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Analytic hierarchy process and dynamical simulation model-based table tennis diameter and appreciation value research

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ABSTRACT

The paper carries out analysis from athletes' experience qualities and audiences' appreciation qualities two aspects, firstly applies analytic hierarchy process, it gets each influence factors weight and athlete experience qualities and each influence factor relationship, it is clear that table tennis diameter increasing has improved athlete experience quality. After that, considering table tennis diameter changes to audience appreciation quality influences. Finally, on the condition that audience is satisfied, establish minimum table tennis falling time mathematical planning model, it gets ω that lets falling time to be minimum. And then in case athlete is satisfied, establish maximum table tennis falling kinetic energy mathematical planning model, it gets ω that lets falling kinetic energy to be maximum. In case that both audience and athlete are satisfied, establish falling time and kinetic energy comprehensive optimization control mathematical planning model, and get best $\omega = 119.757 r / s$, input obtained ω into table tennis dynamical simulation model, and solve best table tennis diameter is: $d = 39.634 mm$.

KEYWORDS

Analytic hierarchy process; Table tennis; Dynamics; Simulation model; Optimization.

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INTRODUCTION

In 2000, international table tennis federation increased international table tennis professional competition official ball diameter from 38mm to 40mm. The aim is to further increases table tennis air resistance during air running, slow down competition's ball running speed, so that achieve the purpose of further increasing and enriching table tennis professional athletes hitting techniques and skills, and finally increase table tennis competitions' overall appreciation. However, since incoming of table tennis "big ball era" up to now, ball diameters disputes never cease. Chinese and foreign coaches and athletes from all walks of life have mixed. It is worth noting that due to professional athletes' height, playing habit, gripping habit differences, their sensitivities to ball diameter changes are also different. Therefore, establish mathematical model to study on table tennis diameter and competition appreciation problem is quite urgent[1,2]. Xu Xiao-Dong (2013) summarized and stated big ball generated impacts and changes on table tennis athlete technical, tactical level, physical quality and table tennis apparatus as well as other aspects[3]. Zhao Qin (2000) carried out preliminary discussion on relative countermeasures about how to adapt to big ball[4]. Feng Ling, Bei Wei-Min, Cui Yuan-Kang (2004)studied on 40 mm table tennis impacts on cutting, and proceeded with preliminary discussion on relative countermeasure of how cutting adapt to big ball[5]. Almost all literatures were considering problems that started from athletes perspective, little people researched on audience appreciation quality, the paper analyzes from athletes' experience qualities and audiences' appreciation qualities two aspects, makes comprehensive consideration and research on table tennis best diameter.

AHP-BASED COMPREHENSIVE EVALUATION MODEL

Model establishment

The model mainly considers athlete experience quality main influence factors that are: technical difficulty, tactical thinking, forehand using times, backhand using times, receiving rate. In order to explain athlete experience quality and technical difficulty, tactical thinking, forehand using times, backhand using times, as well as receiving rate relations, apply analytic hierarchy process getting each influence factor weight[2] as well as experience quality and each influence factor formula, it further gets different table tennis diameters to athlete experience quality influence level.

(1) Hierarchical relations establishment

In order to consider athlete experience quality and technical difficulty, tactical thinking, forehand using times, backhand using times, and receiving rate influence degree. The paper considers to define athlete experience quality as first class indicator, and technical difficulty, tactical thinking, forehand using times, backhand using times[3,4], as well as receiving rate as second class indicators, establish athlete experience quality indicator system structure chart. Among them, A athlete experience quality, B_1 technical difficulty, B_2 tactical thinking, B_3 forehand using times, B_4 backhand using times, B_5 receiving rate.

(2) Judgment matrix and weight vector solution

TABLE 1 : Small ball criterion layer to target layer judgment matrix

A	B_1	B_2	B_3	B_4	B_5
B_1	1	2	4	3	9
B_2	1/2	1	2	4	3
B_3	1/4	1/2	1	2	4
B_4	1/3	1/4	1/2	1	2
B_5	1/9	1/3	1/4	1/2	1

According to analytic hierarchy process nine-scale method, it gets small ball criterion layer to target layer judgment matrix as TABLE 1 show.

