Applied research on biomechanics-based table tennis flight trajectory simulation in teaching

Mingping Fu
Institute of Physical Education, Chuzhou University, Chuzhou 239000, Anhui, (CHINA)

ABSTRACT
The paper firstly starts analyzing from racket hitting to colliding with table, and then to ball flying out of table the whole table tennis flight process, it makes mechanical analysis and modeling, subsequently uses VB writing codes to solve mechanical model and draw out flight route, analyzes different playing methods occurred different arcs, intuitively reflects table tennis flying such physical process and movement rules, let physics to get closer to life, better serve to sports and better let athlete to understand and judge each kind of playing method, coaches’ teaching as well as comprehensive improvements. © 2014 Trade Science Inc. - INDIA

KEYWORDS
VB language; Mechanical model; Flight trajectory; Simulation; Sports teaching.

INTRODUCTION
Table tennis is a sports event that loved by many people, all the more so in China, national people greatly value table tennis, even honor table tennis as national ball. Ping-pong English official name is “table tennis”, it means as “tennis on the table”. Table tennis is originated in 1900; it is famous by giving out sound as “ping-pong” when playing. Up to now, people fondness for table tennis is still on the rise, table tennis is honored as Chinese “national ball”, it has widely mass foundation in our country, however table tennis is an event with certain difficulties, to most of table tennis enthusiasts, due to table tennis motion process is complex and various, for amateur players, it is hard to essentially have a comprehensive understanding on table tennis.

The paper starts from table tennis techniques’ mechanical perspective, for the ball that given initial speed rotational speed, it draws out flight trajectory by VB simulation, and carries out analysis, so that effective understand table tennis hitting process exertion technique and trajectory relations.

FORCE ANALYSIS AND MODELING
After releasing from athlete, table tennis mainly suffers three forces:
(1)Gravity \( G = mg \), in whole flight process, it always suffers gravity influence, gravity direction is vertical and downward;
(2)Air resistance \( F_R \), table tennis diameter is \( D \), rotational angular speed is \( \omega \), air viscosity coefficient is \( \eta \) (can refer to TABLE 1), air density is \( \rho \), table tennis forward translation speed is \( v \), table tennis suffered air resistance size is mainly up to Reynolds number \( R_e \) and translation speed \( v \); Reynolds number is measure of fluid inertia force and viscous force.
ratio, two geometric similar flow fields’ Reynolds numbers are equal, then corresponding micelle inertia force and viscous force ratio is equal. Reynolds number gets smaller, it means that viscous force impacts are more significant, while it gets bigger and then inertia force impacts are more significant. Reynolds number least flowing (as flowing in lubricant film), its viscosity impacts are throughout whole flow field. Reynolds number largest flowing (as general aircraft streaming around), its viscosity impacts are important only in object surface nearby boundary layer or trail.

\[ R_e = \frac{\rho D v}{\eta} \]

We take indoors temperature as 10°C, so viscosity coefficient \( \eta \) is \( 17.8 \times 10^{-6} \), \( \rho = 1.205 \text{ kg/m}^3 \). Therefore, we first calculate table tennis movement Reynolds number:

\[ R_e = \frac{\rho D v}{\eta} = \text{some results} \]

Resistance \( F_R \) size is in direct proportional to translation speed size squares: \( F_R = 2 \frac{A_v}{2} C_D A v^2 \)

### TABLE 1: Viscosity coefficient under different temperatures

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Viscosity coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>17.2 × 10⁻⁶</td>
</tr>
<tr>
<td>10</td>
<td>17.8 × 10⁻⁶</td>
</tr>
<tr>
<td>20</td>
<td>18.3 × 10⁻⁶</td>
</tr>
<tr>
<td>30</td>
<td>18.7 × 10⁻⁶</td>
</tr>
<tr>
<td>40</td>
<td>19.2 × 10⁻⁶</td>
</tr>
<tr>
<td>50</td>
<td>19.6 × 10⁻⁶</td>
</tr>
<tr>
<td>60</td>
<td>20.1 × 10⁻⁶</td>
</tr>
</tbody>
</table>

\( A \) is table tennis cross section area

\( C_D \) is resistance coefficient, according to different Reynolds number ranges, it has:

\[ C_D = \begin{cases} 
\frac{24}{R_e} & (R_e < 1) \\
\frac{24}{R_e} (1 + \frac{R_e^{\frac{1}{6}}}{6}) & (1 < R_e < 1000) \\
0.44(1000 < R_e < 2 \times 10^4)
\end{cases} \]

