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Arrival and departure capacity evaluation in terminal area based on air leg intersection

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ABSTRACT

Capacity evaluation is the key part of airspace fine management. Only the correct evaluation of the runway, terminal area and air route capacity, can be a reasonable scheduling of flight, reduce flight delays and traffic restrictions, also raise airspace utilization rate. On the basis of research flying aircraft characteristics of the terminal air leg intersection, we set up capacity evaluation model respectively in accordance with two flight status such as convergence arrival and dispersal departure. Finally we select the terminal area Wuhan Tianhe and Guangzhou Baiyun airport as a specific terminal area capacity evaluation, combined with its airport rules and flight schedule published data, verify the effectiveness of the evaluation scheme.

KEYWORDS

Capacity evaluation; Terminal area; Convergence flight; Dispersed flight; Arrival and departure.

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INTRODUCTION

Capacity evaluation is the key part of airspace fine management. Only the correct evaluation of the terminal area capacity, air traffic controllers can arrange flight reasonably, effectively reduce flight delay and improve the utilization of terminal airspace. Because so many arrival and departure flights are frequently centralized in such terminal area which is a transition region linking route and airport, moreover convergence and dispersed flights often cause traffic, accurately capacity evaluation of terminal area is hard to achieve. In this field, academic predecessors have put forward some evaluation methods such as based on mathematical statistics^[1], computer simulation^[2-3] and the workload evaluation by radar simulation machine^[4-5]. In 2007, David Lee and Milan Janie had proposed an effective method by analyzing the interaction of runway and corridor capacity to distribute traffic flow ^[6-7]. In 2012, Kim J and Mitchell JSB studied the convective weather forecasting error lead to the sensitivity of airspace capacity prediction results, which means the development of capacity evaluation method came into the fine management direction^[8]. But all these research findings were mainly in the route region or runway, rarely effective program of research in the terminal area.

In view of this, this paper proposes a new evaluation method of terminal area capacity based on mathematics, combined research in characteristics of arrival and departure flights in the terminal area, especially studied two different flight status convergences and dispersed flights how to lead to the sensitivity of airspace capacity prediction results. Certainly, all the steps in this research should abide by ATC spacing requirements, such as CCAR-93 which defined and published the instrument flight procedures and flight schedules ^[9]. Finally we select Wuhan Tianhe airport and Guangzhou Baiyun airport as the specific terminal area, through using the evaluation method to calculate the arrival and departure capacity respectively and to verify results so as to prove the practicability and validity.

MODEL ESTABLISHMENT

Convergence flight capacity model in terminal area

According to the flight characteristics in terminal area, firstly we set up the mathematical model of convergence flight conditions in the abstract.

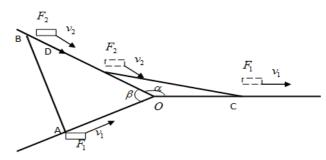


Figure 1 : Convergence flight structure diagram

As showed as Figure 1, we assume that there are two planes which the front one defined F_1 from the *A* to *O* on leg *AO* and the behind defined F_2 from the *B* to *O* on leg *BO*. Obvious the point *O* is the intersection points of the air leg *AO* between *BO*. The plane F_1 speed is v_1 , and v_2 is behalf the speed of F_2 . Further supposing that at time t=0, F_1 at point *O*, F_2 at point *D*, thus the angle between the leg *OC* and *OB* is α , the angle between the leg *OB* and *OA* is β . Set *x* is the horizontal spacing between the two planes, after time *t* the F_1 flight distance is v_1t , the F_2 flight distance is v_2t , according to the expression of cosine theorem in mathematics, the spacing *x* between the two planes can be formulated:

$$x^{2} = (v_{1}^{2} + v_{2}^{2} + 2v_{1}v_{2}\cos\alpha)t^{2} - 2OD(v_{2} + v_{1}\cos\alpha)t + OD^{2}$$
(1)

This is the quadratic polynomial about time t. For the two orders derivative is greater than 0, so that the first derivative is equal to 0 can get the minimum function:

$$t = \frac{OD(v_2 + v_1 \cos \alpha)}{v_1^2 + v_2^2 + 2v_1 v_2 \cos \alpha}$$
(2)

From above formula we know the plus-minus relation of time t depend up by $v_2 + v_1 \cos \alpha$, so on the basis of $v_2 + v_1 \cos \alpha \ge 0$ and $v_2 + v_1 \cos \alpha < 0$, the following may be classified as two situations.

