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Approach with different entropies to the problem of plasma confinement

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Abstract

The equivalence between different kinds of entropies (fuzzy entropy, fractional entropy, Tsallis entropy) is used for description of the problem of optimal plasma confinement at tokamaks. These conditions hold true under assumptions of partial observability or incomplete information.

Key Words

Fuzzy entropy; Tsallis entropy; Fractional entropy; Partial observability; Tokamak.

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INTRODUCTION

Dynamical system can reach „ spontaneously" a steady state exibiting some analogies with a thermodynamic system at a critical point. The system is driven to a stationary state where avalanches are distributed according to a power law. This phenomenon is called Self-Organized Criticality (SOC). The equilibrium condition for a magnetically confined plasma is normally formulated in terms of macroscopic equations. In these equations, the plasma pressure is assumed to be a function of the magnetic flux with continuous derivatives. However, in three-dimensional systems this is not necessarily the case. On the basis of observation scheme, a compensator type is proposed and the conditions that are sufficient for synthesis procedure are given. We have experimental, theoretical and computational methods. Organised research of these methods could provide the future progress. The development of fault diagnosis and sensor validation technique must face : the non-linearity of the process, the uncertain knowledge about the phenomena, often expressed in a linguistic fashion. The central problem is : plasma position, current and shape control.

METHODS OF SOLVING TOKAMAK EQUILIBRIUMS

The variational principle states that the entropy is maximum at equilibrium. It defines a unique probability measure on the phase space, the Gibbs distribution. The stationary regime can be result of specific non- Hamiltonian microscopic dynamics and there is no „natural" Gibbs distribution nor free energy in SOC systems. It is not possible to analyze the out - of - equilibrium stationary state via a local equilibrium hypothesis where one decomposes the system into mesoscopic cells locally at equilibrium.

The thermodynamic formalism may be used to construct the equivalent of finite – volume Gibbs measure where the Hamiltonian is replaced by a dynamically relevant potential. It is possible to generate a finite Markov partition P used for symbolic dynamics.

The invariant probability distribution characterizes the probability of occurrence of any recurrent symbol at stationarity. The attractor is a fractal set. There exists only one maximum or one equilibrium state when we have mixing. This situation corresponds to the absence of a phase transition in statistical mechanics[1].

We discuss the possible interconnection between the equilibrium and anomalous transport.

The Grad - Shafranov equation is the equilibrium equation in ideal magnetohydrodynamics (MHD) for a two dimensional plasma, for example the axisymmetric toroidal plasma in a tokamak.

Interestingly the flux function ψ is both a dependent and an independent variable in this equation

$$\Delta^*\psi = -\mu_0 r^2 \frac{dp}{d\psi} - F\frac{dF}{d\psi} \quad (1)$$

where μ_o is the magnetic permeability, p (ψ) is the presssure, and F (ψ) = r B_Φ, for $\vec{B}$ is the magnetic field.

Two dimensional, stationary, magnetic structures are described by the balance of pressure forces and magnetic forces, i. e. :

$$\nabla p = \vec{j} \times \vec{B} \quad (2)$$

where p is the plasma pressure and $\vec{j}$ is the electric current.

Alternativelly we can consider Vlasov – Poisson – Fokker – Planck (VPFP) equation and the properties of weak solutions

$$\frac{\partial f}{\partial t} + (v\cdot\nabla_x) f + div_v((E_\sigma - \beta v) f) - \sigma\delta_v f = 0 \quad (3)$$

Toroidal and poloidal momentum confinement in neoclassical quasilinear theory in tokamaks and stellarators is under developing. The theory is applied to explain the changing of the toroidal and poloidal flow direction after low mode to high mode (H- mode) transition that is observed in some experiments. Toroidal momentum confinement in tokamaks is most likely to be anomalous. This implies that a quantitative theory for the toroidal flow is difficult if not impossible. Fortunately for H-mode the radial electric field is predominately determined by the poloidal momentum equation or the parallel momentum equation. Poloidal momentum equation can be described fairly by the neoclassical theory.