It solves random consistency rate is $CR_1 = 0.0408 < 0.1$, therefore, criterion layer to target layer consistency test passes.

Then according to analytic hierarchy process nine-scale method, it gets big ball criterion layer to target layer judgment matrix is as TABLE 2 show.

TABLE 2 : Big ball criterion layer to target layer judgment matrix

A	B_2	B_1	B_3	B_4	B_5
B_2	1	3	5	6	2
B_1	1/3	1	3	5	6
B_3	1/5	1/3	1	3	5
B_4	1/6	1/5	1/3	1	3
B_5	1/2	1/6	1/5	1/3	1

It solves random consistency rate is $CR_1 = 0.0579 < 0.1$, therefore, criterion layer to target layer consistency test passes.

Set judgment matrix is a_{ij}, then every line element product:

$$\mathbf{M_i} = \prod_{j=1}^{m} \mathbf{a_{ij}} \tag{1}$$

$$\mathbf{a_i} = \sqrt[m]{\mathbf{M_i}}, \mathbf{i} = (1,2,3,4,\cdots,\mathbf{m}) \tag{2}$$

To vector $\alpha = (\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_m)^T$ it proceeds with normalization, and gets maximum feature value λ_m feature vector:

$$\mathbf{W} = (\mathbf{W_1}, \mathbf{W_2}, \mathbf{W_3}, \cdots, \mathbf{W_m})^{\mathbf{T}} \tag{3}$$

$$\lambda_{\mathbf{m}} = \frac{\mathbf{1}}{\mathbf{m}} \sum_{i=1}^{m} \frac{(\mathbf{AW})_i}{\mathbf{w_i}}, \mathbf{i} = (1,2,\cdots,\mathbf{m}) \tag{4}$$

$$\mathbf{C.I} = \frac{\lambda_{\mathbf{m}} - \mathbf{m}}{\mathbf{m} - 1}, \mathbf{CR} = \frac{\mathbf{C.I}}{\mathbf{RI}} \tag{5}$$

TABLE 3 is all factors to target relative important weight vectors; it calculates judgment matrix consistency indicator CI value. When random consistency ratio $CR < 0.1$, then it is thought that hierarchical single arrangement result has satisfaction consistency, otherwise it needs to adjust matrix elements values.

TABLE 3 : Random consistency indicator RI value

n	1	2	3	4	5	6	7	8	9	10	11
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51

(3) Model solution

To small ball, use MATLAB[5] solving: $\lambda_{\max} = 5.1829, \mathrm{CI} = 0.0457, RI = 0.0408$

Therefore it is thought that it meets consistency indicator, judgment matrix is feasible. Maximum feature values weight vector is $W = (0.4473, 0.2539, 0.1522, 0.0941, 0.0525)$. According to W, it is clear that technical difficulty has maximum influence on athlete experience quality.

To big ball, use MATLAB solving: $\lambda_{\max} = 5.2595, \mathrm{CI} = 0.0649, RI = 0.0579$

Therefore it is thought that it meets consistency indicator, judgment matrix is feasible. Maximum feature values weight vector is $W = (0.5061, 0.2602, 0.1321, 0.0669, 0.0348)$. According to W, it is clear that tactical thinking has maximum influence on athlete experience quality.

