One of the basketball into the box, the author of this paper involves the mechanical problems in the process of the research, the process of the basketball into the box can be divided into the top spin, back spin and side spin three conditions, and with the mechanics of the three ways in synthesis and application of differential equation are detailed analyzed. Finally reached the top spin, back spin to smoothly into the basket you need larger casting speed, sidespin requires greater speed, and the right Angle, so this article will be to the basketball teaching plays a guiding role.

KEYWORDS

Differential Equations; Movement rule; Basketball-shooting analysis; Mechanics model.



INTRODUCTION

Basketball is a sport about field-goal attempts determining the scores. In the course of playing the game, scoring becomes the key to winning championship, so the shooting process is the most important in the whole match. There are many factors which determine the basketball-hitting smoothly. The most common one is scoring again after the collision and the rotation of basketball^[1-4]. Previous studies have been done on shooting and fruitful results have been achieved, for example, Chen Jian proposed basketball shooting model with combination of dynamics and the probability statistics in the analysis of mechanics of rotating and aiming point in basketball. In his study, it is finally concluded that forward spinning and backward spinning depend on basketball player's shooting action^[5-9], and on other factors like air resistance in the process of rotation^[10-13]. Research on side spinning, Huang Shunhe raised the probability of enhancing hitting the hoop when rotating incidence angle is less than the angle of reflection as basket is hitting the ring for the purpose of improving the performance of basketball game.

Based on what is found by the scholars above, this paper studies on several cases when the basketball enter the rim. The solution is concluded to increase field goal attempts.

BASKETBALL- SHOOTING AND MECHANICS ANALYSIS

The course of the basketball into the rim can be divided into striking plate shot, rebound shot, swish shot the basket and so on. After touching the basket, several cases may appear which include forward spinning, backward spinning, and side spinning. The basketball rotating is as shown in Figure 1:



Figure 1: Basketball rotation figure

Mechanical analysis on forward rotation of basketball in shooting

The basketball bounces after the collision with the hoop, thus, the corresponding rebound force f from the rim will appear, so will the force F when the ball gets the force from the rim. The direction of rebounding is the direction of the resultant force between f and F . Furthermore, the direction of rebounding is closer to the direction of vertical force from the rim bouncing, which increases the chance of basketball bumping into the hoop. Figure 2 is shown as follows:

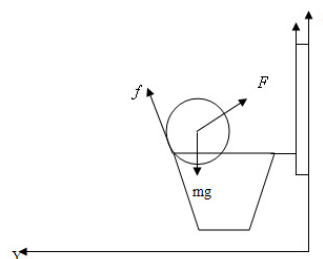


Figure 2 : Schematic before spinning ball bounce along the ring after collision

Besides, the basketball can be more vertical as its rotations speed up so that there is much possibility for the basketball to bump into the rim. After the basketball hits the rim, the resultant force mentioned above will act on the back of the rim so that the chance of ball bouncing and rotating backwards will increase as shown in Figure 3:

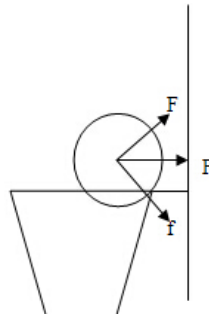


Figure 3 : Rotating the ball hit the rim before the rebound schematic frontier

Mechanical analysis on side spinning of basketball in shooting

The basketball will move in a curved path if it rotates sidewise. Regardless of air resistance, according to the law of angular-momentum conservation and the case where the basketball rotates around *OZ*, the speed and force in the horizontal direction is orthogonal, then F_m can be expressed as shown in the following:

$$F_M = \frac{8}{3} \pi \rho \omega r^3 u \tag{1}$$

In the expression above, when the average velocity can be shown as V , its corresponding velocity component is shown as v , therefore, the differentiation is performed on the process of basketball movement. The formulas can be shown as follows:

In the horizontal direction:

$$M \frac{du}{dt} = -ku^2 \tag{2}$$

In the Plumb Direction:

$$M \frac{d^2}{dt^2} = -Mg \tag{3}$$

$$M \frac{u^2}{R} = \frac{8}{3} \pi \rho \omega r^3 u \equiv G \omega u \left(g = \frac{8}{3} \pi \rho r^3 \right) \tag{4}$$