Empirically we know that we need both fuelling and heating to maintain steady state. This means that a pure heat source cannot maintain the density and pure particle source cannot maintain the temperature.

Over the past decade, step-by-step new regimes of plasma operation have been identified, wherely turbulence can be externally controlled, which led to better and better confinement. Turbulence as a whole is generally not an equilibrium phenomenon. The method for obtaining of adaptive approximatelly recurrence equilibriums is explained.

The notions of the complexity function and entropy functions are introduced to describe systems with nonzero or zero Lyapunov exponents or systems that exibit strong intermittent behaviour with flights, trapping, weak mixing etc.

The new approach to the problem of complexity and entropy covers different limit cases, exponential and polynomial, depending on the local instability of trajectories and the way of the trajectories′ dispersion.

In the framework of mesoscopic nonequilibrium thermodynamics is obtained a generalized Fokker- Planck equation incorporating memory effects through time-dependent coefficients. The nonMarkovian dynamics of anomalous diffusion is discussed. There are many different areas where fractional equations describe real processes.

The equilibrium of the fractal turbulent medium exists for the magnetic field with the power law relation obtaining of a simplified mathematical model describing the dynamics of plasma parameters and the selection of control laws (controllers) stabilizing these parameters[2].

The aim is to obtain a path that with adaptive control technique we arrive to the regime of critical work of fusion reactor that is obtained by uniform stabilization. The regime of long pulse operations could be obtained[3].

SENSORS AND ACTUATORS

In finite dimensions, a simple way of designing a compensator is to construct first a state feedback stabilizer and an obsever for the system, and then to combine the two design the compensator using a feedback of the observer instead of the state. The author believes that, with the development of the technology, in some cases the infinite dimensional compensators can also have applications in practice without the finite dimensional approximations.

Works on sensor scheduling and sensor allocation aimed at finding an optimal sequence of actions (sensor relocations or adjustments) minimize certain cost function. They differ from the optimal observability problem where an optimal policy is sought upon which we choose actions based on real measurements.

We can use an information theoretic measure, estimation entropy to analyze the optimal observability problem. We choose the action based on our past and current observations[4].

A key property of an process is that the information state is a Markov chain. We see that under ergodicity condition (existence of a unique invariant measure), the average cost criterion for the optimal observability problem is the estimation entropy. At any given time only one sensor can be selected for the observation of state. The optimal observability problem is to obtain a policy that we call the scheduled sensor selection policy. The problem is minimizing the estimation entropy over different control policies.

Anomalous transport has been found where turbulent processes of various kinds are responsible for enhanced transport. In recent years tremendous progress has been made to describe the turbulent transport properly in accordance with all the various experimental results over the whole plasma cross section. Therefore, experimental findings are still categorised according to their empirical signatures into confinement „ modes". (the L- mode, the- H mode, E (ELMy H- mode) (ELM-free H- mode)[5].

ANOMALOUS TRANSPORT

Anomalous transport, which is considered to be induced by the plasma turbulence caused various instabilities, limits the energy and particles confinement times in toroidal plasmas. In the fusion-related research using toroidal plasmas, the suppression of turbulence has been main problem to make confinement time longer.

Fluctuations are in many cases driven by the nonlinear instability mechanisms, not by the linear instability mechanisms. Turbulent states do not satisfy the equipartition law. Therefore, even if plasmas are in thermodynamical equilibrium, the conventional statistical theory may not be sufficient. One formal way of the statistical theory is to extend the definition of entropy. As an example, Tsallis statistics is known. In Tsallis statistics, an extended „entropy" is not longer extensive quantity.

The decay of correlations in dynamical systems, or more generally, the rate of approach of a given initial distribution to an invariant one, is an area of long standing interest and research,

Quasi-compactness of Perron- Frobenious operator on the space of function of bounded variation led to an exponential decay of correlations in the case of uniformlly expanding maps on the interval.

There is a general method of deriving statistical limit theorems, such as the central limit theorem and its functional version, in the setting of ergodic measure preserving transformations[6].

This method is applicable in situations where the iterates of discrete time maps a polynomial decay of correlations.