Define athlete experience quality as y, and respectively define technical difficulty, tactical thinking, forehand using times, backhand using times, receiving rate as x_1, x_2, x_3, x_4, x_5,it gets computing relationship is as following :

To small ball:

$$\mathbf{y = 0.4473x_2 + 0.2539x_1 + 0.1522x_3 + 0.0941x_4 + 0.0525x_5} \tag{6}$$

$$\mathbf{y = \frac{1}{19}(1 \times 0.4473 + 2 \times 0.2539 + 4 \times 0.1522 + 3 \times 0.0941 + 9 \times 0.0525) = 0.12203} \tag{7}$$

To big ball:

$$\mathbf{y = 0.5061x_1 + 0.2602x_2 + 0.1321x_3 + 0.0669x_4 + 0.0348x_5} \tag{8}$$

$$\mathbf{y = \frac{1}{17}(1 \times 0.5061 + 3 \times 0.2602 + 5 \times 0.1321 + 6 \times 0.0669 + 2 \times 0.0348) = 0.142247} \tag{9}$$

Because $y = 0.12203 < y = 0.142247$, therefore athlete experience quality improves after changing from small ball to big ball.

TABLE TENNIS DYNAMICAL SIMULATION MODEL

In small ball era, athlete tends to win by service; it greatly reduces mass appreciation quality. And in big ball era, table tennis ball speed and rotational speed are main factors affect competition appreciation, accordingly establish table tennis dynamical simulation model, by which it looks for big ball and small ball's ball speed and rotational speed changes status after diameter changing so as to verify its relation with audience appreciation quality. In the following, it respectively researches on audience appreciation quality influence from table tennis ball speed and table tennis rotational speed two aspects.

Table tennis diameter increasing to speed influence

(1) Model establishment

Rotational table tennis in running process mainly suffers gravity, buoyancy force, additional mass force, air resistance and Magnus force effects. Among them, gravity direction is opposite to buoyancy force, air resistance direction is opposite to table tennis movement direction.

Gravity F_g expression is as following:

$$\mathbf{F_g = mg = \frac{1}{6}\pi\rho_d d^3 g} \tag{10}$$

From which, ρ_d is table tennis density, d is table tennis diameter, buoyancy force F_b is equal to table tennis sphere arranged same volume air mass force, its computation formula is

$$\mathbf{F_b = m_a g = \frac{1}{6}\pi\rho_a d^3 g} \tag{11}$$

From them, ρ_a is air density.
For induction magnetic field generated additional mass force, its size is :

$$\mathbf{F_{m'} = \frac{1}{12}\pi\rho_a d^3 \frac{dv}{dt} = \frac{1}{2} m_a \frac{dv}{dt}} \tag{12}$$

$\frac{1}{2}m_a$ is always recording as m', is called additional mass. Additional mass force can be ignored when table tennis speed hasn't greatly changed.

Spherical object suffered resistance in fluid is equal to the spherical object radius, speed, fluid viscosity and 6π product. The law is called Stokes law. Suffered resistance is called Stokes force. Its F_S computing formula is:

$$\mathbf{F_S = 6\pi r v\mu = 3\pi d v\mu} \tag{13}$$

When a rotational object rotational angular speed vector and object flight speed vector don't overlap, it will produce a horizontal force in rotational angular speed vector and translational speed vector composed plane vertical direction. Under the horizontal force effects, object flight trajectory occurred deviation phenomenon is called Magnus effect. To Magnus effect, by scholars researching, main mechanisms have: 1) asymmetric displacement thickness, 2) asymmetric centrifugal force, 3) asymmetric wall friction stress, 4) asymmetric transition, 5) asymmetric separate and vortex, 6) asymmetric secondary flow. To sphere, Magnus force can be expressed by following formula:

$$\mathbf{F_M = \frac{1}{8}\pi\rho_a d^3 V\omega} \tag{14}$$

Among them, ω is rotational speed.

According to Newton the second law, it can establish rotational table tennis horizontal and vertical dynamics equations:
Horizontal:

$$\mathbf{(m + m')\frac{d^2x}{dt^2} = -3\pi\mu d\frac{dx}{dt} + \frac{1}{8}\pi\rho_a d^3 \frac{dy}{dt}\omega} \tag{15}$$

Vertical:

$$\mathbf{(m + m')\frac{d^2y}{dt^2} = -F_g + F_b - 3\pi\mu d\frac{dy}{dt} - \frac{1}{8}\pi\rho_a d^3 \frac{dx}{dt}\omega} \tag{16}$$

Among them, $F_g = mg, F_b = m_a g$, t is time.