First, while $v_2 + v_1 \cos \alpha \ge 0$, means $t = \frac{OD(v_2 + v_1 \cos \alpha)}{v_1^2 + v_2^2 + 2v_1 v_2 \cos \alpha} \ge 0$, bring time t into formula (1) to get:

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$$x^{2} = OD^{2} \frac{v_{1}^{2} \sin^{2} \alpha}{v_{1}^{2} + v_{2}^{2} + 2v_{1}v_{2} \cos \alpha}$$
(3)

Assume L is the minimum safety space according to ATC requirements, because all above steps is calculated by the minimum function, which means L is equal to x:

$$L^{2} = x^{2} = OD^{2} \frac{v_{1}^{2} \sin^{2} \alpha}{v_{1}^{2} + v_{2}^{2} + 2v_{1}v_{2} \cos \alpha}$$
(4)

Continue solving the above formula to get:

$$OD = \frac{L}{v_1 \sin \alpha} \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos \alpha}$$
(5)

Assume AF act as the minimum spacing of the planes F_1 and F_2 , in this situation while AF reach the minimum it is also equal to x, so we can get:

$$AF = OD = \frac{L}{v_1 \sin \alpha} \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos \alpha}$$
(6)

Then, in this convergence flight condition, the minimal interval time of the planes F_1 and F_2 can be expressed as:

$$T_{\min} = \frac{AF}{v_1}$$
(7)

Because the maximum capacity is the reciprocal relation as the minimal interval time, in this convergence flight condition, so the maximum capacity can be expressed as:

$$C_{\max} = \frac{1}{T_{\min}}$$
(8)

On the other hand, while
$$v_2 + v_1 \cos \alpha < 0$$
, it means $t = \frac{OD(v_2 + v_1 \cos \alpha)}{v_1^2 + v_2^2 + 2v_1v_2 \cos \alpha} < 0$, from Figure 1, according to the expression $x = \sqrt{OB^2 + OA^2 - 2 \times OB \times OA \times \cos \beta}$
of cosine theorem in mathematics, the spacing x can be formulated: $= \sqrt{(v_1t)^2 + (OD - v_2t)^2 + 2v_1t(OD - v_2t)\cos\beta}$
(9)

In this situation, while AF reaches the minimum, it is also equal to x, so can get:

$$AF = \frac{L}{v_1 \sin \beta} \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos \beta}$$
(10)

Dispersed flight capacity model in terminal area

According to the dispersed flight characteristics in terminal area, we set up the mathematical model in the abstract.

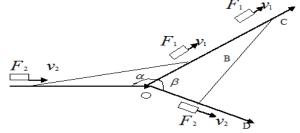


Figure 2 : Dispersed flight structure diagram

As the same way, we assume that there are two planes which the front one defined F_1 from A to O and C on leg AOC and the behind defined F_2 from A to O and D on leg AOD. Obvious the point O is the dispersed intersection points of the air leg AOC between AOD. The plane F_1 speed is v_1 , and v_2 is behalf of the speed of F_2 . At the time is t=0, F_2 reach to point O and F_1 reach point B at this time. After the time t, F_1 flight to point C and the flight distance is v_1t ; F_2 flight to point D and the flight distance is $v_2 t$, according to the expression of cosine theorem, the spacing x can be formulated:

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 $x^{2} = (v_{1}^{2} + v_{2}^{2} - 2v_{1}v_{2}\cos\beta)t^{2}$ $+ 2OB(v_{1} - v_{2}\cos\beta)t + OB^{2}$

This is another quadratic polynomial about time *t*. For the two orders derivative is greater than 0, so that the first derivative is equal to 0 can get the minimum function:

$$t = -\frac{OB(v_1 - v_2 \cos \beta)}{v_1^2 + v_2^2 - 2v_1 v_2 \cos \beta}$$
(12)

On the same principle the plus-minus relation of time t depend on $v_1 - v_2 \cos \beta$, so on the basis of two relations the following may also be classified as two situations.