We let $t = 0$, then $z = 0$, the corresponding elevation of $V = V_0$ can be $\beta = \beta_0$, so:

$$z = (V_0 \sin \beta_0)t - \frac{1}{2}gt^2 \tag{5}$$

Substitute $t = 0$, $u = u_0 = V_0 \cos \beta_0$ into the following formulas:

$$u = \frac{Mu_0}{M + ku_0t} \quad (6)$$

$$s = \frac{M}{k} \ln \frac{M + ku_0t}{Mu_0} \quad (7)$$

$$\alpha = \frac{G}{M} w_0t \quad (8)$$

$$R = -\frac{ds}{d\alpha} = \frac{M^2u_0}{Gw_0(M + ku_0t)} \quad (9)$$

Let speed and rotation angle be simultaneous, then, the solution is:

$$x = \int v \cos \alpha dt = \frac{Mv_0}{Gw_0} \sin \frac{Gw_0}{M} t \quad (10)$$

Combine the formulas with the practice, there is $1 \square ku_0$, by the simultaneous solution, the result is shown as:

$$\left(\frac{Mu_0}{Gw_0}x\right)^2 + \left(1 - \frac{Gw_0}{Mu_0}y\right)^2 = 1 \quad (12)$$

To describe the movement of basketball well, we specialize the parameters, let $G = \frac{8}{3}\pi\rho r^3 = 0.018kg$; $\rho = 1.205kg/m^3(20^\circ C)$; $M = 0.6kg$; $r = 12.13cm$, and substitute them into formula 12, so, during the course of flying, the excused degree at the same level is as follows:

$$y = \frac{Mu_0}{Gw_0} - \sqrt{\left(\frac{Mu_0}{Gw_0}\right)^2 - x^2} = 33.3 \frac{u_0}{w_0} - \sqrt{\left(33.3 \frac{u_0}{w_0}\right)^2 - x^2} \quad (13)$$

From the formulas above, it can be known that there is a positive correlation among the initial speed with the basketball rotating sidewise, the degree of transverse excursion and curvature of moving track, which is determined by the act of players throwing the ball.

Mechanical analysis on back spinning of basketball in shooting

After the collision of the ball and the ball frame, it will suffer the action force F opposite to the original direction after the ball collusive with the rim if the ball does not rotate so that the basket moves in the opposite direction, that is, basketball is bounce back along the original direction. In addition, the basketball also can suffer the reaction force from the hoop in the ring. the action and reaction work together so that the basketball has always been back, the resultant force will direct towards the basket, thus its possibility will increase that the basket bump into the ring as shown in Figure 4:

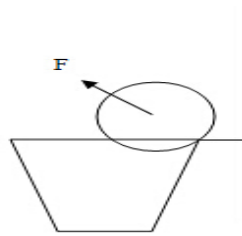


Figure 4 : Ring does not rotate the ball is bounced back schematic

If basketball hits the forefront of the ring after hitting basket, this time it will suffer the force of rebounding f and the rebound force F from the basket, the force in perpendicular direction is the resultant force, and they are very close. Such forces can often make basketball bounce and drop into the basket afterwards, which is shown in Figure 5.

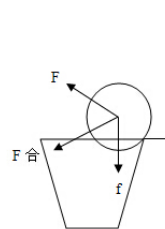


Figure 5 : After rotating the ball hit the rim along the rebound after schematic

And in this way the rotation speed is very fast before the ball drops into the basket, so the greater speed is, the more possibly the basketball drops into the basket. Therefore, there is close connection between the possibilities of basketball into the hoop and the speed of rotation before the basketball enters the hoop, which is shown in Figure 6:

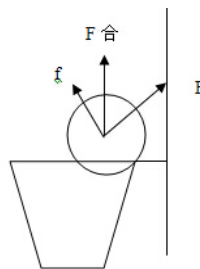


Figure 6 : After rotating the ball hit the rim bounce schematic frontier

From the analysis above, it can be obtained that either forward rotating ball or backward rotating ball; there is much chance for the basketball to drop into the hoop.

