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# Analysis of dance steps based on kinetic differential equations

**Zhanjun Yang**
**Arts Department, Xi'an Physical Education University, Xi'an 710068, (CHINA)**
**E-mail : 13476676325@qqom**

## ABSTRACT

For the "walking step" movement in square dance, this paper conducts kinetic study to explore the mechanical characteristics and the fitness value of the project. First the human body can be seen as system composed of the 17 rigid bodies; it uses the oval shape to present the human body's multi-rigid-body topology, builds systematic kinetic equations for the 17 rigid bodies, and finally conducts the simulation calculation during the walking step process. Through data simulation results, analyzing the kinetic parameters variation for various parts of the human body during "walking step" process provides theoretical discussion for the video learning of square dance. 

## KEYWORDS

Multi-rigid-body system;
Kinetic equation;
Lagrange multipliers;
Gear method.

## INTRODUCTION

In the square dance there is a movement called "walking step", that is the coordinated waking in the stepping dance process with music, the paper studies the kinetic of the movement. For the human multi-rigid-body model, many people have made efforts, and their efforts results have made a contribution on the biomechanical research and video simulation of Square Dance.

In this paper, it studies the walking step movement in Square Dance, establishes a multi-rigid-body model of the human body, then establishes a kinetic equations of the human multi-rigid-body system according rigid body movement, finally explores the kinetic characteristics of Square Dance through the simulation data during human walking step process, and studies the kinetic characteristics in the walking step process by the simulation data of walking step.

## KINETICS MODEL OF HUMAN BODY

According to anatomically principle, the human body is divided into a number of separate rigid bodies. Each rigid body has mass, moment of inertia, centroid and other physical properties of quantity and quality of the heart. The adjacent rigid bodies are connected by hinges. Applying the spring to the connection point, then muscles, ligaments and others are equivalent to dampers to simulate the effect of the soft tissue on the adjacent rigid bodies. Based on the above analysis, the human body can be simplified as a multi-rigid-body system with finite number of freedom degrees, the human receives the combined effects of external and internal forces in the motion process, the key in multi-rigid-body kinetic research is computing a set of constraining force to make the object' exercise be appropriate for a given constraint, and this paper uses R. L. Huston 17 rigid body model.

FULL PAPER

## Model assumption

1 Assuming that the human body is constituted by 17 rigid bodies, namely: head, neck, upper torso, lower torso, pelvis, right hand, left hand, right upper arm, left upper arm, right forearm, left forearm, right thigh, left thigh, right calf, left calf, left foot, right foot;
2 Assuming that the joint is kinematic pair;
3 Assuming that the body will not appear deformations during movement;
4 Assuming that the muscle force does not produce control action on the movement.

## The topology structure of human's multi-rigid-body model

The topology structure of human's multi-rigid-body model is shown in Figure 1:

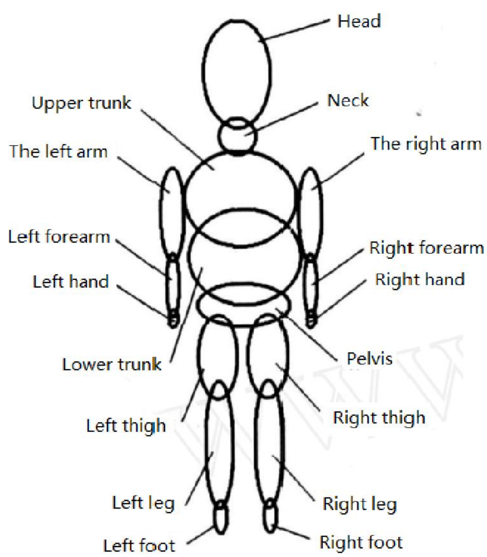


**Figure 1 : Multi-rigid-body model of the human body**

1 In Figure 1, the topological relations between various rigid bodies are as follows:
2 The neck and lower torso have flexible effect and therefore head and neck, neck and upper are connected by the spring, a total of three linear displacements and three angular displacements;
3 The lower torso and upper torso, lower torso and pelvis are connected by the spring, a total of three linear displacements and three angular displacements; Knee and ankle joint are simplified as revolute pair;
4 Waist, shoulder, elbow, wrist, hip and other joints are simplified as spherical pair.