First, while $v_1 - v_2 \cos\beta \le 0$, means $t = -\frac{OB(v_1 - v_2 \cos\beta)}{v_1^2 + v_2^2 - 2v_1v_2 \cos\beta} \ge 0$, bring time t into formula (11) to get:

$$x^{2} = OB^{2} \frac{v_{2}^{2} \sin^{2} \beta}{v_{1}^{2} + v_{2}^{2} - 2v_{1}v_{2} \cos \beta} = L^{2}$$
(13)

Still assume AF act as the minimum spacing of the planes F_1 and F_2 , in this situation while AF reach the minimum it is also equal to x, so continue solving the above formula to get:

$$AF = OB = \frac{L}{v_2 \sin \beta} \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos \beta}$$
(14)

Therefore, in this dispersed flight condition, the minimal interval time of the planes F_1 and F_2 can be expressed as:

$$T_{\min} = \frac{AF}{v_1} \tag{15}$$

Then, in this dispersed flight condition, the maximum capacity can be expressed as:

$$C_{\max} = \frac{1}{T_{\min}}$$
(16)

On the other hand, while $t = -\frac{OB(v_1 - v_2 \cos \beta)}{v_1^2 + v_2^2 - 2v_1v_2 \cos \beta} < 0$, from Figure 2, according to the expression of cosine theorem, the

spacing x and the minimum spacing AF can be formulated in the same step to get:

$$x = \sqrt{OA^{2} + OC^{2} - 2 \times OA \times OC \times \cos \alpha}$$

= $\sqrt{(v_{2}t)^{2} + (OB + v_{1}t)^{2} + 2v_{2}t(OB + v_{1}t)\cos \alpha}$ (17)

$$AF = OB = \frac{L}{v_2 \sin \alpha} \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos \alpha}$$
(18)

MODEL VERIFICATION

With standard instrument flight procedures in terminal area issued by CAAC(STAR and SID), and flight schedule for the data source, we use the above mentioned models to evaluate capacity of two different terminal areas respectively which is in the landing priority situation or in all part of departure situation.

As figure 3 shown above, there are 4 standard approach instrument flight procedures in Wuhan Tianhe terminal area. Among them ZF-1A_2A from north and XSH-2A_3A from east cause a traffic intersection lies in HG point which is located NBD equipment, LKO-1A from south and HZ-1A from west cause the second traffic intersection lies in DA.

According to the provisions of CAAC, the minimal radar spacing in terminal area is 6km.Referring from the Tianhe airport flight schedule; almost more than 97% planes are the large type engaged in civil aviation public transport. In order to simplify calculation, we only consider the C and D types that called large aircraft. According to the Flight Data Recorder (FDR) data acquisition, the mean approach speed of type C plane is averaged 240km/h, type D is 270km/h.

First taken the HG intersection as concerned, this is the most converging conflict point of arrival flights from Beijing and Shanghai. The air leg angle at this point between ZF-1A_2A and XSH-2A_3A is assumed α and β , which is given from

(11)

airport rules by statistics (Table 1):

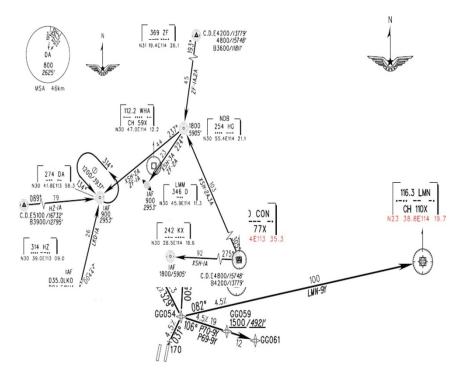


Figure 3 : STARs of runway 20 in Wuhan Tianhe terminal

Table 1 : Arrival leg angle of cross point $HG(\alpha,\beta)$

STARs in HG	ZF	XSH	
ZF	(136°, 0°)	(136°, 112°)	
XSH	(112°, 112°)	(112°, 0°)	