On such a way we have extended central and functional limit theorems for non-invertible ergodic transformations.

APPROACH WITH FUZZY LOGIC CONTROLLERS

An infinite fuzzy logic controller (IFLC) consists of rules considering vector fuzzy values of error e(k) between current set point s(k) and output y(k) and its first difference δ e(k) as input variables and the first difference of control variable $\delta u(k)$ as an output variable:

IF **e(k) is** $E_{1(i)}$ **AND** $\delta e(k)$ **is** $E_{2(i)}$
THEN $\delta u(k)$ **is** $U_{(i)}$ **(i = 1,2,....∞)** (4)

The errors e(k) must be obtained with sensors(observers) (k denotes the time step). We have by the deffuzification distribution method:

$$\delta u(k) = (\delta u^{(1)}(k), ...\delta u^{(6)}(k)$$

$$\delta u^{(r)}(k) = \frac{\sum_{j=1}^{\infty} w_j^{(r)} \; u_j^{(r)}}{\sum_{j=1}^{\infty} w_j^{(r)}}$$

for $\sum_{j=1}^{\infty} w_j^{(r)} \prec \infty, \sum_{j=1}^{\infty} u_j^{(r)} \prec \infty$ **r = 1,2,...6** (5)

We have also

v(k) = IFLC(e(k), δ **e(k)) v(k) = v(k-1)**
+ $\delta u(k)$ **, e(k) = s(k)-y(k)** (6)

With this method, we can approximate any function that is obtained from gyrokinetic theory of Vlasov-Poisson-Fokker-Planck equation. The process has a vector valued state equation y(k + 1) = P(y(k),u(k))[7],[8].

Moreover, the accuracy of the optimized thermodynamic quantities can be evaluated and additional simulations can be carried out when necessary. The reason for chaos could be the extreme sensitivity on initial data or spatiotemporal intermittency that can produce additional complexity. The key for an analytic treatment of such behavior is to introduce probabilistic distribution that could be described with IFLC.

For the most complicated cases of long interactions of the tokamak plasma the approach with long memory effects can be applied via theory of fuzzy neural networks. The goal is, with localized control, overcome the problems of non-equilibrium in plasma. The aim is converting the anomalous diffusion to the exponentially stabilizable diffusion.

We also show the passage from the non- Markovian to the Markovian behaviour in the normal diffusion regime. For times longer then the relaxation time, the correlation function for anomalous diffusion becomes a power law for broad- band noise.

The optimization requires the generation and analysis of a large number of MHD equilibriums. A few thousand equilibriums may have to be computed[9],[10].

Let us consider the models with incomplete information. We suppose that the state of the system at the time t is described by the pair $x_t y_t$ the first of these components becoming known to us and the second not. The actions a_t and the observed states x_t are connected as before by a projection J $(x_{t-1} = J(a_t))$. In this case we also calculate with some uncertainities. We can note that actually all depends about the nature of chaotic attractors, i.e. whether deterministic or stochastic attractor arrives in decision environment.

For deterministic nonlinear case the spatiotemporal behaviour is described by the distribution function s(k) in the Banach space $L_1(\Omega\times V)$. If the process is controllable $\|e(k+1)\|\to 0$, it is easily follows that

$$P(y(k),v(k)) = P_{v(k)}y(k)\to s(k+1) \text{ as } k\to\propto \quad (7)$$

We obtain, according to the contraction rule

$$d\left[P_{v(k)}y(k),P_{v(k+1)}y(k+1)\right]\le \alpha^k d\left[y(0),y(1)\right] \quad (8)$$

where d denotes suitable distance function and $\alpha<1$.

It follows $P_{v(\propto)}y(\propto)=s(\propto)$. On such a way, we can simulate spatiotemporal evolution. The same formalism holds true for local contractive, ergodic Markov processes.

Inside of the theory of self organized criticality we will work actually with the multiple fixed points. For achieving the each fixed point, the condition of local controllability must be true.

In the case of stochastic turbulence, we define the fuzzy information entropy by

$$S=-\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}P_{ij}^{\Delta}\ln P_{ij}^{\Delta} \quad (9)$$

With S(i) we shall denote for each time step i, the fuzzy Jaynes entropy.

