

AN EVALUATION OF OPTICAL PARAMETERS OF HIGH-TEMPERATURE SUPERCONDUCTORS SANJU KUMARI^a, SOBHA RANI^b, S. KUMAR^c and LALIT K. MISHRA^{*}

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ABSTRACT

Frequency dependent scattering rate and effective mass of electron were evaluated for two high Tc-superconductors using extended Drude model at different temperatures. Our theoretically evaluated result for scattering rate is large and effective mass is small in comparison with experimental data for the given temperatures. However, the trend for both the scattering rate and effective mass are in agreement with the experimental data and also with other theoretical workers.

Key words: Frequency dependent scattering rate, Effective mass, High Tc-superconductor, Optical parameters, Dirty limit, K-K relation.

INTRODUCTION

The optical reflectivity measurement is a powerful probe for the study of the electronic structure of solids. It can provide much information on the conduction bands as well as valence bands of the crystals through inter band transition. It is a interesting problem for Bi-based cuprates. How the excitations within Bi_2-O_2 layer affect those within $Cu-O_2$ layer and how the Optical transitions differ from those of Y-Ba-Cu-O and La-Sr-Cu-O. The optical reflectivity spectra of single crystal Bi-based cuprates $Bi_2Sr_2CaCu_2O_{8+x}$ and $Bi_2Sr_2CuO_{6+x}$ were measured¹ in a wide energy range from 0.5 to 40 eV and analyzed through the Kramers-Kronig (KK) relation. The obtained spectra are different from other superconducting cuprates such as $(LaSr)_2CuO_4$ and YBa_2CuO_4 because of existence of characteristic BiO_2 layer. The optical excitation starts from 2 eV or higher energy so it is

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unlikely that the electrons in the BiO_2 layer contribute to low energy excitations or dc conduction in this family of high –Tc superconductors.

Recently², there are two works reported on the optical properties of high Tc superconductors HgBa₂CuO₄ (Hg-1201) (Tc = 97 K) and optimally doped Bi₂Sr₂CaCu₂O₈ (Tc = 88 K)³. In superconductor Hg-1201 in and out of plane optical spectra was presented. In plane normal incidence reflectivity measurement were performed on a Fourier transform spectrometer in the frequency range between 100-7000 cm⁻¹ (12-870 meV). Ellipsometric measurement were also made in the frequency range between 600 and 30,000 cm⁻¹ (0.75-3.72 eV). This measurement directly gives the real and imaginary parts of dielectric function $\varepsilon(\omega)$. Reflectivity was calculated from the pseudo dielectric function. In addition, the c-axis reflectivity $R_c(\omega)$ was measured on ac plane of different samples from 30 to 20,000 cm⁻¹. The c-axis optical conductivity was obtained from Kramers-Kronig variational analysis⁴.

In this paper, we report the evaluation of optical parameters namely the frequency dependent scattering rate $\frac{1}{\tau(\omega)}$ and effective mass $\frac{m^*(\omega)}{m_b}$ of Hg-1201 and Bi 2212. Using the extended Drude model⁵ and taking the contribution from inter band transition in the infra red region $\varepsilon_{\infty,IR}$, we have evaluated the above optical parameters.

Mathematical formulae used in the evaluation

We have used extended Drude-Lorentz model⁵ for the evaluation of frequency dependent optical parameters $\frac{1}{\tau(\omega)}$ and $\frac{m(\omega)}{m_b}$. We have also used a term $\mathcal{E}_{\infty,IR}$ which is a contribution to the dielectric function in the infrared region arising from inter-band transitions. This term has not been taken into account earlier in the single component approach. Drude theory was the theory of non-interacting electrons which assumes a frequency independent scattering rate [$\frac{1}{\tau}$ = constant] is given by –

$$\sigma(\omega) = \frac{1}{4\pi} \frac{\omega_p^2}{\frac{1}{\tau} - i\omega} \qquad \dots (1)$$