(3) Table tennis spinning generated Magnus force \( F_m \)

Magnus effect is a kind of viscosity effect; it is generated when spinning objects move in viscosity fluid, table tennis air movement process can be regarded as even distribution sphere movement process in fluid. Generally speaking, object movement in fluid will suffer lift force, resistance and lateral pressure effects, it is when table tennis rotates, circulation generates in its surrounding boundary layer, front incoming backflow and circulation common acting result is same direction of incoming flow and circulation one side flow speed quickens, and in the other side of reverse direction, flow speed slows down. According to Bernoulli principle, flow speed quicken one side pressure reduces, flow speed slowing down’s side pressure rises, two side pressure gap generated effect on table tennis that becomes Magnus force.

By Zhukovski circulation theory, it can solve Magnus force as: \( F_m = \frac{8}{3} \pi \rho D v \)

Let \( G = \frac{8}{3} \pi \rho D v \) = some results, and then: \( F_m = G \omega v \)

Concrete direction can be defined by right hand rule:

Right hand thumb, index finger and middle finger are in paired vertical, thumb points to rotational angular speed direction, index finger points to movement speed direction, and then middle finger direction is the direction of Magnus force.

(4) Air buoyancy \( F_U \), direction is vertical and upward, by dynamics, it easily gets: \( F_U = \frac{1}{6} \pi \rho g D^3 \)

### MODEL SOLUTIONS

Take table tennis bottom side as \( x \) axis, vertical table direction as \( y \) axis, bottom side central point as

![Figure 1: Table tennis table schematic diagram](image-url)
origin \( O \) to construct plane, through point \( O \), it makes \( xOy \) plane vertical line as \( z \) axis, establish space rectangular coordinate system, as Figure 1.

According to Newton’s Second Law, we can get table tennis force chart, as Figure 2.

\[
\begin{align*}
F_y &= F_m \cos \theta \sin \varphi + (\sin \alpha \sin \theta \cos \varphi + \cos \alpha \cos \varphi)F_M \\
F_y &= -F_z \sin \varphi + \frac{1}{6} \pi r D I - mg \\
F_y &= F_r \cos \theta \cos \varphi + F_m (\sin \alpha \sin \theta \sin \varphi - \cos \alpha \sin \varphi)
\end{align*}
\]

\[
\begin{align*}
\frac{dv_x}{dt} &= v_x \\
\frac{dv_y}{dt} &= v_y \\
\frac{dv_z}{dt} &= v_z
\end{align*}
\]

Among them, \( m \) is table tennis mass, \( v_x, v_y, v_z \) respectively represent table tennis movement speed components in \( x, y, z \) axis; \( \theta \) is track angle, is formed angle by table tennis speed direction and \( xz \) plane; \( \varphi \) is azimuth angle, is formed angle by table tennis speed projection in \( xz \) plane and \( z \) axis positive direction; \( \alpha \) is angle of heel, is formed angle by \( F_M \) force and its component in \( xOz \) plane (as Figure 2); \( x, y, z \) represent table tennis located space position; in above equation \( v_x, v_y, v_z, x, y, z, \theta, \alpha, \varphi \) are functions of time \( t \), therefore, after giving initial values, we can get and solve \( v_x, v_y, v_z \), and further solve \( x, y, z \), so that can define table tennis trajectory;

Therefore, by table tennis movement equation model, it needs to first define some parameters, these parameters include:

- When leave form racket or service devices, table tennis space position \( S(x, y, z) \);
- Translation motion initial speed \( v_0 \);
- Rotation speed \( \omega \);
- Initial moment track angle \( \theta_0 \), azimuth angle \( \varphi_0 \) and heel angle \( \alpha_0 \);

According to translation motion speed \( v_0 \) and \( \theta_0 \), \( \varphi_0 \) and \( \alpha_0 \) as well as others three angles, it can get table tennis components in three coordinate axis under initial state, parameter relations are as following:

\[
v_0 = \sqrt{v_{0x}^2 + v_{0y}^2 + v_{0z}^2} \\
\tan \varphi_0 = \pm \frac{v_{0y}}{v_{0x}} \\
\tan \theta_0 = \pm \frac{v_{0z}}{v_{0x}} \sin \varphi_0
\]