COMPUTATION OF THE BASKETBALL HITTING THE RIM

We let the basketball hits the upper backporch at the speed of v_c , and its mass is M , the radius of the basketball is r and the declination is α , which is shown in Figure7:

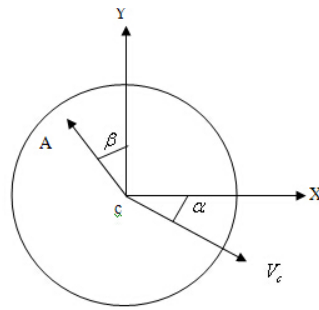


Figure 7 : angle of declination condition after the ball hitting the upper backporch

Meanwhile, let the speed of colliding point after the basketball hits the hoop reconstruct to the contrary, in which the angular velocity is ω_0 , and β is the given angle. At the moment of the ball hitting the hoop, the velocity of the center of mass of the basketball is shown by \vec{V}_c , whose the two projection anises can be defined as follows:

$$V_{cn} = -(V_c \sin \alpha \cos \beta + V_c \cos \alpha \sin \beta) = -V_c \sin(\alpha + \beta) \tag{14}$$

$$V_{cz} = V_c \sin \alpha \cos \beta - V_c \cos \alpha \sin \beta = -V_c \cos(\alpha - \beta) \tag{15}$$

The angular velocity about the axis through the center of mass is rotated counter-clockwise and we use ω to represent it, The angular velocity after hitting the ring is expressed as P_c , angle of declination along straight line between them is indicated as φ , as shown in Figure 8:

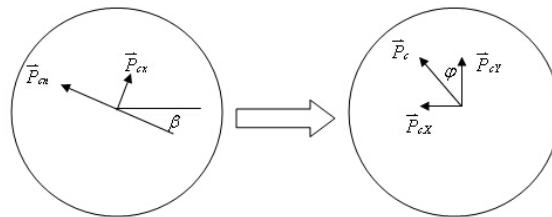


Figure 8 : angle of declination change after hitting the upper backporch

According to Figure 8, we can get the following formulas based on the equation of impulse of the correlative center of mass and the law of impulse of center of mass as follows:

$$mP_{cz} - mP_{cz} = Q_z \tag{16}$$

$$mP_{cn} - mP_{cn} = Q_n \tag{17}$$

$$J_c \omega - J_c \omega_0 = Q_z r \tag{18}$$

Wherein $J_c = \frac{2}{3}mr^2$.

Suppose the basketball hits the point A on the rim and respective speed before and after the collision can be expressed as V_{Az} , V_{An} and P_{Az} , P_{An} , according to the assumption of the factors, we can obtain the equations: $P_{An} = -V_{An} = 1$.

$$e' = -\frac{P_{Az}}{V_{Az}} = 0 \tag{19}$$

$$P_{Az} = 0 \tag{20}$$

Regard the course of Collision the hoop as a hitting of a rigid body, when the ball hits the ring at the beginning, the equation of kinematics corresponding with it can be as shown: $\overline{V}_A = \overline{V}_c + \overline{V}_{Ac}$

Wherein, tangential direction is the direction of $V_{Ac} = r\omega_0$.

$$V_{Az} = V_{cz} + r\omega_0 \tag{21}$$

$$V_{An} = V_{cn} = -(V_c \sin \alpha \cos \beta + V_c \cos \alpha \sin \beta) \tag{22}$$

Similarly, at the moment when the ball is about to end its collision with the ring, the equation of kinematics can be expressed as: $\overline{P}_A = \overline{P}_c + \overline{P}_{Ac}$

Wherein, tangential direction is the direction of $V_{Ac} = r\omega$

$$P_{Az} = P_{cz} + r\omega \tag{23}$$

$$P_{An} = P_{cn} \tag{24}$$

Put the three formulas above into the two former formulas, it will happen like this:

$$P_{cn} = -P_{cn} \tag{25}$$

$$P_{cz} + r\omega = 0 \tag{26}$$

By simultaneousness, each value for the formulas above can be shown as follows:

$$P_{cn} = -V_{cn} = V_c \sin \alpha \cos \beta + V_c \cos \alpha \sin \beta = V_c \sin(\alpha + \beta) \tag{27}$$

$$P_{cz} = \frac{3V_{cz} - 2r\omega_0}{5} = \frac{-3V_c \cos(\alpha - \beta) - 2r\omega_0}{5} \tag{28}$$

$$\omega = -\frac{P_{cz}}{r} = \frac{3V_c \cos(\alpha + \beta) + 2r\omega_0}{5r} \quad (29)$$

$$Q_n = -2mV_{cn} = 2m(V_c \sin\alpha \cos\beta + V_c \cos\alpha \sin\beta) = 2mV_c \sin(\alpha + \beta) \quad (30)$$

$$Q_z = -\frac{2}{5}m(V_{cz} + 2r\omega_0) = -\frac{2}{5}m[-V_c \cos(\alpha - \beta) + 2r\omega_0] \quad (31)$$