 ω_p is the bare plasma frequency of the free charge carriers. This assumption does not hold in the system where the charge carriers interact with bosonic spectrum or where strong correlations are important. In order to evaluate physical properties of high Tc superconductors Allen and Mikkelsen⁵ extended the Drude model by including a frequency dependent scattering rate -

$$\sigma(\omega,T) = \frac{1}{4\pi} \frac{\omega_p^2}{\frac{1}{\tau(\omega,T)} - i\omega \frac{m^*(\omega,T)}{m_b}} \qquad \dots (2)$$

m^{*} is the effective mass and m_b is the band mass. $\frac{1}{\tau(\omega,T)}$ and $\frac{m^*(\omega,T)}{m_b}$ obey Kramers –Kroing (KK) relations⁴. $\frac{1}{\tau(\omega,T)}$ and $\frac{m^*(\omega,T)}{m_b}$ are simply related to the real and imaginary part of $\frac{1}{\sigma(\omega)}$. One can also express these terms in terms of dielectric function $\varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega)$ where $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$ are the real and imaginary part of the dielectric function, we have

$$\frac{1}{\tau(\omega)} = \frac{\omega_p^2}{\omega} \frac{\varepsilon_2(\omega)}{\left[\varepsilon_{\infty,R} - \varepsilon_1(\omega)\right]^2 + \varepsilon_2^2(\omega)} \qquad \dots (3)$$

$$\frac{m^*(\omega)}{m_b} = \frac{\omega_p^2}{\omega} \frac{[\varepsilon_{\infty,IR} - \varepsilon_1(\omega)]}{[\varepsilon_{\infty,IR} - \varepsilon_1(\omega)]^2 + \varepsilon_2^2(\omega)} \qquad \dots (4)$$

Where $\mathcal{E}_{\infty,IR}$ is the contribution to the dielectric function in the infrared region arising from inter-band transition. The choice of $\mathcal{E}_{\infty,IR}$ is not so important at low energies where $|\mathcal{E}_1| \square \mathcal{E}_{\infty,IR}$ but becomes important at high energies. One also writes

$$\varepsilon_{\infty,IR} = \varepsilon_{\infty} + \Sigma_j S_j \qquad \dots (5a)$$

Where

$$S_j = \frac{\omega_{p,j}^2}{\omega_{0,j}^2} \qquad \dots (5b)$$

These are the oscillator strength of the inter-band transition obtained from Drude Lorentz fit. The contribution to the dielectric function from the polarizability of oxygen is calculated using the Clausis-Mossotti relation^{7,8}.

$$\varepsilon_{\infty,R} = 1 + \frac{4\pi N \frac{\alpha}{V}}{1 - \frac{4\pi}{3} N \frac{\alpha}{V}} = 1 + \frac{\alpha_0}{1 - \gamma \alpha_0} \qquad \dots (6)$$

Where N is the number of oxygen per unit cell. V is the unit volume and α is the polarizability of the oxygen atoms. $\alpha_0 = \frac{4\pi N\alpha}{V}$. For high Tc-superconductor HgBa₂CuO_{4+ δ} (Tc = 97 K) putting α = 3.88 × 10⁻⁸ cm³ and unit cell parameter for Hg -1201 of a × b × c = 3.85 × 3.85 × 9.5 Å and four oxygen atom per unit cell, we find $\varepsilon_{\infty,IR}$ = 3.56 and for superconductor Bi₂Sr₂CaCu₂O₈ (Bi 2212, Tc = 88K) $\varepsilon_{\infty,IR}$ = 4.5.

The other important quantity is the in-plane reflectivity at low temperature. For frequency $\omega \prec \frac{1}{\tau}$, one uses Hagen-Ruben approximation⁹ to describe the reflectance.

$$R(\omega) = 1 - 2(\frac{2\omega}{\pi\sigma_0})^{1/2} \qquad ...(7)$$

where σ_0 is the dc conductivity. $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$ are calculated from K-K relation¹⁰

$$\varepsilon_1(\omega) - 1 = \frac{2}{\pi} P \int_0^\infty \left(\frac{x \varepsilon_2(\omega)}{x^2 - \omega^2}\right) dx \qquad \dots (8)$$

and

$$\varepsilon_2(\omega) = -\frac{2}{\pi} p \int_0^\infty \frac{\varepsilon_1(x)}{(x^2 - \omega^2)} dx + \frac{4\pi\sigma_0}{\omega} \qquad \dots (9)$$

Equations (8) and (9) cannot be applied directly since both ε_1 and ε_2 depend on the unknown phase θ of the complex reflectivity.

$$r = \frac{1 - \sqrt{\varepsilon}}{1 + \sqrt{\varepsilon}} = \sqrt{R} \exp(i\theta) \qquad \dots (10)$$