For the fuzzy exponentially stabilizable process, on each i-th time step, the optimal result is obtained by maximum entropy method. As final criteria we take the maximal amount of mathematical expectations over tree of events[11].

But, necessary is better understanding the nature of the process Instead of fuzzy entropy with exponential stabilization as it is done in the case of Landau damping[12], we should apply the approach with fractional entropies.

INTERPRETATION OF EXPERIMENTS WITH ENTROPIES

Research activities are focused on developing new analysis and design methods for fractional order controllers as an extension of classical control theory[13]. Actually, the PD^{μ} proportional and derivative fractional order controller itself is an infinite dimensional linear filter due to the fractional order differentiator μ. Finite dimensional approximation of the FOC should utilized in a proper range of frequency of practical interest. In the case of bifurcations and fractals we can apply our methods of IFLC.

In the classical statistical physics and the information theory the close relation with Boltzmann-Shannon entropy has been well established to offer elementary and clear understandings. Tsallis opened new perspectives for the power entropy to elucidate non-equilibrium states in statistical physics, and these give the strong influence on the research for non- extensive and chaotic phenomenon.

The Tsallis entropy for the continuous case is

$$S_q(f)=\frac{1}{q-1}\left\{1-\int f(x)^q\,dx\right\} \quad (10)$$

where the probability density function f(x) is proportional to the power entropy.

The projective power entropy is[14].

$$H_{\gamma}(f)=-\frac{1}{\gamma(1+\gamma)}\left\{\int f(x)^{1+\gamma}\,dx\right\}^{\frac{1}{1+\gamma}} \quad (11)$$

Tsallis entropy can be derived as a result of incomplete information condition.

Given a random variable X which takes on the values $x_1,x_2,\dots,x_m,\dots$ with the probabilities $p_1,p_2,\dots p_m\dots,\ p_1+p_2+\dots+p_m+\dots=1$, its fractional entropy of order α is defined by the expression,

$$H_{\alpha}(X)=-\sum_{i=1}^{\infty}p_i(Ln_{\alpha}p_i)^{1/\alpha} \quad (12)$$

where we have got that $Ln_{\alpha}p$ denotes the inverse of the Mittag – Leffler function, clearly $p=E_{\alpha}(Ln_{\alpha}p)$ and with the convention $0(Ln_{\alpha}0)^{1/\alpha}=0,(Ln_{\alpha}1)^{1/\alpha}=0$

We have $H_{\alpha}(X)=\sum_{i=1}^{\infty}p_i(Ln\frac{1}{p_1})^{1/\alpha}$ (13)

This entropy has properties similar to those of the entropies of Renyi and Tsallis in which the parameter could be thought of as measuring an information loss. Assume that the process is observed with some defect in observation. An approach to describe this phenomenon is to select a model in the form

Observed information $=\alpha\ln(1/p_i), 0\prec\alpha\prec 1$

Therefore the entropy $\langle\alpha\ln(1/p))\rangle$, that is to say

$$H_{obs}(X)=\sum_{i=1}^{\infty}p_i\ln(1/p_i^{\alpha})=-\sum_{i=1}^{\infty}p_i\ln p_i^{\alpha} \quad (14)$$

In the setting of fractional entropy, according to Jaynes maximum entropy, our purpose will be determine the probability distribution p(x) which maximizes the entropy $\langle -Ln_{\alpha}\ p\rangle^{1/\alpha}$ that the mathematical expectations have given values[15],[16].

When $\alpha=1$, $E_{\alpha}(\cdot)$ turns to be exp(.) and we obtain exactly the well known result corresponding to the normal law. On the modeling standpoint according to the maximum entropy principle, in the presence of defect in observation (or fuzzy observation), the probability density defined by the Mittag-Leffler function would be more relevant than the Gaussian law.