To sum up: The freedom degree of the human body model is 17 × 6-4 × 5-9 × 3 = 55.

## Kinetic equation

In the multi-rigid-body model system, the centroid of any rigid body $\mathbf{B}_i(\mathbf{i}=1,2,3,\cdots,17)$ in the inertial coordinate system is $\mathbf{r}_i=(\mathbf{x}_i,\mathbf{y}_i,\mathbf{z}_i)^T$, the Euler angle of the relative inertial coordinate system is $\mathbf{p}_i=(\boldsymbol{\varphi}_i,\boldsymbol{\theta}_i,\boldsymbol{\phi}_i)^T$, the generalized coordinate is $\mathbf{q}_i=(\mathbf{r}_i^T,\mathbf{p}_i^T)^T$, the angular velocity $\omega_i$ of the rigid body $\mathbf{B}_i(\mathbf{i}=1,2,3,\cdots,17)$ is shown in formula (1) below:

$$\boldsymbol{\omega}_i=\mathbf{B}_i\dot{\mathbf{p}}_i \tag{1}$$

The $\mathbf{B}_i$ Formula (1) represents the coordinate transformation matrix between reference coordinate system and the inertial system of the rigid body's centroid is shown in formula (2) below:

$$\mathbf{B}_i=\begin{bmatrix}\sin\theta\sin\phi & 0 & \cos\theta\\ \cos\theta\sin\phi & 0 & -\sin\theta\\ \cos\phi & 1 & 0\end{bmatrix} \tag{2}$$

The matrix form of rigid $B_i$ is in formula (3) as below:

$$\mathbf{T}_i=\frac{1}{2}\dot{\mathbf{r}}_i^T\mathbf{m}_i\dot{\mathbf{r}}+\frac{1}{2}\boldsymbol{\omega}_i^T\mathbf{J}_i\boldsymbol{\omega}_i^T \tag{3}$$

In Formula (3) $\dot{\mathbf{r}}_i$ represents the centroid speed of the rigid body, $\vec{\mathbf{r}}_i$ represents the coordinate array in the inertial base, $\boldsymbol{\omega}_i$ represents the angular velocity of the rigid body, $\vec{\omega}_i$ represents the coordinate array in the connected system, $m_i$ represents the $3\times3$ mass diagonal matrix of the rigid body, $J_i$ represents the moments of inertia of the relative centroid for the rigid body, $\vec{J}_i$ represents the inertia matrix means in the connected system, by the formula (1) and formula (3) the Euler angles' expression form of kinetic energy can be drawn in the formula (4) below:

$$\mathbf{T}_i=\frac{1}{2}\dot{\mathbf{r}}_i^T\mathbf{m}_i\dot{\mathbf{r}}+\frac{1}{2}\dot{\mathbf{p}}_i^T\mathbf{B}_i^T\mathbf{J}_i\mathbf{B}_i\dot{\mathbf{p}} \tag{4}$$

Write the six multipliers' Lagrange equations corresponding to the six generalized coordinates for each rigid body, as shown in formula (5) below:

$$\frac{\mathbf{d}}{\mathbf{dt}}\begin{bmatrix}\left(\dfrac{\partial \mathbf{T_i}}{\partial \dot{\mathbf{r}}_\mathbf{i}}\right)^\mathbf{T} \\ \left(\dfrac{\partial \mathbf{T_i}}{\partial \dot{\mathbf{p}}_\mathbf{i}}\right)^\mathbf{T}\end{bmatrix} - \begin{bmatrix}\left(\dfrac{\partial \mathbf{T_i}}{\partial \dot{\mathbf{r}}_\mathbf{i}}\right)^\mathbf{T} \\ \left(\dfrac{\partial \mathbf{T_i}}{\partial \dot{\mathbf{p}}_\mathbf{i}}\right)^\mathbf{T}\end{bmatrix} + \begin{bmatrix}\phi_{\mathbf{ri}}^\mathbf{T}\lambda \\ \phi_{\mathbf{pi}}^\mathbf{T}\lambda\end{bmatrix} = \begin{bmatrix}\mathbf{Q_{ri}} \\ \mathbf{Q_{pi}}\end{bmatrix} \quad (5)$$