 $R(\omega)$ is the normal-incidence reflectivity -

$$\ln r(\omega) = \ln \sqrt{R(\omega) + i\theta(\omega)} \qquad \dots (11)$$

$$\theta(\omega) = -\frac{2\omega}{\pi} P \int_{0}^{\infty} \frac{\ln\sqrt{R(x)}}{(x^2 - \omega^2)} dx + \theta(0) \qquad \dots (12)$$

These equations are used to compute $\theta(\omega)$ from $R(\omega)$. Now, one can restore dielectric function by inverting equation (10) -

$$\mathcal{E} = \frac{(1-r)^2}{(1+r)^2}$$
 ...(13)

Now, one can also calculate the real part of the dielectric function in the superconducting state ¹¹ by using the formulae

$$\varepsilon_1(\omega) = -\frac{\omega_{p_{ls}}^2}{\omega^2} \qquad \dots (14)$$

Where $\omega_{p,s}$ is in-plane super fluid plasma frequency¹². In case of superconductor HgBa₂CuO_{4+ δ} (Tc = 97 K). $\omega_{p,s}$ = 9600 ± 400 cm⁻¹ (1.2 ± 0.05 eV). In case of superconductor Bi₂Sr₂CaCu₂O₈ (Tc = 88 K) $\omega_{p,s}$ = 9500 cm⁻¹.

RESULTS AND DISCUSSION

We have evaluated frequency dependent scattering rate $\frac{1}{\tau(\omega)}$ using equation (3) at different temperature for two high Tc-superconductor HgBa₂CuO_{4+ δ} (Tc = 97 K) and $Bi_2Sr_2CaCu_2O_8$ (Tc = 88K). The results are shown in Table 1 and 2 along with the experimental data^{2,3,13}. Our theoretical results for HgBa₂CuO_{4+ δ} indicate that the values of $\frac{1}{\tau(\omega)}$ is large in comparison to experimental data for all temperatures 250 K, 200 K, 100 K and 50 K between $\omega = 100$ to 4000 cm⁻¹. On the other hand the values for Bi₂Sr₂CaCu₂O₈ is lower against the experimental data. In the experimental analysis of the $\frac{1}{\tau(\omega)}$ data¹³, it was mentioned that scattering rate is strongly suppressed for temperatures below Tc which is indicative of the opening of the gap. ARPES¹⁴ measurements on Hg-1201 show a maximum gap value, there is some uncertainty in this value because no quasi-particle peak is observed around the anti nodal direction¹². From the optical measurements, it is difficult to extract the gap, s-wave BCS superconductor in the dirty limit show an onset in the absorption associated with the superconducting gap at 2 Δ . It was observed that the onset seen in cuprates is shifted due to interaction of the electrons with the magnetic resonance¹⁵. Due to this, one feels that the evaluated results of $\frac{1}{\tau(\omega)}$ do not match with the magnitude of the experimental data. However the trend that $\frac{1}{\tau(\omega)}$ increases with ω for a given temperature has been noticed in our calculation for both superconductors Hg-1201 and Bi-2212. We have also evaluated frequency dependent effective mass $\frac{m^*(\omega)}{m_b}$ at different temperatures for both superconductors using equation (4). The results are shown in Table 4 and 5. Our theoretically evaluated results show that up to $\omega = 600 \text{ cm}^{-1}$, $\frac{m^*(\omega)}{m_b}$ increases with ω and after that it decreases with $\omega = 4000 \text{ cm}^{-1}$. In case of superconductor Hg-1201 the magnitude of $\frac{m^*(\omega)}{m_b}$ is lower than the experimental data² and for superconductor Bi-2212 the evaluated results are larger with experimental data¹³. However the trend is in agreement with the experimental data^{2,13}. The argument of this mismatch is the same as we have mentioned above. It has been pointed out that at onset of superconducting gap, K-K relations⁴ gives maximum value $\frac{m^*(\omega)}{m_b}$. There is a quite uncertainty in the experimental data. In another work J. J. McGuire¹⁶ studied the infra-red dissipation in the ab-plane scattering rate. They observed two separate effects. At high temperature there is a broad dispersion of scattering rate below 1000 cm⁻¹ and at low temperature a sharp structure is seen which is associated with the scattering from a mode at 300 cm⁻¹ in under doped high Tc-superconductor Bi₂Sr₂CaCu₂O₈. The various parameters used in the evaluation are shown in Table 1.