Afterwards, let the coordinates rotate, and transform then into XOY , the corresponding P_{cx} , P_{cy} will change, then the transformed formulas are shown as follows:

$$P_{cx} = P_{cn} \cos\beta - P_{cz} \sin\beta = V_c \sin(\alpha + \beta) \cos\beta + \frac{3V_c \cos(\alpha - \beta) + 2r\omega_0}{5} \sin\beta \quad (32)$$

$$P_{cy} = P_{cn} \sin\beta + P_{cz} \cos\beta = V_c \sin(\alpha + \beta) \sin\beta - \frac{3V_c \cos(\alpha - \beta) + 2r\omega_0}{5} \cos\beta \quad (33)$$

$$\operatorname{tg} \varphi = \frac{P_{cx}}{P_{cy}}$$

Besides, the center of mass of the basketball is bound to be studied, the radius of the standard basketball is shown as $L = 22.5\text{cm}$, and then, the time consumed in horizontal direction can be expressed as follows:

$$t_1 = \frac{L}{P_{cx}} = \frac{L}{V_c \sin(\alpha + \beta) \cos\beta + \frac{3V_c \cos(\alpha - \beta) + 2r\omega_0}{5} \sin\beta} \quad (34)$$

The movement in a vertical direction can be expressed as follows:

$$Y = -\frac{1}{2}gt^2 + P_c \cos\varphi \cdot t \quad (35)$$

Then, if $Y = 0$, the corresponding time is t_2 , so:

$$\left(-\frac{1}{2}gt_2 + P_{cy}\right)t_2 = 0 \quad (36)$$

$$t_2 = 0 \quad \text{or} \quad t_2 = \frac{2P_{cy}}{g} = \frac{2V_c \sin(\alpha + \beta) \sin\beta}{g} - \frac{6V_c \cos(\alpha - \beta) + 4r\omega_0}{5g} \cos\beta$$

Through calculation, it can be obtained as follows:

First, if $t_2 > t_1$, the basketball will not enter the hoop after the ball hits the rim.

Secondly, if $t_2 \leq t_1$, his basketball will enter the hoop after the ball hits the rim.

CONCLUSION

This paper studies on the major problems for shooting in a basketball game. The mechanical models of basketball forward spinning and backward spinning are brought out. This problem about shooting is mainly related to the strength of basketball player throwing the ball, namely, the speed of the ball moving. Also, side rotation is related to other factors like the projection angle. This model has a guiding significance for basketball players and coaches; therefore, the model is great importance to the development of basketball career.

REFERENCES

- [1] Chen Jian, Yao Song-ping; Journal of Shanghai Physical Education Institute, **33(5)**, (2009).
- [2] Wang Luning et al.; China Sport Science, **19(3)**, 37-40 (1999).
- [3] Mao Jie, Shan Shuguang; Journal of Wuhan Institute of Physical Education, **46(2)**, 70-73, 87 (2012).
- [4] Zhang Lei. China Sport Science and Technology, **42(1)**, 50-52 (2006).
- [5] Fu Fan-fei, Yu Zhen-feng, Zhang Quan-ning; Shandong Sports Science & Technology, **28(2)**, 24-25 (2006).
- [6] Wang Luning et al.; China Sport Science, **19(3)**, 37-40 (1999).
- [7] Li Ji-hui; Journal of Shenyang Sport University, **25(2)**, 63-66 (2006).
- [8] Yang Yue-qing, Rao Han-fu, Len Ji-lan; Journal of Hubei Sports Science, **22(2)**, 204-205 (2003).
- [9] B.Zhang; S.Zhang; G.Lu; Journal of Chemical and Pharmaceutical Research, **5(9)**, 256-262 (2013).
- [10] B.Zhang; International Journal of Applied Mathematics and Statistics, **44(14)**, 422-430 (2013).
- [11] B.Zhang; H.Yue; International Journal of Applied Mathematics and Statistics, **40(10)**, 469-476 (2013).
- [12] B.Zhang, Y.Feng; International Journal of Applied Mathematics and Statistics, **40(10)**, 136-143 (2013).
- [13] Xiaomin Zhang; Journal of Chemical and Pharmaceutical Research, **5(12)**, 8-14 (2013).