The Lagrange multiplier array $\lambda$ of Formula (5) is shown in formula (6) below:

$$\lambda = [\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_{6\times17}]^\mathbf{T} \quad (6)$$

The $\mathbf{Q_{ri}}, \mathbf{Q_{pi}}$ in Formula (5) corresponds to the generalized array of $\mathbf{r_i}, \mathbf{p_i}$, $\phi_{\mathbf{ri}}^\mathbf{T}, \phi_{\mathbf{pi}}^\mathbf{T}$ represents the partial derivative of the left parts of the constraint equations to the generalized coordinates.

Generalized angular momentum matrix is defined in formula (7) as below:

$$\mathbf{\Gamma_i} = \frac{\partial \mathbf{T_i}}{\partial \mathbf{p_i}} = \mathbf{B_i^T J_i B_i \dot{p}_i} \quad (7)$$

The formula (5) can be converted to formula (9) by the formula (8):

$$\begin{cases}\dfrac{\mathbf{d}}{\mathbf{dt}}\left[\left(\dfrac{\partial \mathbf{T_i}}{\partial \dot{\mathbf{r}}_\mathbf{i}}\right)^\mathbf{T}\right] = \mathbf{m_i \ddot{r}} \\ \left(\dfrac{\partial \mathbf{T_i}}{\partial \dot{\mathbf{r}}_\mathbf{i}}\right)^\mathbf{T} = \mathbf{0}\end{cases} \quad (8)$$

$$\begin{cases}\mathbf{m_i \ddot{r}} + \phi_{\mathbf{ri}}^\mathbf{T}\lambda = \mathbf{Q_{ri}} \\ \dot{\mathbf{\Gamma}}_\mathbf{i} - \dfrac{\partial \mathbf{T_i}}{\partial \mathbf{p_i}} + \phi_{\mathbf{pi}}^\mathbf{T}\lambda = \mathbf{Q_{pi}}\end{cases} \quad (9)$$

Through the seventeen formulas (9) produced by the seventeen rigid bodies, we can draw the unified matrix form of system kinetic equations as shown in formula (10) below:

$$\begin{cases}\mathbf{M\ddot{r}} + \phi_\mathbf{r}^\mathbf{T}\lambda - \mathbf{Q_r} = \mathbf{0} \\ \dot{\mathbf{\Gamma}} - \dfrac{\partial \mathbf{T}}{\partial \mathbf{p}} + \phi_\mathbf{p}^\mathbf{T}\lambda - \mathbf{Q_p} = \mathbf{0} \\ \mathbf{\Gamma} - \mathbf{B^T JB\dot{p}} = \mathbf{0}\end{cases} \quad (10)$$

To sum up: According to the kinetic equations and constraint equations we obtain complete and systematic algebraic-differential equations; for such equations we had better use order-variable and step-variable integration methods, namely Gear method, which applies both to solve the non-rigid differential equation, but also applies to analog rigid system.

## EMPIRICAL ANALYSIS

Taking the a dancer of square dance for example, we conduct simulation calculation during the walking process on the dancers' various parts' mass of the body, moment of inertia and, the constraint angle of