Table 1

 $\varepsilon_{\infty} = 2.53$, $\varepsilon_{\infty,IR} = 3.56$ (HgBa₂CuO_{4+ δ}, Tc = 97K) $\varepsilon_{\infty,IR} = 4.5$ (Bi₂Sr₂CaCu₂O₈, Tc = 88K)

 $\omega_{p,s}$ (in plane super fluid plasma frequency) = 9600 ± 400 cm⁻¹ (1.2 ± 0.065 eV) HgBa₂CuO₄₊₆. Tc = 97K) $\omega_{p,s}$ = 9500 cm⁻¹ (Bi 2212) (Tc = 88K).

Table 2: An evaluated result of frequency dependent scattering rate $[\tau(\omega)]^{-1}$ at different temperature for superconductor HgBa₂CuO_{4+ δ}

Wave number	$[\tau(\omega)]^{-1} \ge 10^3 \text{ cm}^{-1}$								
	T = 250 K		200 K		100 K		50 K		
cm ⁻¹	Theory	Expt.	Theory	Expt.	Theory	Expt.	Theory	Expt.	
100	0.524	0.439	0.505	0.405	0.453	0.398	0.422	0.377	
200	0.656	0.563	0.624	0.526	0.505	0.456	0.486	0.432	
400	0.738	0.678	0.708	0.682	0.654	0.603	0.584	0.529	
600	0.849	0.775	0.822	0.713	0.722	0.698	0.653	0.640	
800	0.967	0.859	0.943	0.886	0.896	0.754	0.732	0.695	

Cont...

Wave number cm ⁻¹ 1000 1500 2000 3000 4000		$[\tau(\omega)]^{-1} \ge 10^3 \text{ cm}^{-1}$								
	T = 2	T = 250 K		200 K		100 K		50 K		
cm ⁻¹	Theory	Expt.	Theory	Expt.	Theory	Expt.	Theory	Expt.		
1000	1.128	1.098	1.106	0.986	0.954	0.863	0.853	0.782		
1500	2.074	1.863	1.963	1.055	1.133	0.955	0.936	0.845		
2000	2.365	2.059	2.124	1.354	1.586	1.124	1.139	0.957		
3000	2.967	2.353	2.759	1.798	2.054	1.863	1.386	1.116		
4000	3.547	2.958	3.128	2.154	2.846	2.058	1.754	1.458		

Table 3: An evaluated result of frequency dependent scattering rate $[\tau(\omega)]^{-1}$ at different temperature for superconductor Bi₂Sr₂CaCu₂O₈ (2212) Tc = 88 K

Wave	$[\tau(\omega)]^{-1} \ge 10^3 \text{ cm}^{-1}$								
number	T = 300 K		200 K		150	150 K		50 K	
cm ⁻¹	Theory	Expt.	Theory	Expt.	Theory	Expt.	50 Theory 0.055 0.074 0.083 0.094 0.106 0.118 0.122 0.144 0.158 0.167	Expt.	
100	0.183	0.204	0.154	0.162	0.098	0.107	0.055	0.067	
200	0.196	0.223	0.167	0.174	0.115	0.112	0.074	0.088	
400	0.206	0.236	0.188	0.193	0.128	0.137	0.083	0.097	
500	0.217	0.247	0.195	0.205	0.139	0.144	0.094	0.105	
600	0.222	0.255	0.204	0.214	0.146	0.152	0.106	0.112	
800	0.238	0.263	0.215	0.222	0.155	0.167	0.118	0.126	
1000	0.246	0.270	0.224	0.237	0.167	0.178	0.122	0.134	
1500	0.287	0.292	0.249	0.256	0.199	0.207	0.144	0.155	
1700	0.305	0.316	0.256	0.267	0.215	0.226	0.158	0.166	
2000	0.346	0.352	0.264	0.277	0.224	0.237	0.167	0.174	

Wave	$\frac{m^*(\omega)}{m_b}$							
number cm ⁻¹	T = 2	50 K	200	K	100 K			
•	Theory	Expt.	Theory	Expt.	Theory	Expt.		
100	3.20	3.52	2.53	2.66	2.46	2.54		
200	3.86	3.97	2.95	3.03	2.58	2.68		
400	4.52	4.72	3.84	3.64	2.67	2.77		
600	5.50	5.80	4.50	4.72	2.86	2.94		
800	4.30	4.54	4.28	4.34	2.48	2.53		
1000	3.75	3.86	3.86	3.92	2.34	2.39		
1500	3.22	3.57	3.67	3.84	2.21	2.26		
2000	3.15	3.32	3.48	3.56	2.16	2.20		
2500	2.86	2.97	3.22	3.37	2.10	2.15		
3000	2.73	2.86	2.89	2.92	2.08	2.10		
3500	2.67	2.74	2.75	2.84	2.05	2.09		
4000	2.58	2.64	2.65	2.70	2.02	2.04		