In motion process, corresponding parameters constantly updating also adopts the formula to obtain; air resistance \( F_R \) along \( x, y, z \) axis components are respectively:

\[
\begin{align*}
F_{R_x} &= \frac{1}{2} C_D \rho A v v_x \\
F_{R_y} &= \frac{1}{2} C_D \rho A v v_y \\
F_{R_z} &= \frac{1}{2} C_D \rho A v v_z
\end{align*}
\]

As Figure 3 set initial speed \( v_0 \) components along coordinate axis are respectively \( v_{0x}, v_{0y}, v_{0z} \), its included
angle with $xOy$ plane is $\theta$, its projection in $xOy$ and $x$ axis included angle is $\beta$. Ball rotational axis $w$ and $z$ axis included angle is $\phi$. As following Figure 4:

Rotation frequency vector $\vec{w} = (0, w\sin\phi, w\cos\phi)$, therefore it can get Magnus force components in coordinate axis are respectively:

$$F_{xw} = C_l\rho D^3\gamma(v_z\sin\phi - v_y\cos\phi);$$
$$F_{yw} = C_l\rho D^3\gamma(v_z\sin\phi);$$
$$F_{zw} = C_l\rho D^3v_y\sin\phi;$$

Therefore, it easily gets $x, y, z$ axis respectively resultant forces are

$$F_x = m\frac{dv_x}{dt} = C_l\rho D^3\gamma(v_z\sin\phi - v_y\cos\phi) - \frac{1}{2} C_d\rho Av v_x;$$
$$F_y = m\frac{dv_y}{dt} = C_l\rho D^3\gamma(v_z\sin\phi) - \frac{1}{2} C_d\rho Av v_y;$$
$$F_z = m\frac{dv_z}{dt} = -mg + \frac{1}{6} \pi \rho g D^2 - C_l\rho D^3\gamma(v_z\sin\phi) - \frac{1}{2} C_d\rho Av v_z;$$

Speed $v$ components along coordinate axis are:

$$\frac{dx}{dt} = v_x = v\cos\theta\cos\beta$$
$$\frac{dy}{dt} = v_y = v\cos\theta\sin\beta$$
$$\frac{dz}{dt} = v_z = v\sin\theta$$

**UTILIZE VB TO SOLVE MOTION EQUATION AND DRAW TRAJECTORY CURVE**

Set up parameter: Table tennis mass $m = 2.7 g$, diameter $D = 40mm$, air density $\rho = 1.205 \frac{kg}{m^3}$, lift force coefficient $C_l = \frac{1}{2}(2.022 + 0.981(\Sigma \omega))$, resistance coefficient $C_d = \frac{2.2}{m^{1.936}}$, resistance coefficient $C_d = \frac{2.2}{m^{1.936}}$. 

![Figure 4: Magnus force coordinate axis](image)

**Figure 6: Hanging high rotating arc ball motion trajectory chart**

**Figure 5: Ordinary shot motion trajectory chart**

**Figure 7: Forward Arcing balls motion trajectory chart**
Full Paper

Applied research on biomechanics-based table tennis flight trajectory simulation

FULL PAPER

BTAIJ, 10(6) 2014

F

ULL

P

APER

1458

BioTechnology

An Indian Journal

ci

t

= 0.508 + (1

22.053 + 4.196(\frac{\nu}{\nu_0})^2)^{\frac{1}{\gamma}}

sliding friction coefficient \( \mu = 0.15 \), restitution coefficient \( \epsilon = 0.8 \). Initial state: \( x \) direction initial position is \( 0m \), initial speed is \( 4m/s \), \( y \) initial position is \( 0.3m \), initial speed is \( 2.5m/s \), table tennis rotation speed \( \nu = 100\text{run/s} \). After that, it can input corresponding motion trajectory in VB. As Figure 5-7 by changing codes initial conditions, the paper implements different motion trajectories simulation, and outputs important moment timing, position, initial ball angle, which is very beneficial to athlete master table tennis motion rules.

CONCLUSION

By force analysis, the paper establishes table tennis motion trajectory model, simulation obtained trajectory compares to actual flight, it is well matching to actual trajectory, but table tennis such motion is broad and profound, the paper can only partially reflect problems, the paper only can use concrete analysis of the problem to let physics to get closer to life, and let physics to better apply into table tennis and serve to universities’ table tennis teaching.

ACKNOWLEDGMENT


Chuzhou University for teaching and research projects focus on: “Practice of Anhui Normal University Sports classroom teaching strategies.” Item Number: 2013jyz007.

REFERENCES