If the front and the behind flights are composed of different type C and D class, therefore there are four different approach situations. Considering the most common situation is that, in the actual control environment controller usually apply speed adjust to avoid catching up or pull out the space. So as to simplify the calculation, we assign 240km/h and

270km/h to v_1 and v_2 separately. According formula (1)-(6) mentioned before, set *L*=6km, while $t \ge 0$, $v_2 + v_1 \cos \alpha \ge 0$, the minimum spacing of the planes F_1 and F_2 is:

$$AF = \frac{L}{v_1 \sin \alpha} \sqrt{v_1^2 + v_2^2 + 2v_1v_2 \cos \alpha}$$
$$= \frac{L}{\sin \alpha} \sqrt{2 + 2\cos \alpha}$$

Thus: $\alpha_1 = 136^\circ$, $AF_1 = 6.4728$; $\alpha_2 = 112^\circ$, $AF_2 = 7.2397$.

Then synthesis $t \ge 0$ and t < 0 two kinds of circumstances, rounding up for *AF* and taking a larger value in the two case, we can get the flight separation matrix AF_{ij}^{CC} and its values in different situations.(*CC* means the type C plane after another type C, 1 means the intersection point, *i* and *j* means the two standard approach instrument flight procedures):

$$AF_{lij}^{CC} = \begin{bmatrix} 7 & 7 \\ 8 & 8 \end{bmatrix}, AF_{lij}^{CD} = \begin{bmatrix} 7 & 7 \\ 8 & 8 \end{bmatrix}, AF_{lij}^{DC} = \begin{bmatrix} 7 & 7 \\ 8 & 8 \end{bmatrix}, AF_{lij}^{DC} = \begin{bmatrix} 7 & 7 \\ 8 & 8 \end{bmatrix}$$

According formula (7), the minimal interval time matrix can get:

$$T_{1ij}^{CC} = \begin{bmatrix} 1.75 & 1.75 \\ 2 & 2 \end{bmatrix}, \ T_{1ij}^{CD} = \begin{bmatrix} 1.75 & 1.75 \\ 2 & 2 \end{bmatrix}, \ T_{1ij}^{DC} = \begin{bmatrix} 1.56 & 1.56 \\ 1.78 & 1.78 \end{bmatrix}, \ T_{1ij}^{DD} = \begin{bmatrix} 1.56 & 1.56 \\ 1.78 & 1.78 \end{bmatrix}$$

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Referring to the Tianhe airport flight schedule, the probability of approach in intersection HG along different standard approach instrument flight procedures is:

$$p_{1ij} = \begin{bmatrix} 0.35 & 0.21 \\ 0.33 & 0.11 \end{bmatrix}$$

So taking the time matrix results above into calculation, then the average minimal interval time in this convergence situation can get:

$$T_1^{CC} = \sum_{i=1}^2 \sum_{j=1}^2 (T_1^{CC}(ij) \times p_{1ij}(ij)) = 1.86$$

In other case, concerned different type C and D class, we set v_1 and v_2 to different average approach speed in calculation, as the same steps can get:

$$T_1^{CD} = 1.86$$
; $T_1^{DD} = T_1^{DC} = 1.66$

Secondly, taken another intersection DA as concerned, this is the most converging conflict point of arrival flights from Chengdu and Guangzhou. The air leg angle at this point between HZ-1A and LKO-1A is assumed α and β , which is given from airport rules by statistics (Table 2):

Table 2: Arrival leg angle of cross point $DA(\alpha,\beta)$

STARs in DA	LKO	HZ
LKO	(66°, 0°)	(66°, 69°)
HZ	(135°, 69°)	(135°, 0°)