Table 4: An evaluated result of frequency dependent effective mass $\frac{m^*(\omega)}{m_b}$ at different temperature for superconductor HgBa₂CuO_{4+ô} (Tc = 97 K)

Table 5: An evaluated result of frequency dependent effective mass $\frac{m^*(\omega)}{m_b}$ at different temperature for superconductor Bi₂Sr₂CaCu₂O₈ (2212) Tc = 88 K

Wave	$\frac{\underline{m^{*}(\omega)}}{\underline{m_{b}}}$								
number cm ⁻¹	T = 300 K		200	K	150 K				
CIII	Theory	Expt.	Theory	Expt.	Theory	Expt.			
100	2.12	2.08	3.46	3.55	3.68	3.34			
200	2.34	2.14	3.37	3.48	3.60	3.27			
400	2.67	2.18	3.30	3.42	3.52	3.15			

						Cont
Wave			m [*] (m	$\frac{(\boldsymbol{\omega})}{b}$		
number cm ⁻¹	T = 300 K		200 K		150 K	
CIII	Theory	Expt.	Theory	Expt.	Theory	Expt.
500	2.86	2.23	3.22	3.36	3.47	3.09
600	2.92	2.29	3.15	3.22	3.33	3.00
800	2.94	2.34	2.92	3.05	3.17	2.97
1000	2.99	2.38	2.75	2.95	3.09	2.84
1200	3.04	2.42	2.64	2.86	2.95	2.73
1500	3.07	2.56	2.53	2.74	2.87	2.65
1700	3.09	2.67	2.47	2.58	2.72	2.54
2000	3.12	2.75	2.38	2.49	2.63	2.44
3500	3.15	2.79	2.45	2.86	2.24	2.12
4000	3.23	2.98	2.54	2.36	2.44	2.29

REFERENCE

- 1. I. Terasaki, S. Tajima, H. Eisaki, H. Takagi, K. Uchinokura and S. Uchida, Phys. Rev., **B41**, 865 (1990).
- E. Van Heumen, R. Lorentz, A. B. Kuzmenko, F. Carbone, D. Vander Marel, X. Zhao, G. Yu, Y. Cho, N. Barisie, M. Greven, C. C. Homes and S. V. Dordevic Phys. Rev., B75, 054522 (2005).
- 3. A. B. Kuzmenko, H. J. A. Molegraaf, F. Carbone and D. Vander Marel, Phys. Rev., **B75**,144503 (2005).
- 4. A. B. Kuzmenko, Rev. Sci. Instrum., 76, 083108 (2005).
- 5. J. W. Allen and J. C. Mikkelsen, Phys. Rev. B15, 2952 91977).
- 6. D. L. Cox and N. Grewe, Z. Phys., **B71**, 321 (1988).
- A. El Azarak, R. Nahoum, N. Bontemps, M. Guilloux-Viry and H. Raffy, Phys. Rev., B49, 9846 (1994).

- 8. D. Vander Marel, H. J. A. Molegraaf, J. Zaanen, F. Carbone, A. Damascelli, H. Eisaki, M. Greven, P. H. Kes and M. Lie, Nature (London), **425**, 271 (2003).
- 9. F. Wooten, Optical Properties of Solids, Academic Press, New York (1972).
- 10. A. B. Kuzmenko, E. A. Tishchenko and A. S. Krechetov, Opt. Spectrosc., 84, 402 (1998).
- 11. C. C. Homes, S. V. Dordevic, T. Valla and M. Strongin, Phys. Rev., **B75**, 134517 (2005).
- 12. F. Carborne, A. B. Kuzmenko, H. J. A. Molegraaf and E. Giannini, Phys Rev., **B76**, 024502 (2006).
- 13. T. Timusk, Solid State Commun., **127**, 337 (2003).
- 14. Y. Ohash and A. Griffen, Phys. Rev. Lett. (PRL), 92, 110401 (2004).
- 15. J. Hwang, T. Timusk and G. D. Gu, Nature (London), 427, 714 (2004).
- 16. J. J. McGuire, M. Windt, T. Timusk and V. Viallet, Phys. Rev., B62, 8711 (2000).

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