(2) Model solution

Due to additional mass force can be neglected when table tennis speed changes are not big, to small ball and big ball, all can neglect m' .

To small ball:

Known that small ball horizontal direction accelerated speed $a_x = \frac{d^2x}{dt^2} = 14.5581m/s^2$, vertical direction accelerated speed $a_y = \frac{d^2y}{dt^2} = -50.6362m/s^2$. Big ball rotational speed $w = 116r/s$, big ball mass $m = 2.7g$, diameter $d = 40mm$, $\mu = 1.86 \times 10^{-5} Ns/m^3$, $\rho_a = 1.293kg/m^3$.

To big ball:

Known that big ball horizontal direction accelerated speed $a_x = \frac{d^2x}{dt^2} = 15.416m/s^2$, vertical direction accelerated speed $a_y = \frac{d^2y}{dt^2} = -44.254m/s^2$. Small ball rotational speed $w = 133.5r/s$, small ball mass $m = 2.5g$, diameter $d = 38mm$, $\mu = 1.86 \times 10^{-5} Ns/m^3$, $\rho_a = 1.293kg/m^3$.

Calculation adopted initial condition is as TABLE 4 show.

TABLE 4 : Model initial parameter

Parameter	Value
Big ball rotational speed	$116.5r/s$
Small ball rotational speed	$133.5r/s$
Big ball	$\mathrm{m = 2.7g, d = 40mm}$
Small ball	$\mathrm{m = 2.5g, d = 38mm}$
Environment	$\mathrm{T = 20\ ^\circ C, P = 1atm}$
Air density	$\rho_a = 1.293kg/m^3$
Viscosity	$\mu = 1.86 \times 10^{-5} Ns/m^3$

Input small ball known data into rotating table tennis horizontal direction and vertical direction dynamical equations, and get $v_x = \frac{dx}{dt} = 12m/s$, $v_y = \frac{dy}{dt} = 13m/s$ and $v = 17.6918m/s$

Input big ball known data into rotating table tennis horizontal direction and vertical direction dynamical equations, and get $v_x = \frac{dx}{dt} = 13m/s$, $v_y = \frac{dy}{dt} = 11m/s$ and $v = 17.0294m/s$.

(3) Table tennis diameter increasing influences on rotational speed

If athlete hits different sizes two balls with same way and equal size force, because big ball and small ball rotational inertia are different, then ball movement state changes will have obvious differences, two balls rotational inertia computing formulas are:

$$\mathbf{I_2 = 2/3m_2r_2^2 = 7.9312(g \cdot cm^2)} \tag{17}$$

$$\mathbf{I_1 = 2/3m_1R_2^2 = 6(g \cdot cm^2)} \tag{18}$$

Calculate according to small ball rotational speed is 50 turn/second, according to moment of momentum theorem, it can calculate big ball rotational speed ω_2 is:because: $M \cdot t = I_1\omega_1$ $M \cdot t = I_2\omega_2$ then: $I_1\omega_1 = I_2\omega_2$ $\omega_2 = I_1\omega_1 / I_2 = 40.59(r/s\ )$ two balls angular speed difference is:

$$\mathbf{\Delta\omega = \omega_1 - \omega_2 = 9.4112(r/s)} \tag{19}$$

By calculation, it is clear when hitting different sizes two balls with same way, big ball rotational speed reduces $9.4112(r/s)$ (near to1/5) to small ball rotational speed.

(4) Result analysis.

Big ball speed $v = 17.0294 m/s$ and small ball speed $v = 17.6918 m/s$, big ball speed is smaller than small ball speed. Comparing with using small ball, every round competition time will be extended, competition intense degree will be increased, and audience appreciation quality is greatly improved.