As the same way, the minimal interval time matrix can get while in intersection DA:

	25	25	25]		222	222	222]		222	222	222]
$I_{ij}^{C} = I_{ij}^{D} =$	3	3	3 ,	$T_{j}^{DC} =$	244	244	244	$T_{ij}^{DD} =$	267	267	267
	1.75	1.75	1.75		1.56	1.56	1.56		1.56	1.56	1.56

Referring to the Tianhe Airport flight schedule, the probability of approach in intersection DA along different standard approach instrument flight procedures is:

 $p_{CC}=0.48$, $p_{CD}=0.21$, $p_{DC}=0.20$, $p_{DD}=0.11$

Finally, compare the result in HG, the reasonable average minimal interval time matrix should be:

 $T_{ij}^{CC}(1,1) = \max(T_{ij}^{CC}, T_1^{CC})$

Taking the time matrix results above into calculation, then the average minimal interval time in this convergence situation can get:

$$\begin{aligned} \overline{T}_{ij} &= p_{CC} \times T_{ij}^{CC} + p_{CD} \times T_{ij}^{CD} + p_{DC} \times T_{ij}^{DC} + p_{DD} \times T_{ij}^{DD} \\ &= \begin{bmatrix} 2.41 & 2.41 & 2.41 \\ 2.85 & 2.85 & 2.85 \\ 1.69 & 1.69 & 1.69 \end{bmatrix} \end{aligned}$$

That is means based on minimal time interval all arrival flights average approach time is:

$$\overline{T} = \sum_{i=1}^{3} \sum_{j=1}^{3} (p_{ij} \times \overline{t_{ij}}) = 2.36$$

According to formula (8), the maximum capacity in the landing priority situation is:

$$C_{\max} = \frac{1}{\overline{T}_{\min}} = 0.42$$

Therefore, the result 0.42 means the maximum capacity in landing priority situation in Wuhan terminal is 25 sorties per hour.

As far as departure situation, we select Guangzhou Baiyun terminal area (Figure 4) to study departure capacity in dispersed flights situation. There are 4 standard departure instrument flight procedures in there. All the 4 SIDs such as P68-9Y,YIN-9Y,LMN-9Y and P70/69-9Y flight pass through the same cross intersection point GG054.

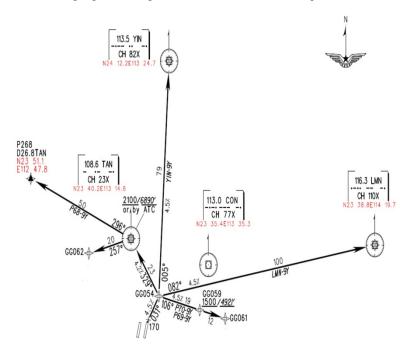


Figure 4 : SIDs of runway 02R in GuangZhou Baiyun terminal area

The air leg angle at this point between the front and the behind is also assumed α and β , which is given from airport rules. Because all 4 SIDs pass GG054, it is easy to get the angle:

 $\alpha_1 = 118^\circ$, $\alpha_2 = 154^\circ$, $\alpha_3 = 129^\circ$, $\alpha_4 = 105^\circ$

On the basis of the provisions of CAAC, the minimal radar spacing in terminal area is 6km. Still only concerned type C and D class planes to simple calculation. According to the flight data recorder (FDR) data acquisition, the mean departure speed of type C plane is averaged 260km/h, type D is 290km/h.