**TABLE 1 : The basic parameters of the human body model**

| Various segments | Quality | Moment of rotational inertia | | |
|---|---|---|---|---|
| | | $I_{xx}$ | $I_{yy}$ | $I_{zz}$ |
| Head | 4.22 | 301.65 | 317.60 | 191.41 |
| Neck | 1.30 | 25.94 | 25.94 | 18.51 |
| the upper arm | 1.50 | 118.00 | 121.80 | 15.82 |
| Forearm | 0.75 | 30.63 | 29.59 | 7.25 |
| Hand | 0.38 | 5.23 | 5.228 | 2.81 |
| pelvis | 8.25 | 1330.2 | 1183.80 | 389.46 |
| thigh | 8.16 | 866.67 | 883.66 | 142.38 |
| Calf | 2.33 | 175.35 | 179.12 | 19.71 |
| Foot | 0.87 | 89.40 | 100.41 | 22.88 |
| Upper torso section | 10.13 | 1200.01 | 695.17 | 1076.31 |
| Lower torso section | 6.97 | 1731.70 | 1584.20 | 798.66 |

**TABLE 2 : The constraint angle of lower limb joint**

| Joint name | Pelvis and thigh are connected | | | Knee joint | Ankle joint |
|---|---|---|---|---|---|
| Angle | $\theta_C$ | $\phi_C$ | $\psi_C$ | $\phi_K$ | $\psi_A$ |
| the axis | $Z_C$ | $X_C$ | $Y_C$ | $X_K$ | $X_A$ |
| Min | -30° | -120° | -40° | 0° | -30° |
| Max | 50° | 15° | 60° | 135° | 30° |

**TABLE 3 : The constraint angle of upper limb joint**

| Name | Pelvis and thigh are connected | | | Head and neck are connected | | | Wrist joint | | Neck and torso are connected | |
|---|---|---|---|---|---|---|---|---|---|---|
| Angle | $\theta_H$ | $\phi_H$ | $\psi_H$ | $\theta_N$ | $\phi_N$ | $\psi_N$ | $\phi_W$ | $\psi_W$ | $\phi_B$ | $\psi_B$ |
| the axis | $Z_H$ | $X_H$ | $Y_H$ | $Z_N$ | $X_N$ | $Y_N$ | $X_W$ | $Y_W$ | $X_B$ | $Y_B$ |
| Min | -40° | 100° | -90° | -90° | -50° | -30° | -70° | -10° | -30° | -60° |
| Max | 100° | 50° | 90° | 90° | 50° | 30° | 70° | 30° | 30° | 0° |

upper and lower limb joint. The calculation results include the basic parameters of the human body model, the constraint angle of lower limb joint, the constraint angle of upper limb joint and height variation of the supporting leg.

The animation key frame of dancers' walking a cycle is shown in Figure 2:

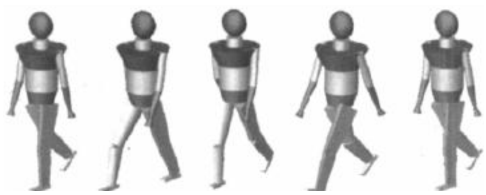

**Figure 2 : The animation key frame of walking a cycle**

The height variation of the dancer's supporting leg in Figure 2 in a cycle is shown in Figure 3:

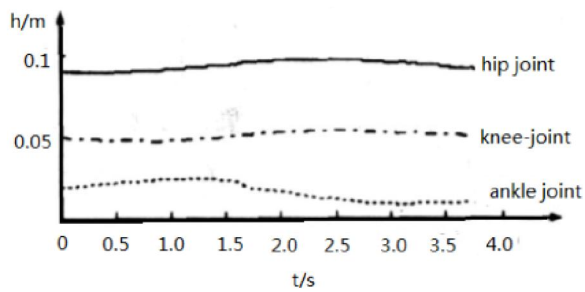


**Figure 3 : Height variation of the supporting leg**

## CONCLUSIONS

The actual data analysis shows that, in accordance with kinetic equations and constraint equations we obtain complete and systematic algebraic-differential equations use order-variable and step-variable integration methods, namely Gear method, which well explains the kinetic characteristics of the square dancers during walking process. This method applies both to solve the non-rigid differential equation, but also applies to analog rigid system; the simulation calculation shows that the quantity variation of various segments during the walking step process is reasonable. The established mathematical model can not only well explain the kinetic characteristics of Square Dance during the walking step process, but also can be applied to the studies of other human body kinetics.

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