Small ball rotational speed is 50 turn/s, and big ball rotational speed is 40.59 turn/s, according to Bernoulli's theorem, high speed rotating ball's flight trajectory in the air is a curve not a straight line, curve crooked level is up to eccentric force leads to ball produced rotational speed and seed compound vector size and direction. Therefore, rotational speed reduction will affect ball assaulting, increase table tennis competition round numbers, and let audience appreciation quality improve.

Best diameter model establishment and solution

On the condition that athlete is satisfied, establish table tennis maximum falling kinetic energy mathematical planning model, and get maximum falling kinetic energy ω, input obtained ω into table tennis dynamical simulation model, and get maximum table tennis diameter. After that, on the condition that audience is satisfied, establish minimum table tennis falling time planning model, and get minimum falling time ω, input obtained ω into table tennis dynamical simulation model, and get minimum table tennis diameter. To get best table tennis diameter, it should consider conditions that both audience and athlete are satisfied, therefore establish falling time and kinetic energy comprehensive optimization control mathematical planning model, and get best ω, input it into table tennis dynamical simulation model, and get best table tennis diameter.

Establish as Figure 1 showed space coordinate system, table midpoint is coordinate origin o, from perspective of hitter, right hand direction is positive direction of x_1 axis, the forward is positive direction of x_2 axis, the upward is positive direction of x_3 axis.

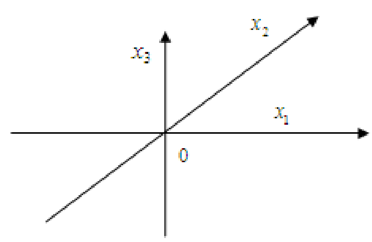

Figure 1: Space coordinate system

v_{20}, v_{30}, table tennis initial rotational speed value is: $\omega_{10}, \omega_{20}, \omega_{30}$.

Let: $x_{10}(t) = \omega_{10}$; $x_{11}(t) = \omega_{20}$; $x_{12}(t) = \omega_{30}$

Model establishment

$$\min f = t_f(v_{10}, v_{20}, v_{30}, \omega_{10}, \omega_{20}, \omega_{30}) \tag{20}$$

$$g_1 = v_{max} - \left(\sqrt{v_0^2} + \frac{2}{3}\frac{r^2\omega_0^2}{\sqrt{v_0^2}}\right) \geq 0 \tag{21}$$

$$g_2 = 1 - \frac{2}{3} r \sqrt{\frac{\omega_0^2}{\sqrt{v_0^2}}} \geq 0 \tag{22}$$

$$\mathbf{h_1 = v_0 \omega_0 = 0} \quad \mathbf{(23)}$$

In formula, $v_0 = (v_{10}, v_{20}, v_{30}), \omega_{10} = (\omega_{10}, \omega_{20}, \omega_{30})$, g_1 represents maximum swinging speed constraint, g_2 represents hitting point cannot be out of table tennis: h_1 represents orthogonal relation between table tennis initial mass center speed and angular speed.

Model solution

When table tennis initial position is: $\begin{matrix} x_1(0)=0, x_2(0)=-10, x_3(0)=-1, \\ x_4(0)=0, x_5(0)=0, x_6(0)=0, \end{matrix}$,且 $v_{\max} = 200$ 时,

It solves:

$$\mathbf{t_f = 0.09443} \quad \mathbf{(24)}$$

$$\mathbf{v_0 = \{0,158.444,29.355\}} \quad \mathbf{(25)}$$

$$\mathbf{\omega_0 = \{-738.984,0,0\}} \quad \mathbf{(26)}$$

$$\omega = \frac{738.984}{2\pi} = 117.613 r/s \quad \mathbf{(27)}$$

Input $\omega = \frac{738.984}{2\pi} = 117.613 r/s$ into table tennis dynamical simulation model, and can get maximum diameter is: $d = 39.8734mm$.