According formula (11)-(14) mentioned before in chapter 2, concerned with plus-minus relation of time t, the minimum spacing of the two dispersed planes named AF is:

$$AF = \frac{L}{v_1 \sin \alpha} \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos \alpha}$$
$$= \frac{L}{\sin \alpha} \sqrt{2 + 2\cos \alpha}$$

So take the value of α in above, can get:

AF1=6.9999, AF2=6.1579, AF3=6.6477, AF4=7.5630

Then synthesis $t \ge 0$ and t < 0 two kinds of circumstances, rounding up for AF and taking a larger value in the two cases, we can get the flight separation matrix AF_{lii}^{CC} and its values in different situations.

$$AF_{ij}^{CC} = \begin{bmatrix} 7 & 7 & 7 & 8 \\ 7 & 7 & 7 & 8 \\ 7 & 7 & 7 & 8 \\ 7 & 7 & 7 & 8 \\ 7 & 7 & 7 & 8 \end{bmatrix} \quad AF_{ij}^{CD} = \begin{bmatrix} 7 & 6 & 7 & 8 \\ 7 & 6 & 7 & 8 \\ 7 & 6 & 7 & 8 \\ 7 & 6 & 7 & 8 \end{bmatrix} \quad AF_{ij}^{DC} = \begin{bmatrix} 8 & 7 & 8 & 9 \\ 8 & 7 & 8 & 9 \\ 8 & 7 & 8 & 9 \\ 8 & 7 & 8 & 9 \end{bmatrix} \quad AF_{ij}^{DD} = \begin{bmatrix} 7 & 7 & 7 & 8 \\ 7 & 7 & 7 & 8 \\ 7 & 7 & 7 & 8 \\ 7 & 7 & 7 & 8 \\ 7 & 7 & 7 & 8 \end{bmatrix}$$

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Follow the formula (15), the minimal interval time matrix can get:

	[1.62					1		1.62						1.86		1			1.66
TCC	1.62	1.62	1.62	1.85	T^{CD} –	1.62	1.38	1.62	1.85	T^{DC} –	1.66	1.45	1.66	1.86	$T_{ij}^{DD} =$	1.45	1.45	1.45	1.66
$I_{ij} =$	1.62	1.62	1.62	1.85	$I_{ij} =$	1.62	1.38	1.62	1.85	$I_{ij} =$	1.66	1.45	1.66	1.86	1 _{ij} —	1.45	1.45	1.45	1.66
	1.62	1.62	1.62	1.85		1.62	1.38	1.62	1.85		1.66	1.45	1.66	1.86		1.45	1.45	1.45	1.66

Referring to the Baiyun Airport flight schedule, the probability of departure along different standard instrument flight procedures is:

$$p_{\rm CC}=0.38$$
, $p_{\rm CD}=0.25$, $p_{\rm DC}=0.23$, $p_{\rm DD}=0.14$.

Taking the time matrix results above into calculation, then the average minimal interval time in this dispersed situation can get:

$$\overline{T}_{ij} = p_{CC} \times T_{ij}^{CC} + p_{CD} \times T_{ij}^{CD} + p_{DC} \times T_{ij}^{DC} + p_{DD} \times T_{ij}^{DD}$$

$$= \begin{bmatrix} 1.61 & 1.51 & 1.61 & 1.83 \\ 1.61 & 1.51 & 1.61 & 1.83 \\ 1.61 & 1.51 & 1.61 & 1.83 \\ 1.61 & 1.51 & 1.61 & 1.86 \end{bmatrix}$$

That is means based on minimal time interval all departure flights average release time is:

$$\overline{T} = \sum_{i=1}^{4} \sum_{j=1}^{4} (p_{ij} \times \overline{t_{ij}}) = 1.63$$

According to formula (16), the maximum capacity in the all part departure situation is:

$$C_{\max} = \frac{1}{\overline{T}_{\min}} = 0.6135$$

Therefore, the result 0.6135 means the maximum capacity in departure situation in Guangzhou terminal is 36 sorties per hour.

CONCLUSIONS

On the basis of research flying aircraft characteristics of the terminal air leg intersection, we set up capacity evaluation model respectively in accordance with two flight status such as convergence arrival and dispersal departure. Through analysing the standard instrument flight procedure in Wuhan Tianhe and Guangzhou Baiyun terminal area, as well as two airport flight schedule, combined with the ATC interval rules and actual flight data, we respectively evaluate the maximum capacity of the two terminals in different situations. With the evaluation results is corresponding to the actual terminal area capacity, so it has proved the effectiveness and correctness of the evaluation scheme.

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