CONCLUSIONS

Non-extensive statistical mechanics appears as a powerful way to describe complex systems. Tsallis entropy, the main core of this theory has been remainded as an unproven assumption. In the paper[17] is derived Tsallis entropy using the incomplete information theory. In the case of transport theory we define Tsallis entropy as $S_q = -k\frac{1-\sum_{i=1}^{\infty} p_i^q}{1-q}$ where k is a positive constant, p_i stands for probability of occupation of i-th state of the system, ∞ counts the known microstates of the system and q is a positive real parameter.

Tsallis entropy is non-extensive, which means that if two identical systems combine, the entropy of combined system is not equal to summation of entropy of its subsystems. The possibility of existence of infinite states of the statistical system is necessary for obtaining the desired results from the theoretical convergence theorems of functional analysis[18].

We gave the approach with fuzzy logic controllers and fractional entropy as equivalent formulation of the problems of non-equilibrium statistical mechanics.

REFERENCES

[1] B.Cessac, Ph.Blanchard, T.Kruger, J.L.Meunier; Self-organized criticality and thermodynamic formalism, Journal of Statistical Physics, 5/6,115, 1283-1326 (2004).
[2] D.Rastovic; Fractional fokker-planck equations and artificial neural networks for stochastic control of tokamak, J.Fusion Energy, **3**, **27**, 182-187 (2008).
[3] D.Rastovic; Analytical description of long-pulse tokamaks, J.Fusion Energy, **2**,**27**, 285-291 (2008).
[4] M.Rezaeian, B.N.Vo, J.S.Evans; The optimal observability of partially observable markov decision processes: Discrete state space, IEEE Trans Automatic Control, **12**,**55**, 2793-2798 (2010).
[5] D.Rastovic; On stochastic control of tokamak and artificial intelligence, J.Fusion Energy, **4**,**26**, 337-342 (2007).
[6] Tyran M.Kaminska; An invariance principle for maps with polynomial decay of correlations, Commun.Math.Phys., **260**, 1-15 (2005).
[7] D.Rastovic; Infinite fuzzy logic L1 controllers, Fuzzy Sets and Systems, **1**, **72**, 75-77 (1995).
[8] D.Rastovic; Controlled fusion and vector-valued infinite fuzzy logic controllers, Advances in Modelling & Analysis A. Mathematical, Gen.Math Modelling, **1**, **36**, 15-20 (1999).
[9] A.Murari, G.Vagliasindi, P.Arena et al.; Prototype of an adaptive disruption predictor for JET based on Fuzzy logic and regression trees, Nucl.Fusion, **48**, 035010(10pp) (2008).
[10] A.Murari, J.Vega, G.A.Ratta et al.; Unbiased and non_supervised learning methods for disruption prediction at JET, Nucl.Fusion, **49**, 055028(11pp), (2009).
[11] D.Rastovic; Targeting and synchronization at tokamak with recurrent artificial neural networks, Neural Comput & Applic, DOI 10.1007/s00521-011-0527-4, (online) (2011).
[12] D.Rastovic; Tokamak design as one sustainable system, Neural Network World, **6**, 493-504 (2011).
[13] H.S.Li, Y.Luo, Y.Q.Chen; A fractional order proportional and derivative (FOPD) motion controller: Tuning rule and experiments, IEEE Trans on control systems technology, **2**, **18**, 516-520 (2010).
[14] S.Eguchi, O.Komori, S.Kato; Projective power entropy and maximum tsallis entropy distributions, Entropy, **13**, 1746-1764 (2011).
[15] G.Jumarie; Path probability of random fractional systems defined by white noises in coarse-grained time, Application of Fractional Entropy, Fractional Differential Calculus, **1**,**1**, 45-87 (2011).
[16] D.Rastovic; Fractional variational problems and particle in cell gyrokinetic simulations with fuzzy logic approach for tokamaks, Nucl.Tech.& Radiation Protection, **2**, **24**, 138-144 (2009).
[17] A.H.Darooneh, G.Naemi, A.Mehri, P.Sadeghi; Tsallis entropy, Escort probability and the incomplete information theory, Entropy, **12**, 2497-2503 (2010).
[18] D.Rastovic; Simulation and control of turbulence at tokamaks with artificial intelligence methods, Journal of Modern Physics, **3**, 1856-1869 (2012).