Table tennis maximum falling kinetic energy mathematical planning model:

(1) Model establishment

$$\min f = \frac{1}{T(t_f)}, \quad \mathbf{(28)}$$

$$g_1 = v_{\max} - \left(\sqrt{v_0^2} + \frac{2}{3}\frac{r^2\omega_0^2}{\sqrt{v_0^2}}\right) \ge 0 \quad \mathbf{(29)}$$

$$g_2 = 1 - \frac{2}{3} r \sqrt{\frac{\omega_0^2}{\sqrt{v_0^2}}} \ge 0 \quad \mathbf{(30)}$$

$$\mathbf{h_1 = v_0 \omega_0 = 0} \quad \mathbf{(31)}$$

(2) Model solution

When initial position coordinate is: $\begin{matrix} x_1(0)=0, x_2(0)=-10, x_3(0)=-1, \\ x_4(0)=0, x_5(0)=0, x_6(0)=0, \end{matrix}$,and $v_{\max} = 200$,it solves:

$$\mathbf{t_f = 0.15681s} \quad \mathbf{(32)}$$

$$\mathbf{v_0 = \{0,94.233,78.271\}} \quad \mathbf{(33)}$$

$$\omega_0 = \{-909.924,0,0\} \tag{34}$$

$$\omega = \frac{909.924}{2\pi} = 144.819 r/s \tag{35}$$

Input $\omega = \frac{909.924}{2\pi} = 144.819 r/s$ into table tennis dynamical simulation model, and can get maximum diameter is: $d = 37.2015mm$.
Falling time and kinetic energy comprehensive control mathematical planning model:

Model establishment

$$\min f = k\frac{t_f}{0.09443} + (1-k)\frac{46.653}{T(t_f)} \tag{36}$$

$$g_1 = v_{max} - (\sqrt{v_0^2} + \frac{2}{3}\frac{r^2\omega_0^2}{\sqrt{v_0^2}}) \geq 0 \tag{37}$$

$$g_2 = 1 - \frac{2}{3} r\sqrt{\frac{\omega_0^2}{\sqrt{v_0^2}}} \geq 0 \tag{38}$$

$$h_1 = v_0\omega_0 = 0 \tag{39}$$

Model solution

k value range is $0 \sim 1$, when $k = 1$, it is time optimal control, when $k = 0$ it is kinetic energy optimal control, during model calculating $k = 10\%$.

When initial coordinate position is: $\begin{matrix} x_1(0) = 0, x_2(0) = -10, x_3(0) = -1, \\ x_4(0) = 0, x_5(0) = 0, x_6(0) = 0, \end{matrix}$, and $v_{max} = 200$, it solves:

$$t_f = 0.11964s \tag{40}$$

$$v_0 = \{0,153.013,44.049\} \tag{41}$$

$$\omega_0 = \{-752.455,0,0\} \tag{42}$$

$$\omega = \frac{752.455}{2\pi} = 119.757 r/s \tag{43}$$

Input $\omega = \frac{752.455}{2\pi} = 119.757 r/s$ into table tennis dynamical simulation model, and can get best diameter is: $d = 39.634mm$.

CONCLUSION

Apply analytic hierarchy process analyzing athlete experience quality influences factors, decompose a complicated qualitative analysis problem into several factors that can make quantitative analysis and solve, obtained result is relative intuitional. Utilize table tennis dynamical simulation model more vividly combining ideal state mechanical research with practical mechanical trajectory, it describes table tennis mechanical state in specific running, obtained conclusion is correct and conforms to practice. Apply falling time and kinetic energy comprehensive optimization control mathematical planning model, combine table tennis speed with rotational speed, on the premise audience appreciation quality is ensured, let athlete experience quality arrive at maximum, and finally get table tennis best diameter. For table tennis dynamical simulation model, it can promote to research on loop flight trajectory and flight speed as well as different rotational speeds impacts on loop such problems.

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